

USGS notation
for:

12-1

$$x = X_f - x_p$$

← shift to principal point

$$y = Y_f - y_p$$

↙ radial lens distortion

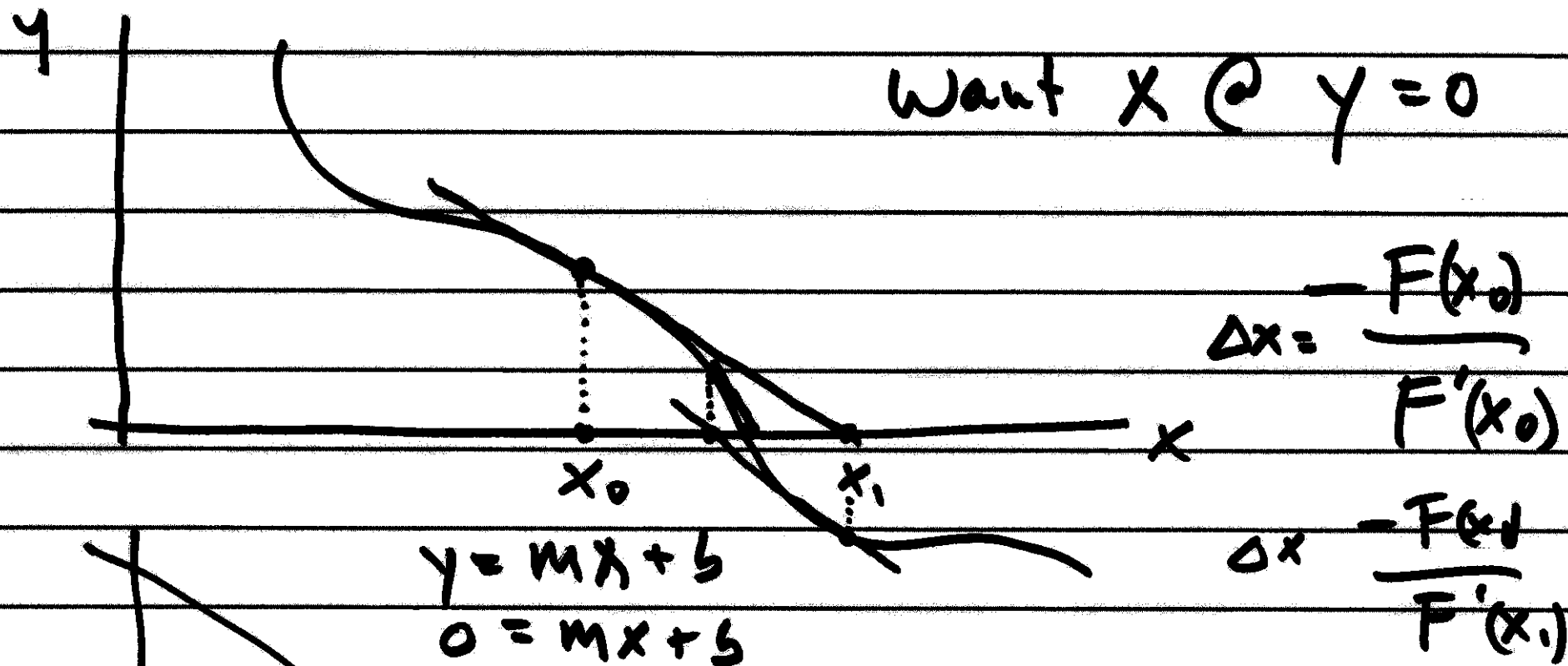
$$dr = \underline{K_0} r + K_1 r^3 + K_2 r^5 + \underline{K_3} r^7$$

$$dx = (1 + P_3 r^2 + P_4 r^4) (P_1 (r^2 + 2x^2) + 2P_2 xy)$$

$$dy = (1 + P_3 r^2 + P_4 r^4) (2P_1 xy + P_2 (r^2 + 2y^2))$$

decentering distortion ↗

Newton Iteration 1D



$$y = mx + b$$

$$0 = mx + b$$

$$mx = -b$$

$$x = \frac{-b}{m}$$

$$x_{i+1} = x_i + \frac{-F(x_i)}{\underbrace{F'(x_i)}_{\Delta x}}$$

repeat iteration steps until
 Δx small

2 NL equations in 2 unknowns

$$F_1(x_1, x_2) = 0$$

$$F_2(x_1, x_2) = 0$$

Taylor Series

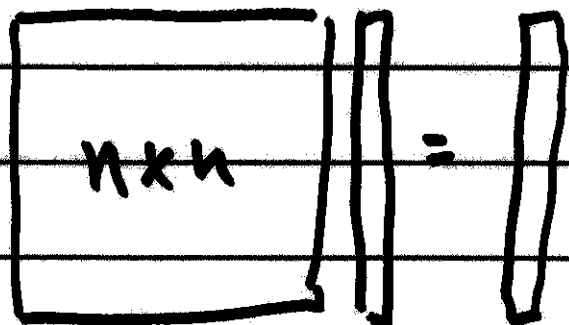
$$0 = F_1(x_1, x_2) = F_1(x_1^0, x_2^0) + \frac{\partial F_1}{\partial x_1} \Delta x_1 + \frac{\partial F_1}{\partial x_2} \Delta x_2 + \dots$$

$$0 = F_2(x_1, x_2) = F_2(x_1^0, x_2^0) + \frac{\partial F_2}{\partial x_1} \Delta x_1 + \frac{\partial F_2}{\partial x_2} \Delta x_2 + \dots$$

$$\begin{bmatrix} -F_1(x_1^0, x_2^0) \\ -F_2(x_1^0, x_2^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \end{bmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix}$$

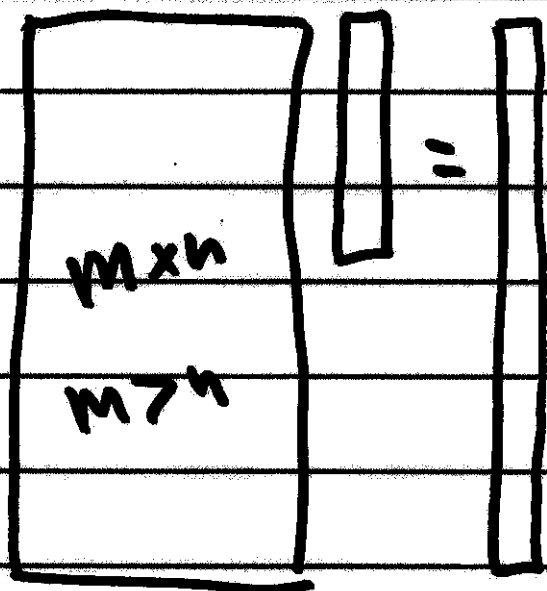
$$-F^0 = J \Delta$$

$$\Delta = -J^{-1} F^0, \quad \boxed{\Delta = -J^{-1} F^0}$$



A hand-drawn diagram representing a square matrix equation. On the left is a square box labeled $n \times n$. To its right is a vertical rectangle representing a vector, followed by an equals sign, and another vertical rectangle representing a vector on the right-hand side.

uniquely determined
system



A hand-drawn diagram representing a rectangular matrix equation. On the left is a tall rectangular box labeled $m \times n$ and $m > n$. To its right is a short vertical rectangle representing a vector, followed by an equals sign, and another tall vertical rectangle representing a vector on the right-hand side.

over determined
system

$$\bar{F}(\hat{\ell}, x) = \hat{\ell} - G(x) = 0$$

$$0 = F(\hat{\ell}, x) \approx F(\ell^0, x^0) + \boxed{\frac{\partial F}{\partial \ell}} \Delta \ell + \boxed{\frac{\partial F}{\partial x}} \Delta x$$

$$c, n$$

$$c, u$$

$$I_n \begin{bmatrix} \frac{\partial F}{\partial x_1} & \frac{\partial F}{\partial x_2} & \dots & \frac{\partial F}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_c}{\partial x_1} & \dots & \dots & \frac{\partial F_c}{\partial x_n} \end{bmatrix}$$

$$0 = \underbrace{F(\ell^0, x^0)}_{\ell^0 - G(x^0)} + \Delta \ell + B \Delta x$$

$$\ell^0 - G(x^0)$$

12-7

$$x^0 + \Delta x = \hat{x}$$

$$l^0 + \Delta l = \hat{l}, \quad l + v = \hat{l}$$

$$l^0 + \Delta l = l + v, \quad \Delta l = \underline{v + (l - l^0)}$$

$$0 = l^0 - G(x^0) + \Delta l + B \Delta$$

$$0 = \textcircled{l^0} - G(x^0) + \underset{\uparrow}{v + l - \textcircled{l^0}} + B \Delta$$

$$0 = \underbrace{l - G(x^0)}_{F(l, x^0)} + v + B \Delta$$

$$F(l, x^0)$$

$$\underline{\underline{v + B\Delta = f}}, \quad f = -F(q, x^0)^{12-8}$$

$$\Delta = (B^T W B)^{-1} B^T W f$$

$$v = f - B\Delta, \quad \hat{x} = l + v$$

NL LS algorithm (Ind. Obs.) 12-9

1. $F(\hat{Q}, x) = \hat{y} - G(x) = 0$

2. initial approx x^0

3. obtain weights $w_i = \frac{\sigma_0^2}{\sigma_i^2}$

n : obs

$n=c$ cond eq.

$u = n_0$ par.

→ 4. eval. B, f

$$B = \frac{\partial F}{\partial x} \Big|_{x_0} \quad f = -F(\hat{Q}, x_0)$$

$$W = \begin{pmatrix} w_1 & & \\ & w_2 & \\ & & \ddots \\ \phi & & & w_n \end{pmatrix}$$

5. solve linear LS problem

$$\Delta = (B^T W B)^{-1} B^T W f$$

6. update par.

no $x_0(\text{new}) = x_0(\text{old}) + \Delta$

7. check convergence

8. $v = f - B\Delta, \hat{Q} = \hat{Q} + v$

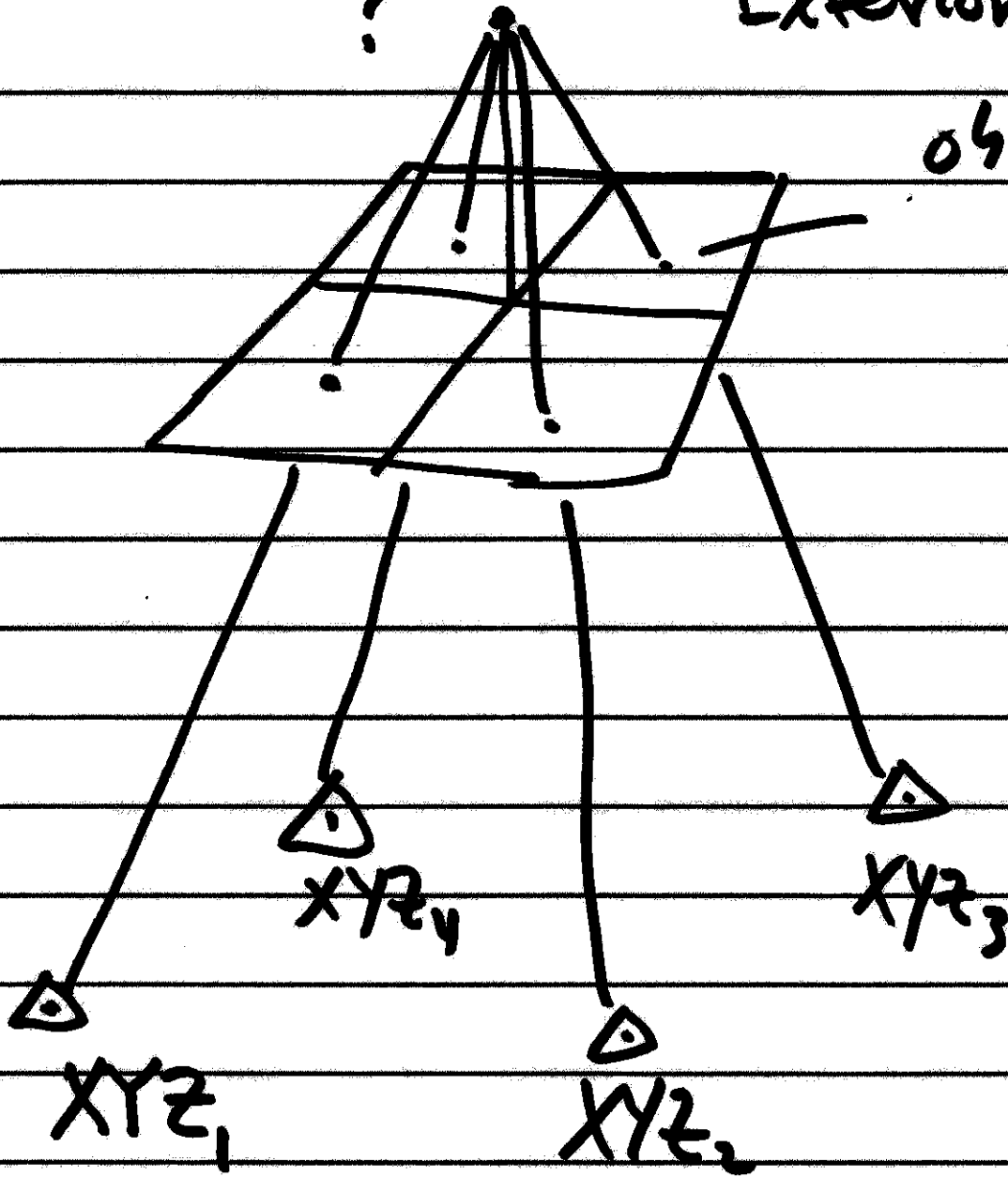
Exterior Orientation: 12-10

?

observed
 xy

Resection
Problem

X, Y, Z, ω, ϕ, K



GCP

$$\underbrace{x-x_0}_{x_c} = -f \frac{M_1 x}{M_3 x}$$

$$\underbrace{y-y_0}_{y_c} = -f \frac{M_2 x}{M_3 x}$$

$F_x = x_c + f \frac{u}{w} = 0$
$F_y = y_c + f \frac{v}{w} = 0$

ω, α, k M
 x_c, y_c, z_c
 $\rightarrow xyz$