

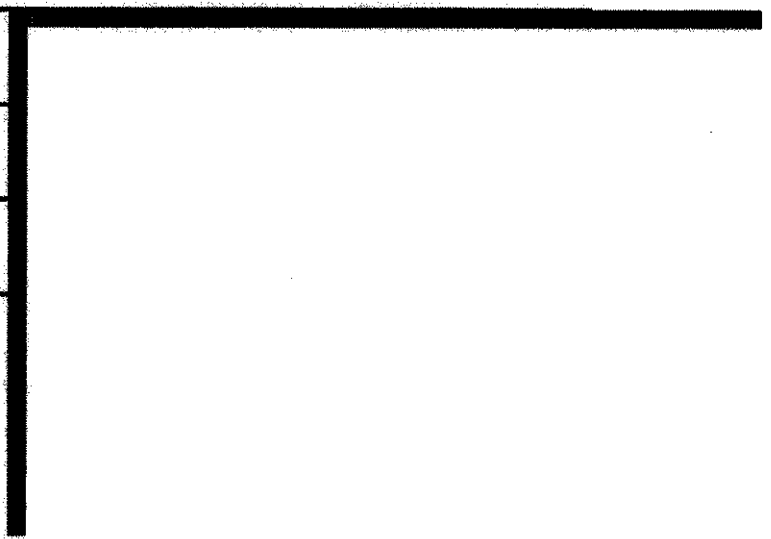
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} +400.5 \\ +300.5 \end{pmatrix} + \begin{pmatrix} \cos 90 & \sin 90 \\ -\sin 90 & \cos 90 \end{pmatrix} \begin{pmatrix} z \\ s \end{pmatrix}$$

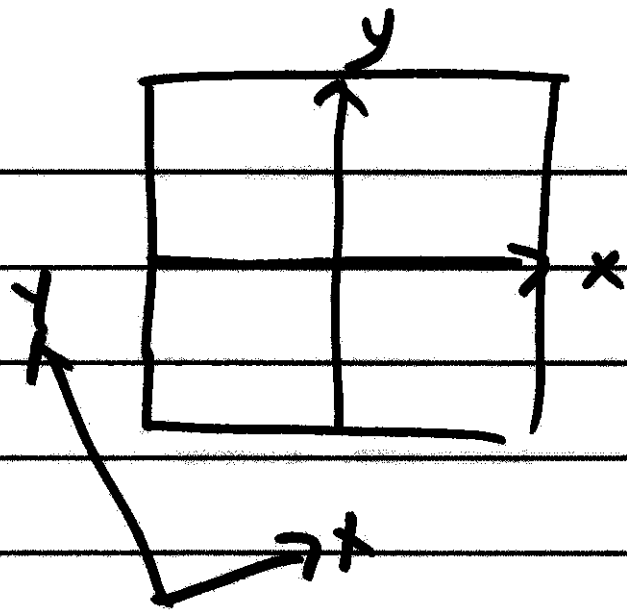
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -400.5 \\ 300.5 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} z \\ s \end{pmatrix}$$

Nominal
Principal
Point
System



Pixel
Coordinates





$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{estimator} \quad 9-2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \end{pmatrix} + \begin{pmatrix} \quad \\ \quad \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{application}$$

$$x = t + mX$$

$$x - t = mX$$

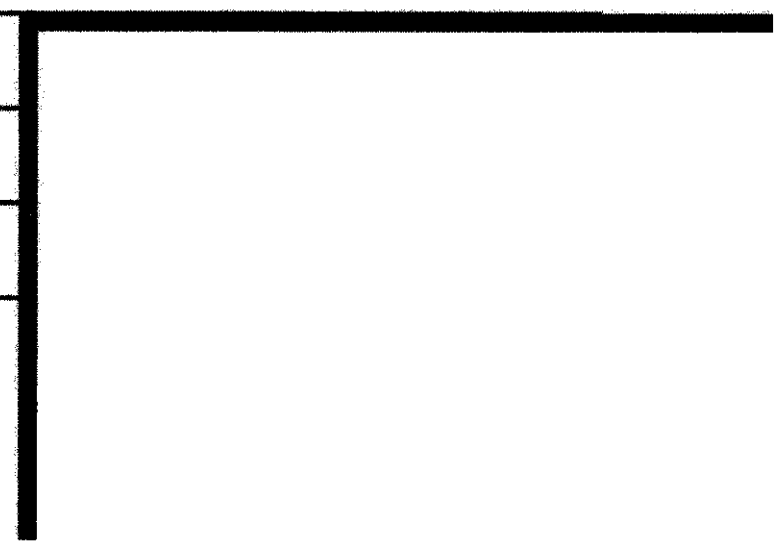
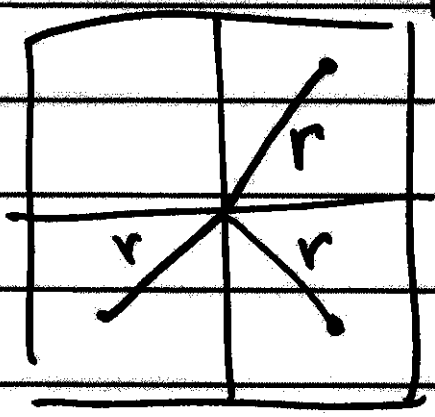
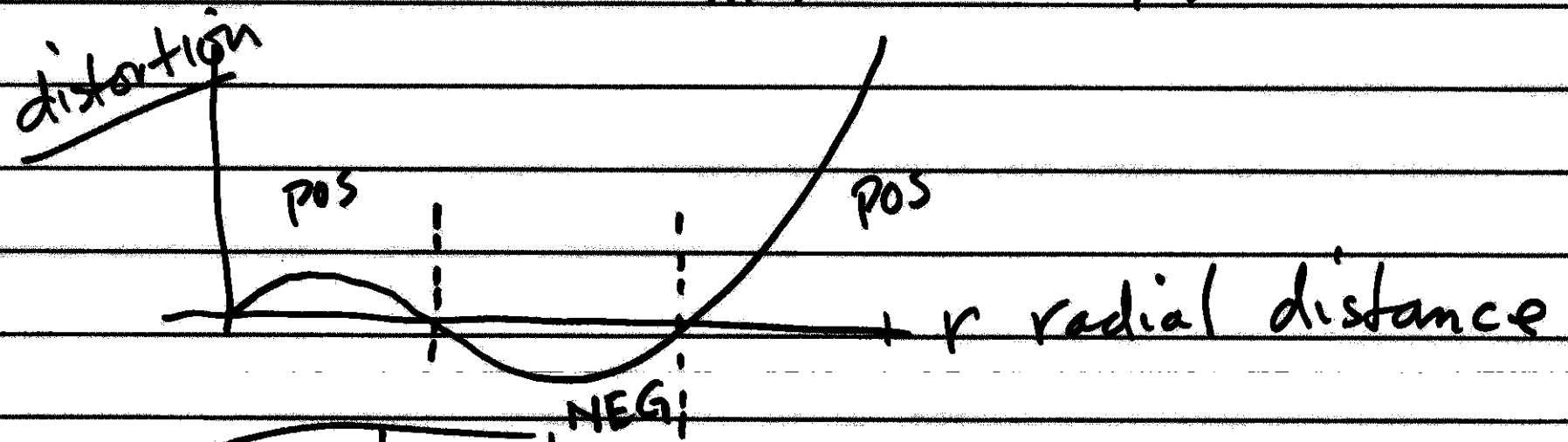
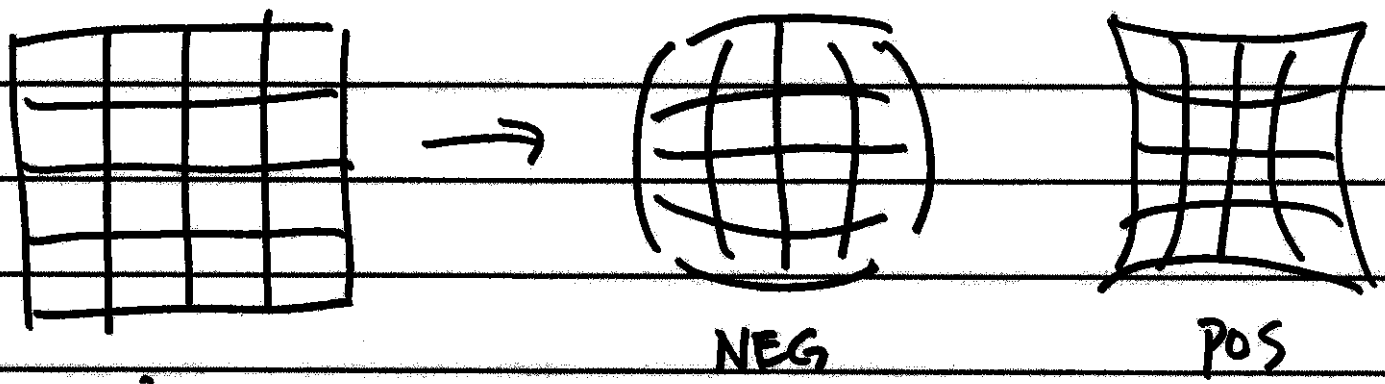
$$m^{-1}(x - t) = X$$

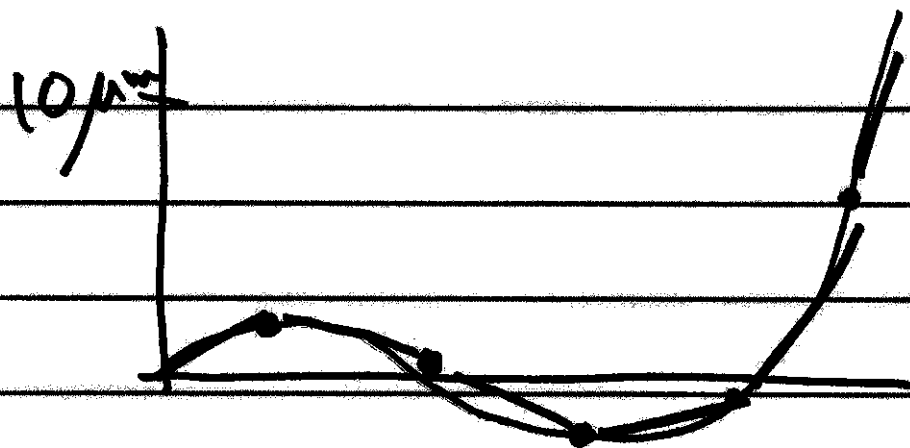
Meas. \leftarrow Cal.
(estimation)

Cal. \leftarrow Meas.

(application)

$$X = \boxed{-m^{-1}t} + \boxed{m^{-1}}x$$



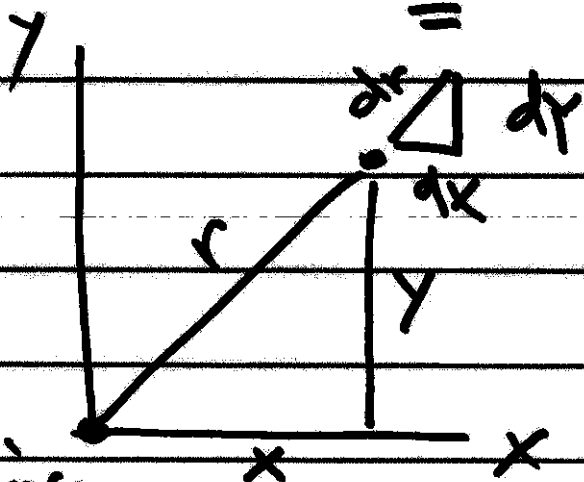


to get dr : 9-4

1. compute r
 evaluate $k_1 r^3 + k_2 r^5 \dots$

2. linear interpolation

$$dr = \underbrace{k_1}_{=} r^3 + \underbrace{k_2}_{=} r^5 + \underbrace{k_3}_{=} r^7 \dots$$



$$\frac{dr}{r} = \frac{dx}{x} = \frac{dy}{y}$$

Princ.
Pt.

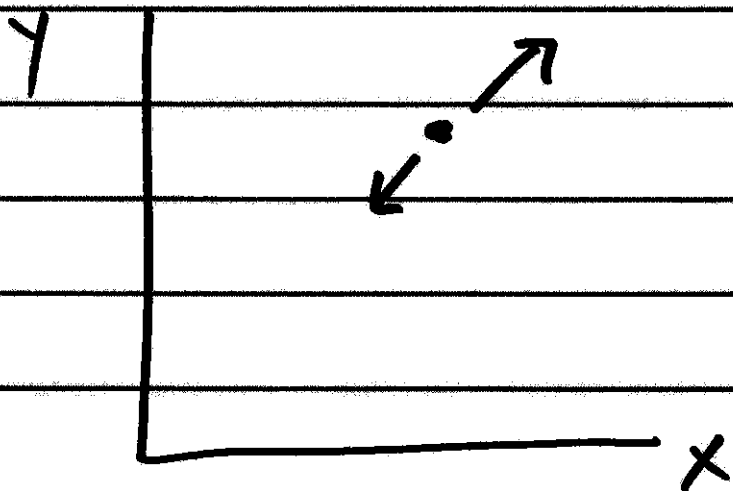
$$dx = x \frac{dr}{r}, \quad dy = y \cdot \frac{dr}{r}$$

9-5

$$dx = \frac{x(k_1 r^3 + k_2 r^5 + k_3 r^7)}{r}$$

$$dx = x(k_1 r^2 + k_2 r^4 + k_3 r^6)$$

$$dy = y(\quad \quad \quad)$$



$$X_c = X - dx_{\text{dist}}$$

$$X_c = X + dx_{\text{corr}} \quad 9-6$$

OR

$$Y_c = Y - dy_{\text{dist}}$$

$$Y_c = Y + dy_{\text{corr}}$$

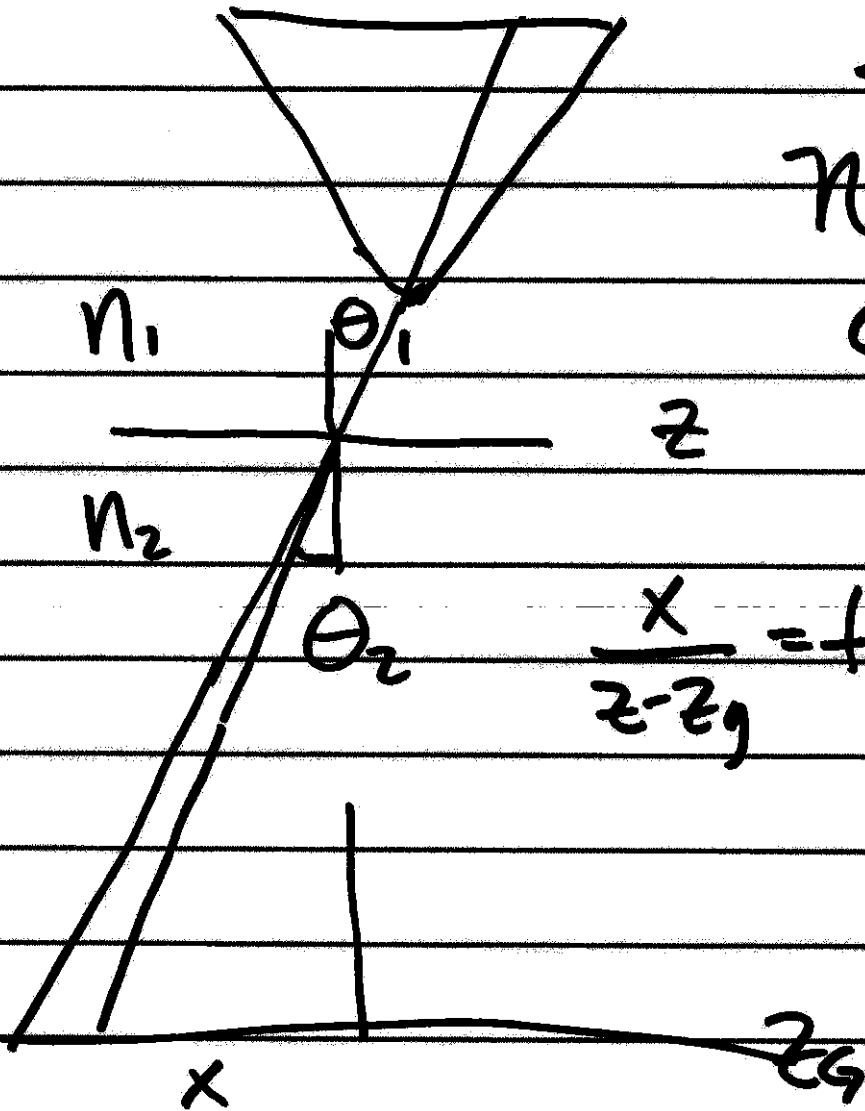
atmospheric refraction

9-7

ρ density Schut
PERS

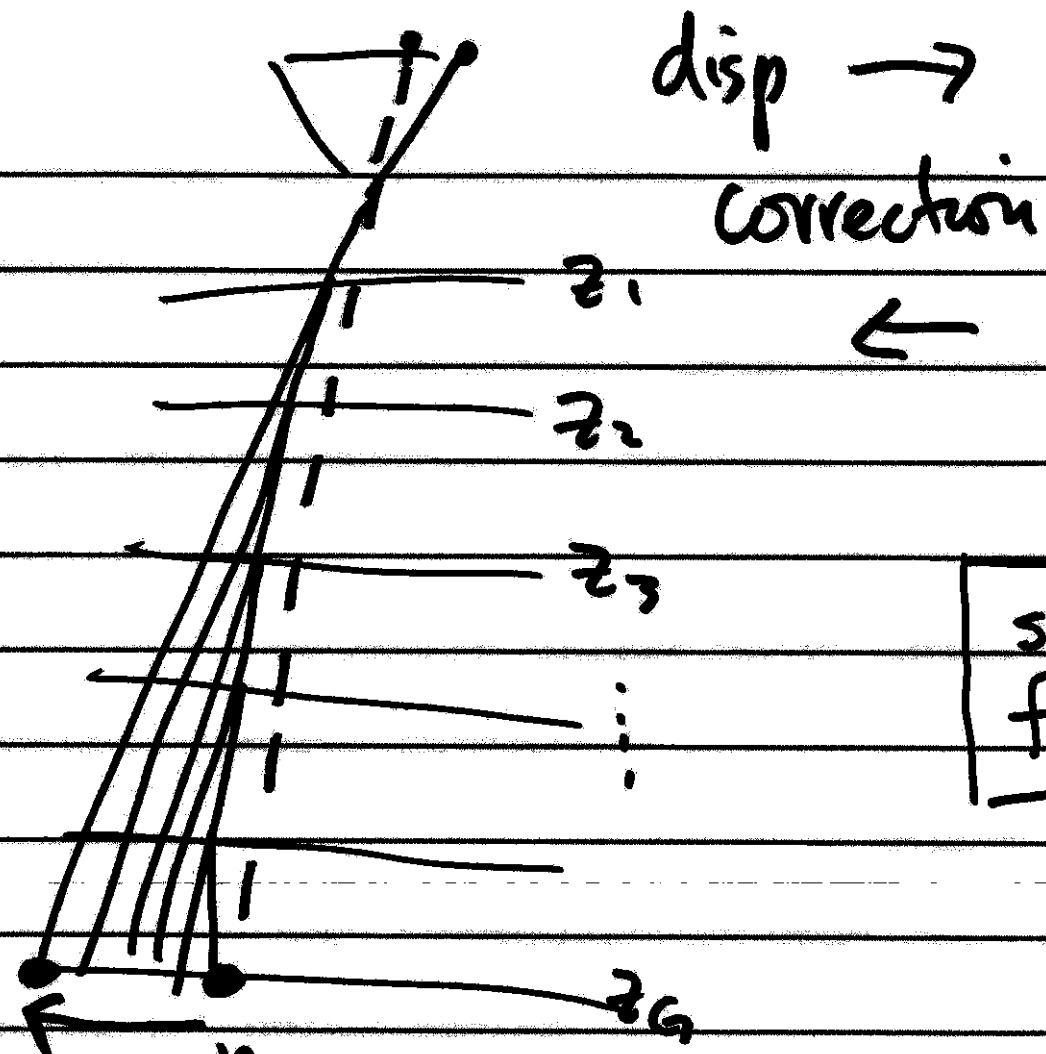
$$n^2 = 1 + 2c\rho$$

$$c = .0002261$$



$$\frac{x}{z-z_g} = \tan \theta_1$$

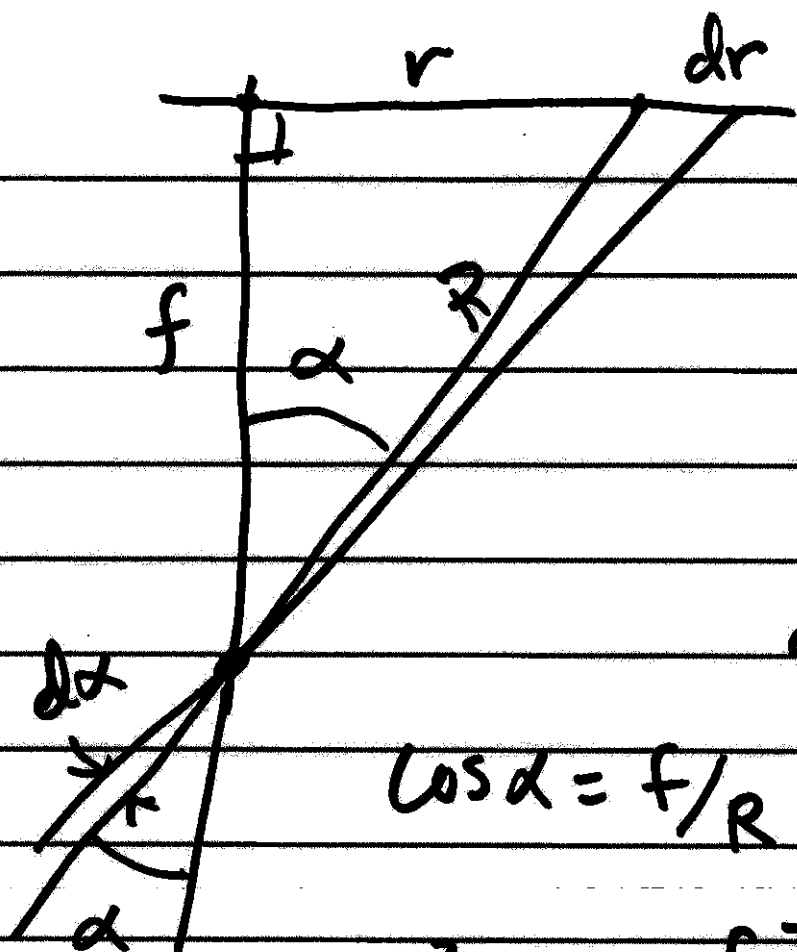
$$dx_1 = (z-z_g) \tan \theta_1 - (z-z_g) \tan \theta_2$$



see slightly revised formula below

$$dX = \sum_{i=1}^n (z_i - z) [\tan \theta_i - \tan \theta_{i-1}]$$

$$dX = \sum_{i=1}^n (z_i - z_G) [\tan \theta_i - \tan \theta_{i+1}]$$



$$\frac{r}{f} = \tan \alpha$$

$$r = f \cdot \tan \alpha$$

$$\frac{dr}{d\alpha} = f \cdot \frac{1}{\cos^2 \alpha}$$

$$dr = d\alpha \cdot f \cdot \frac{1}{\cos^2 \alpha}$$

$$\cos \alpha = \frac{f}{R} = \frac{f}{\sqrt{f^2 + r^2}}$$

$$\cos^2 \alpha = \frac{f^2}{f^2 + r^2}$$

$$\frac{1}{\cos^2 \alpha} = \frac{f^2 + r^2}{f^2} = \boxed{1 + \frac{r^2}{f^2}}$$

9-10

$$dr = d\alpha \cdot f \cdot \left(1 + \frac{r^2}{f^2}\right)$$

$$d\alpha = \underbrace{K}_{=} \cdot \tan \alpha \quad \tan \alpha = \frac{r}{f}, \quad K \cdot f$$

$$K = \left(\frac{2410 H}{H^2 - 6H + 250} - \frac{2410 h}{h^2 - 6h + 250} \left(\frac{h}{H}\right) \right) \times 10^{-6}$$

H flying height platform

h elevation of point

(K_m)

9-11

$$dr = k \left(r + \frac{r^3}{f^2} \right)$$

$$\frac{dx}{x} = \frac{dr}{r}$$

$$dx = x \cdot \frac{dr}{r}$$

make sure it's negative

$$x_c = x - dx_{\text{dist}} + dx_{\text{AR}}$$

$$y_c = y - dy_{\text{dist}} + dy_{\text{AR}}$$

$$x = -f \frac{m_{11}(x-x_c) + m_{12}(y-y_c) + m_{13}(z-z_c)}{m_{31}(x-x_c) + m_{32}(y-y_c) + m_{33}(z-z_c)} + x_0 \quad 9-12$$

$$y = -f \frac{m_{21}(x-x_c) + m_{22}(y-y_c) + m_{23}(z-z_c)}{\quad \quad \quad} + y_0$$

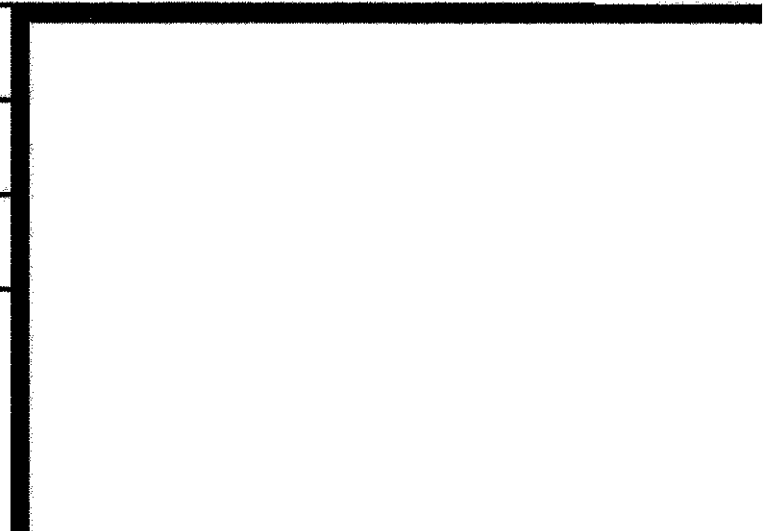
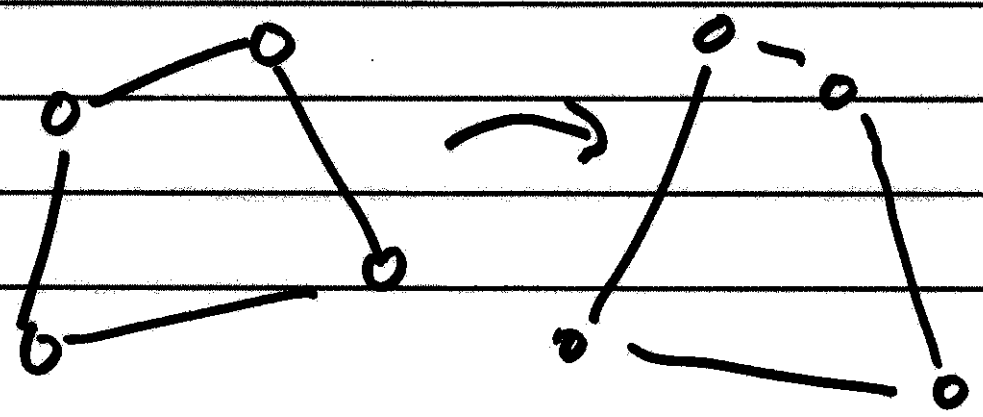
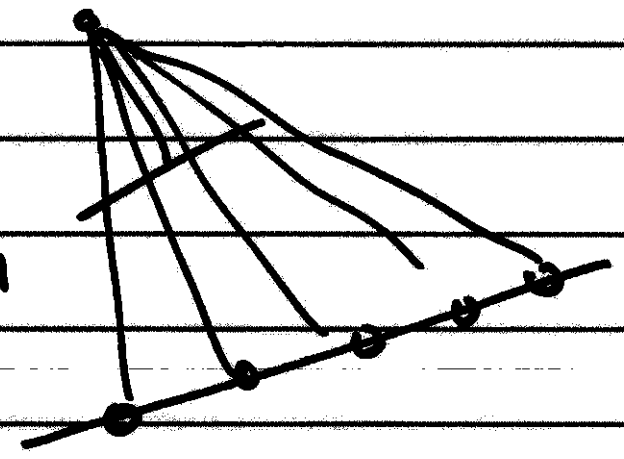
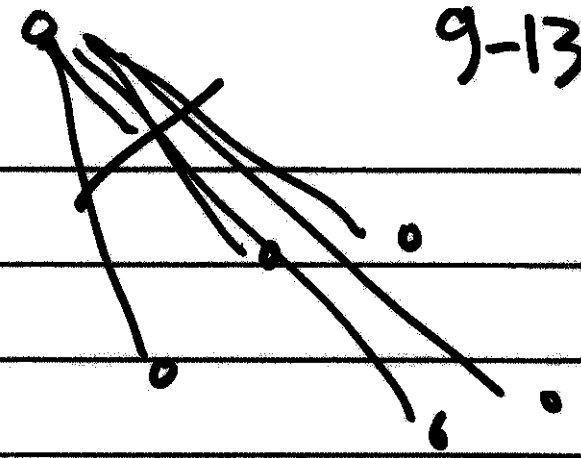
$$\underbrace{(m_{31}x + m_{32}y + m_{33}z)}_{\text{ground point}} - \underbrace{(m_{31}x_c + m_{32}y_c + m_{33}z_c)}_{\text{exp. sta}}$$

$$z = e_0 + e_1 x + e_2 y$$

$$x = \frac{a_0 + a_1X + a_2Y}{c_1X + c_2Y + 1}$$

$$y = \frac{b_0 + b_1X + b_2Y}{c_1X + c_2Y + 1}$$

8 parameter transformation
projectivity equations



$$x c_1 X + x c_2 Y + x = a_0 + a_1 X + a_2 Y$$

$$y c_1 X + y c_2 Y + y = b_0 + b_1 X + b_2 Y$$

$$x = a_0 + a_1 X + a_2 Y - x c_1 X - x c_2 Y$$

$$y = b_0 + b_1 X + b_2 Y - y c_1 X - y c_2 Y$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 & -xX & -xY \\ 0 & 0 & 0 & 1 & X & Y & -yX & -yY \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix}$$

each point : 2 equations

8 unknowns

4 pts \Rightarrow unique solution

simple rectification

9-15

(tilt only)

ortho rectification

(tilt + terrain)

9-17

Exterior Orientation X, Y, Z, ω, ϕ, K

Interior Orientation x_0, y_0, f

DLT: do not enforce

$$z = e_0 + e_1 x + e_2 y$$

$$x = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

$$y = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

(x_0, y_0)