USGS notation for:

$$x = x_f - x_p$$

shift to principal point

$$y = y_f - y_p$$

radial lens distortion

$$dr = \underline{\underline{K_0 r}} + K_1 r^3 + K_2 r^5 + \underline{K_3 r^7}$$

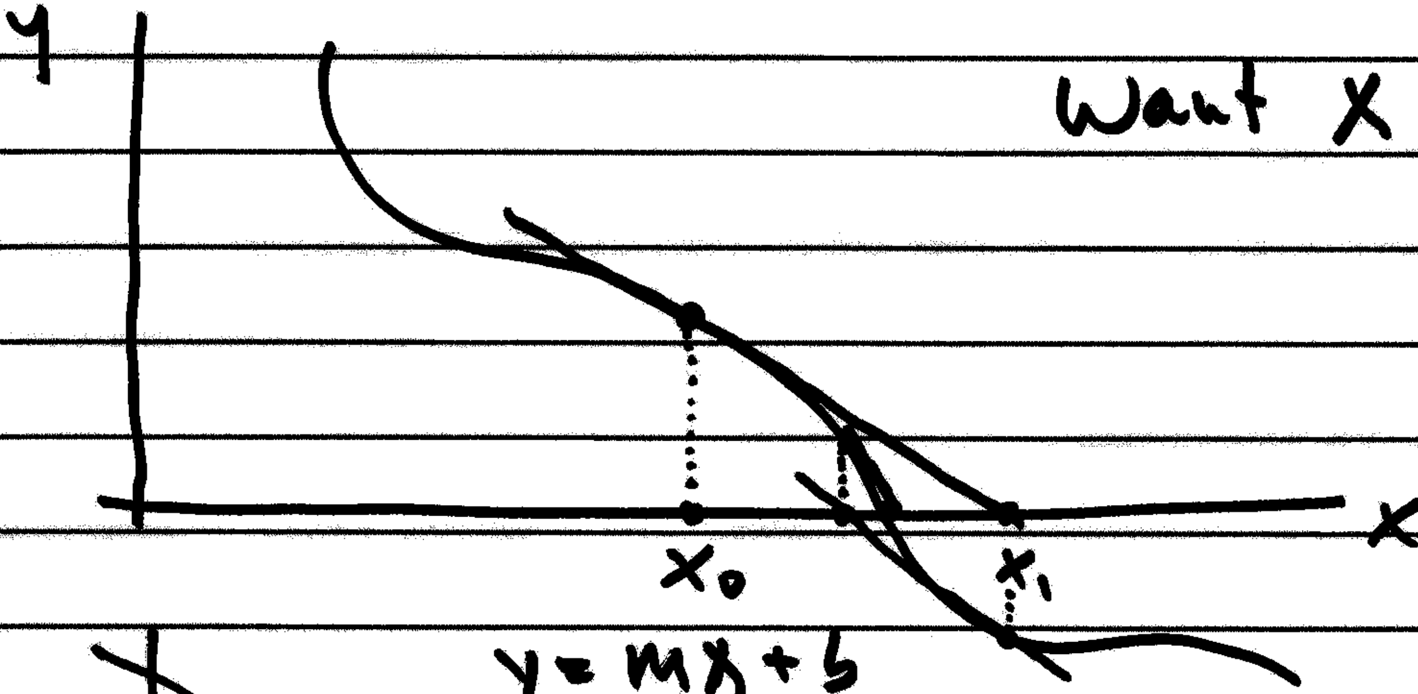
$$dx = (1 + P_3 r^2 + P_4 r^4)(P_1(r^2 + 2x^2) + 2P_2 xy)$$

$$dy = (1 + P_3 r^2 + P_4 r^4)(2P_1 xy + P_2(r^2 + 2y^2))$$

decentering distortion

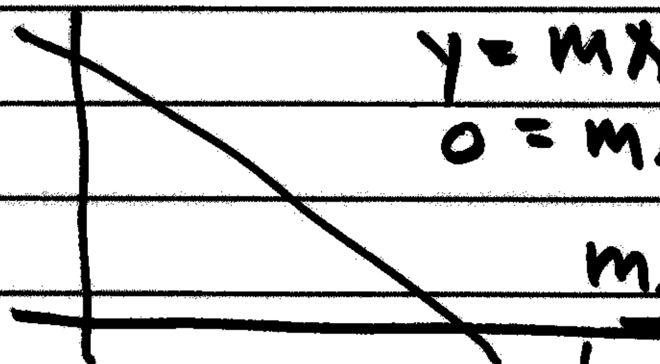
Newton Iteration 1D

Want x @ $y = 0$



$$\Delta x = \frac{-F(x_0)}{F'(x_0)}$$

$$\Delta x = \frac{-F(x_1)}{F'(x_1)}$$



$$y = mx + b$$

$$0 = mx + b$$

$$mx = -b$$

$$\boxed{x = -b/m}$$

$$x_{i+1} = x_i + \underbrace{\frac{-F(x_i)}{F'(x_i)}}_{\Delta x}$$

repeat iteration steps until
Δx small

2 NL equations in 2 unknowns

$F_1(x_1, x_2) = 0$

$F_2(x_1, x_2) = 0$

Taylor Series

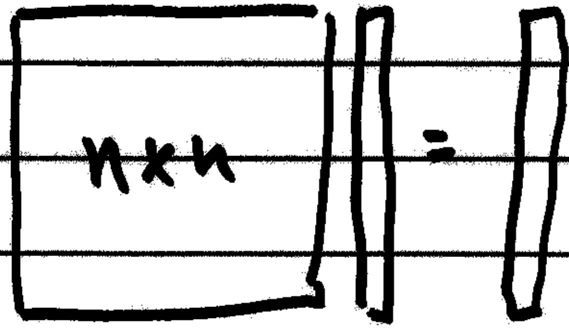
$$0 = F_1(x_1, x_2) = F_1(x_1^0, x_2^0) + \frac{\partial F_1}{\partial x_1}\Delta x_1 + \frac{\partial F_1}{\partial x_2}\Delta x_2 + \cdots$$

$$0 = F_2(x_1, x_2) = F_2(x_1^0, x_2^0) + \frac{\partial F_2}{\partial x_1}\Delta x_1 + \frac{\partial F_2}{\partial x_2}\Delta x_2 +$$

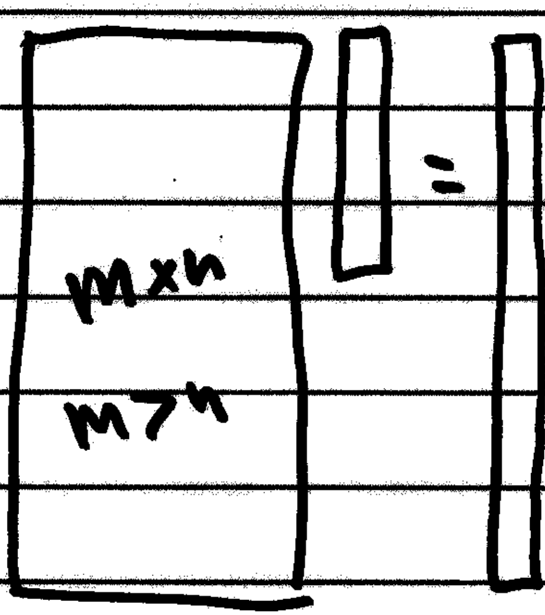
$$\begin{bmatrix} -F_1(x_1^0, x_2^0) \\ -F_2(x_1^0, x_2^0) \end{bmatrix} = \begin{bmatrix} \partial F_1/\partial x_1 & \partial F_1/\partial x_2 \\ \partial F_2/\partial x_1 & \partial F_2/\partial x_2 \end{bmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix}$$

$$-F^0 = J\Delta$$

$$\Delta = -J^{-1}F^0, \quad \boxed{\Delta = -J^{-1}F^0}$$



uniquely determined system



over determined system

$$F(\hat{\ell}, x) = \hat{\ell} - G(x) = 0$$

$$0 = F(\hat{\ell}, x) \approx F(\ell^0, x^0) + \underbrace{\boxed{\frac{\partial F}{\partial \ell}}}_{c,n} \Delta \ell + \underbrace{\boxed{\frac{\partial F}{\partial x}}}_{c,u} \Delta x$$

$$I_n$$

$$B = \begin{bmatrix} \partial F_1/\partial x_1 & \partial F_1/\partial x_2 & \cdots & \partial F_1/\partial x_u \\ \vdots & & & \\ \partial F_c/\partial x_1 & \cdots & & \partial F_c/\partial x_u \end{bmatrix}$$

$$0 = \underbrace{F(\ell^0, x^0)}_{\ell^0 - G(x^0)} + \Delta \ell + B \Delta x$$

$$x^0 + \Delta x = \hat{x}$$

$$\ell^0 + \Delta\ell = \hat{\ell} \quad , \quad \ell + v = \hat{\ell}$$

$$\ell^0 + \Delta\ell = \ell + v , \quad \Delta\ell = \underline{v + (\ell - \ell^0)}$$

$$0 = \ell^0 - G(x^0) + \Delta\ell + B\Delta$$

$$0 = \cancel{\ell^0} - G(x^0) + v + \ell - \cancel{\ell^0} + B\Delta$$

$$0 = \underbrace{\ell - G(x^0)}_{F(\ell, x^0)} + v + B\Delta$$

$$\underline{\underline{v + B\delta = f}}, \quad f = -F(\ell, x^0)$$

$$\delta = (B^T W B)^{-1} B^T W f$$

$$v = f - B\delta, \quad \hat{\ell} = \ell + v$$

NL LS algorithm (Ind. Obs.)

1. $F(\hat{\ell}, x) = \hat{\ell} - G(x) = 0$
2. initial approx x^0
3. obtain weights $w_i = \frac{\sigma_0^2}{\sigma_i^2}$
4. eval. B, f

   $B = \frac{\partial F}{\partial x}\Big|_{x_0}$ $f = -F(\ell, x^0)$
5. solve linear LS problem

   $\Delta = (B^T W B)^{-1} B^T W f$
6. update par.

   $X_0(\text{new}) = X_0(\text{old}) + \Delta$
7. check convergence — no: back to 4.
8. $v = f - B\Delta$, $\hat{\ell} = \ell + v$

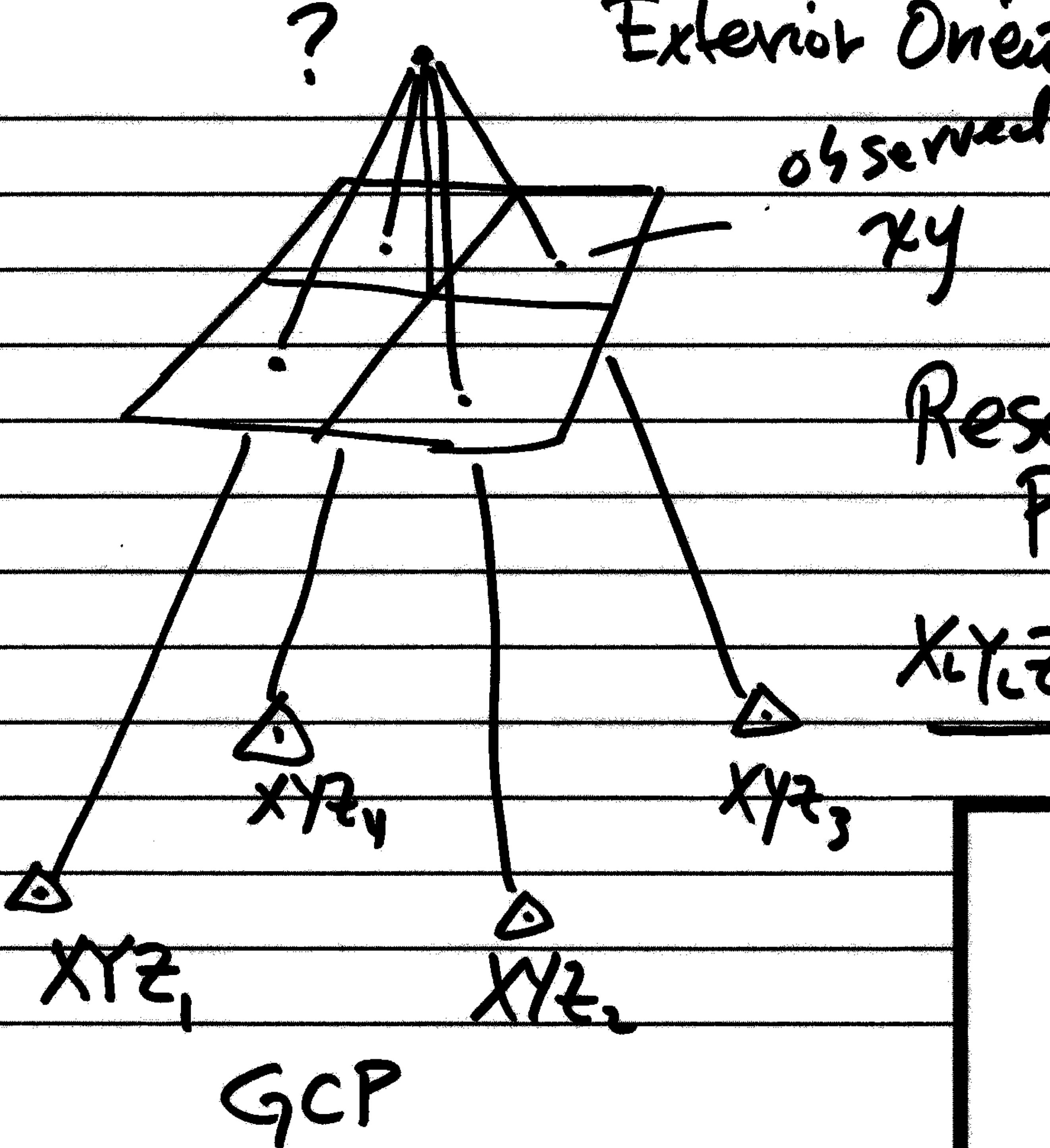
n: obs

$n = c$ cond eq.

$u = n_0$ par.

$$W = \begin{pmatrix} w_1 & & & \phi \\ & w_2 & & \\ & & \ddots & \\ \phi & & & w_n \end{pmatrix}$$

Exterior Orientation : 12-10



Resection Problem

$X_L Y_L Z_L \ \omega \ \phi \ \kappa$

$$\underbrace{x - x_0}_{X_c} = -f \frac{M_1 X}{M_3 X} \qquad \underbrace{y - y_0}_{Y_c} = -f \frac{M_2 X}{M_3 X}$$

$$\boxed{\begin{aligned} F_x &= x_c + f \frac{u}{w} = 0 \\ F_y &= y_c + f \frac{v}{w} = 0 \end{aligned}}$$

ω, φ, κ $\quad M$

$X_c\, Y_c\, Z_c$

XYZ