$$x = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

$$y = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{''}$$

DLT: Direct Linear Transformation

note: x_0, y_0 omitted

$$L_9 X_x + L_{10} Y_x + L_{11} Z_x + x = L_1 X + L_2 Y + L_3 Z + L_4$$

$$\underbrace{L_9 X_y + L_{10} Y_y + L_{11} Z_y} + y = L_5 X + L_6 Y + L_7 Z + L_8$$

$$x = L_1 X + L_2 Y + L_3 Z + L_4 - L_9 X_x - L_{10} Y_x - L_{11} Z_x$$

$$y = \underbrace{L_5 X + L_6 Y + L_7 Z + L_8 - L_9 X_y - L_{10} Y_y - L_{11} Z_y}$$

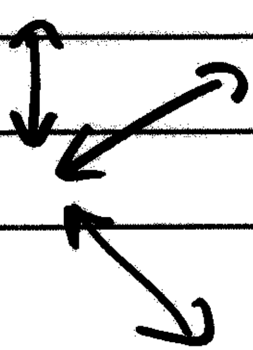
$$\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -Xx & -Yx & -Zx \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -Xy & -Yy & -Zy \end{bmatrix}}_{2\times 11} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \\ L_9 \\ L_{10} \\ L_{11} \end{bmatrix}$$

11 unknowns

each point $\Rightarrow$ 2 equations

unique solution $5\frac{1}{2}$ points

important : Distributed



$$P = -\left[1 / (L_9^2 + L_{10}^2 + L_{11}^2) \right]^{1/2}$$

$$L_1' = -P \cdot L_1$$

$$L_2' = -P \cdot L_2$$

$$L_3' = -P \cdot L_3$$

$$C_x = \left[(L_1')^2 + (L_2')^2 + (L_3')^2 \right]^{1/2}$$

$$L_5' = -P \cdot L_5$$

$$L_6' = -P \cdot L_6$$

$$L_7' = -P \cdot L_7$$

$$C_y \left[(L_5')^2 + (L_6')^2 + (L_7')^2 \right]^{1/2}$$

$$m_{31} = -PL_9$$

$$m_{32} = -PL_{10}$$

$$m_{33} = -PL_{11}$$

$$m_{11} = L_1' / C_x$$

$$m_{12} = L_2' / C_x$$

$$m_{13} = L_3' / C_x$$

$$m_{21} = L_5' / C_y$$

$$m_{22} = L_6' / C_y$$

$$m_{23} = L_7' / C_y$$

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

c_x, c_y

$$f = \frac{c_x + c_y}{2}$$

$$\begin{bmatrix} -m_{31} & -m_{32} & -m_{33} \\ m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} P \\ P \cdot L_4 / c_x \\ P \cdot L_8 / c_y \end{bmatrix}$$

$$\phi = \sin^{-1}(m_{31})$$

$$\kappa = \tan^{-1}\left(\frac{-m_{21}/\cos\phi}{m_{11}/\cos\phi} \right)$$

$$\omega = \tan^{-1}\left(\frac{-m_{32}/\cos\phi}{m_{33}/\cos\phi} \right)$$

RPC

RFM

Adjustment of Geospatial Observations

Advanced Geospatial Estimation

Analysis + Adjustment of Survey Measurements
Mikhail + Gracie

mikhail@ecn.purdue.edu
observations + Least Squares
— Mikhail

mathematical model — functional (equations)
mathematical model — stochastic

constant — observation (σ) — unknown

n : # of measurements
$-\ n_0$: minimum # of meas. to fix model

r : redundancy

how to adjust?

$$\ell + v = \hat{\ell}$$

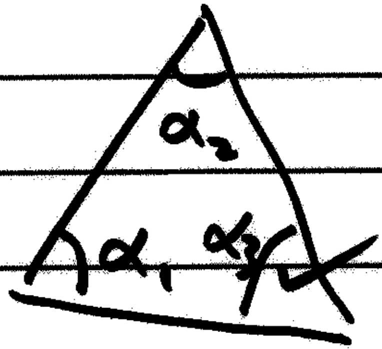
$$\sum_i v_i^2 \rightarrow \text{minimum}$$

$$\sum_i W_i V_i^2 \rightarrow \text{minimize} \quad \underline{v^T W v}$$

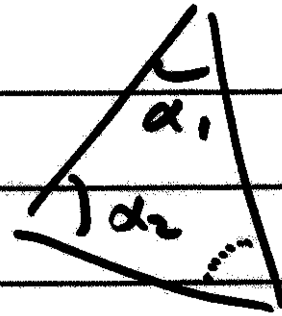
$$W_i = \frac{\sigma_0^2}{\sigma_i^2}$$

σ_0^2 ← variance of "typical" observation

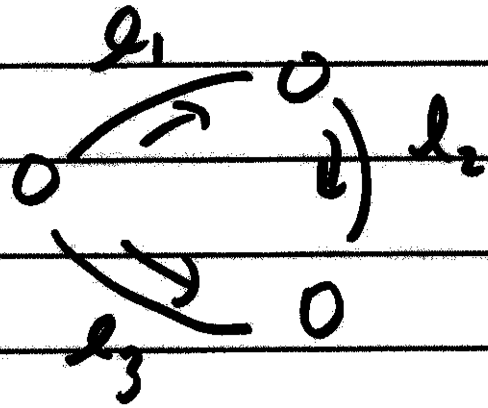
$$W = \begin{bmatrix} w_1 & & & & \\ & w_2 & & & \\ & & w_3 & & \\ & & & \ddots & \\ & & & & w_n \end{bmatrix} \qquad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$



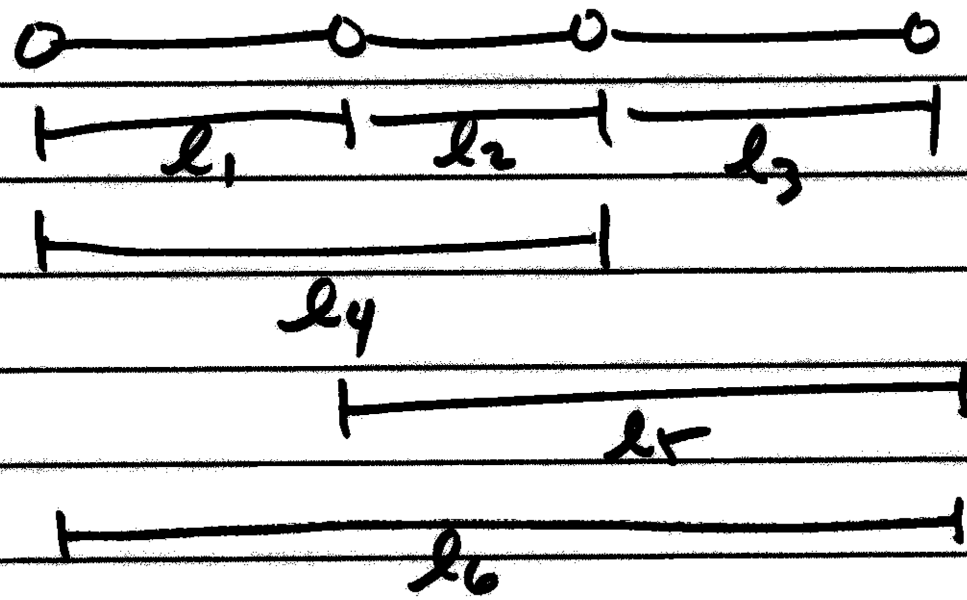
$n = 3$
$n_0 = 2$
$r = 1$



$n = 2$
$n_0 = 2$
$r = 0$



$n = 3$
$n_0 = 2$
$r = 1$



$n = 6$

$n_0 = 3$

$r = 3$

if add additional unknowns

$\#$ unknowns $= u$

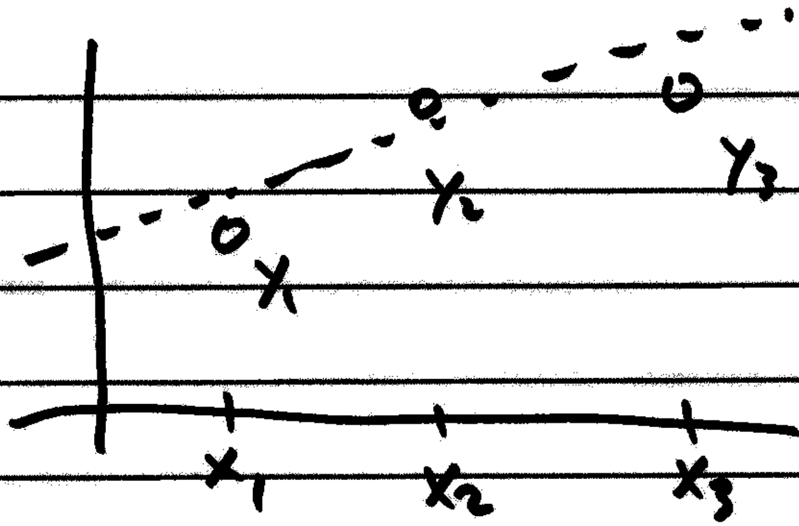
$\#$ equations $= r + u$

(C)

$$\hat{l}_4 = \hat{l}_1 + \hat{l}_2$$

$$\hat{l}_5 = \hat{l}_2 + \hat{l}_3$$

$$\hat{l}_6 = \hat{l}_1 + \hat{l}_2 + l_3$$



$n = 3$

$n_0 = 2$

$r = 1$

$$\frac{\hat{y}_2 - \hat{y}_1}{x_2 - x_1} = \frac{\hat{y}_3 - \hat{y}_1}{x_3 - x_1}$$

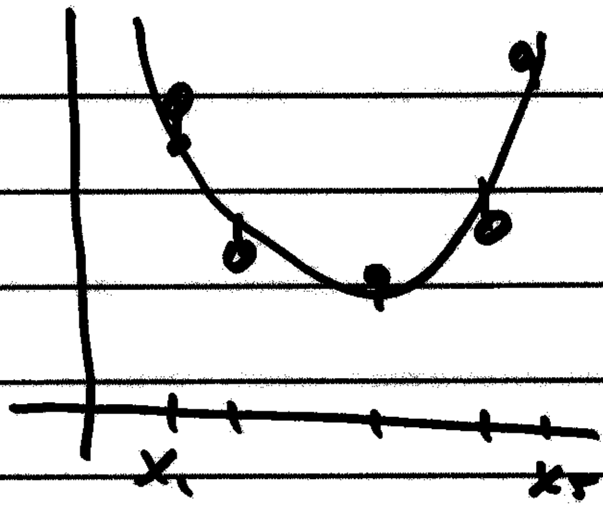
$$y = \underline{\underline{m}}\,x + \underline{\underline{b}} \quad , \; u = 2$$

$$c = r + u = 1 + 2 = 3$$

$$y_1 + v_1 = m x_1 + b$$

$$y_2 + v_2 = m x_2 + b$$

$$y_3 + v_3 = m x_3 + b$$



$Y = a_0 + a_1 X + a_2 X^2$ 10-12

$n = 5$

$\underline{n_0 = 3}$

$r = 2$

3 parameters: a_0, a_1, a_2

$u = 3$

$c = 2 + 3 = 5$

$$
\begin{aligned}
Y_1 + V_1 &= a_0 + a_1 X_1 + a_2 X_1^2 \\
Y_2 + V_2 &= a_0 + a_1 X_2 + a_2 X_2^2 \\
&\vdots \\
Y_5 + V_5 &= a_0 + a_1 X_5 + a_2 X_5^2
\end{aligned}
$$

$$V_1 - a_0 - a_1 x_1 - a_2 x_1^2 = -y_1$$

$$\vdots \qquad\qquad \vdots$$

$$V_5 - a_0 - a_1 x_5 - a_2 x_5^2 = -y_5$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_5 \end{bmatrix} + \begin{pmatrix} -1 & -x_1 & -x_1^2 \\ -1 & -x_2 & -x_2^2 \\ \vdots & \vdots & \vdots \\ -1 & -x_5 & -x_5^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -y_1 \\ -y_2 \\ \vdots \\ -y_5 \end{pmatrix}$$