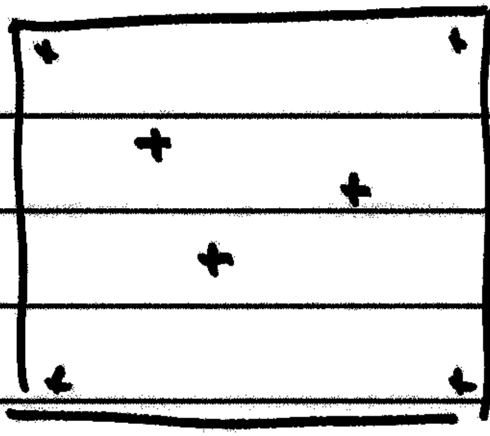


X	Y	ℓ	s
X_1	Y_1	ℓ_1	s_1
X_2	Y_2	ℓ_2	s_2
$\vdots$		$\vdots$	
X_4	Y_4	ℓ_4	s_4
constants		observations	

4-parameter transform

$$\ell_i + V_{\ell_i} = a X_i + b Y_i + C$$

$$s_i + V_{s_i} = -b X_i + a Y_i + d$$

$$V_{l_i} - aX_i - bY_i - c = -l_i$$

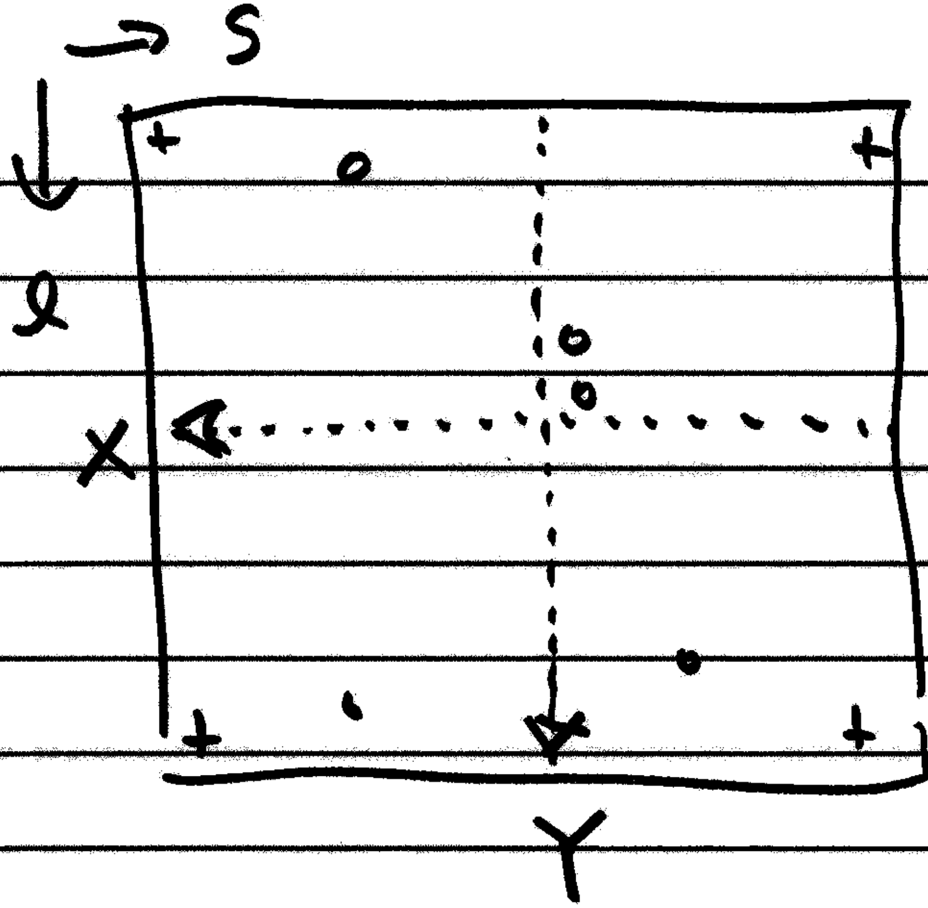
$$V_{s_i} + bX_i - aY_i - d = -s_i$$

$$\begin{bmatrix} V_{l_i} \\ V_{s_i} \end{bmatrix} + \begin{bmatrix} -X_i & -Y_i & -1 & 0 \\ -Y_i & X_i & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{pmatrix} -l_i \\ -s_i \end{pmatrix}$$

$$V + B\Delta = f$$

$$\begin{bmatrix} V_{l_1} \\ V_{s_1} \\ V_{l_2} \\ V_{s_2} \\ V_{l_3} \\ V_{s_3} \\ V_{l_4} \\ V_{s_4} \end{bmatrix} + \begin{bmatrix} -X_1 & -Y_1 & -1 & 0 \\ -Y_1 & X_1 & 0 & -1 \\ -X_2 & -Y_2 & -1 & 0 \\ -Y_2 & X_2 & 0 & -1 \\ & \vdots & & \\ & & & \\ -X_4 & -Y_4 & -1 & 0 \\ -Y_4 & X_4 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -l_1 \\ -s_1 \\ -l_2 \\ -s_2 \\ \vdots \\ \\ -l_4 \\ -s_4 \end{bmatrix}$$

$$\underset{n,1}{V} + \underset{n,4}{B}\,\underset{4,1}{\Delta} = \underset{n,1}{f}$$



ℓ, s

$\downarrow$

$X_f \, Y_f$

$\downarrow$

$X_c \, Y_c$

$\downarrow$ L.D.

$\downarrow$ A.R.

$X_c' \, Y_c'$

$$x - x_0 = -f \frac{M_1 X}{M_3 X}, \quad y - y_0 = -f \frac{M_2 X}{M_3 X}$$

$$\boxed{\begin{aligned} F_x &= x - x_0 + f \frac{u}{w} = 0 \\ F_y &= y - y_0 + f \frac{v}{w} = 0 \end{aligned}}$$

ω, φ, K

$X_L\ Y_L\ Z_L$

$$\frac{\partial F_x}{\partial p} = f\left(\frac{w \frac{du}{dp} - u \frac{dw}{dp}}{w^2}\right)$$

$$= \frac{f}{w}\left(\frac{du}{dp} - \frac{u}{w}\frac{dw}{dp}\right)$$

$$\frac{\partial F_y}{\partial p} = f\left(\frac{w\frac{\partial v}{\partial p} - v\frac{\partial w}{\partial p}}{w^2}\right)$$

$$= \frac{f}{w}\left(\frac{\partial v}{\partial p} - \frac{v}{w}\frac{\partial w}{\partial p}\right)$$

$$u = m_{11}(X - X_L) + m_{12}(Y - Y_L) + m_{13}(Z - Z_L)$$

$$v = m_{21}(X - X_L) + m_{22}(Y - Y_L) + m_{23}(Z - Z_L)$$

$$w = m_{31}(X - X_L) + m_{32}(Y - Y_L) \ldots$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

$$M = M_K M_\varphi M_\omega$$

$$\frac{\partial M}{\partial \omega} = M_K M_\varphi \cdot \frac{\partial M_\omega}{\partial \omega}$$

$$M_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}$$

$$\frac{\partial M_\omega}{\partial \omega} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\omega & \cos\omega \\ 0 & -\cos\omega & -\sin\omega \end{bmatrix}$$

$$M = M_K M_\varphi M_\omega$$

$$\frac{\partial M}{\partial \varphi} = M_K \underline{\frac{\partial M_\varphi}{\partial \varphi}} \cdot M_\omega$$

$$M_\varphi = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & & \cos\varphi \end{bmatrix}$$

$$\frac{\partial M_\varphi}{\partial \varphi} = \begin{pmatrix} -\sin\varphi & 0 & -\cos\varphi \\ 0 & 0 & 0 \\ \cos\varphi & 0 & -\sin\varphi \end{pmatrix}$$

$$\frac{\partial M}{\partial \kappa} = \frac{\partial M_\kappa}{\partial \kappa} M_\varphi M_\omega$$

$$M_\kappa = \begin{pmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\frac{\partial M_\kappa}{\partial \kappa} = \begin{pmatrix} -\sin\kappa & \cos\kappa & 0 \\ -\cos\kappa & -\sin\kappa & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$B: \begin{array}{c} \\ \\ \text{pt. 2} \\ \\ \text{pt. 3} \\ \\ \end{array} \begin{bmatrix} \frac{\partial F_{x_1}}{\partial \omega} & \frac{\partial F_{x_1}}{\partial \varphi} & \frac{\partial F_{x_1}}{\partial \kappa} & \frac{\partial F_{x_1}}{\partial X_L} & \frac{\partial F_{x_1}}{\partial Y_L} & \frac{\partial F_{x_1}}{\partial Z_L} \\ \frac{\partial F_{y_1}}{\partial \omega} & \frac{\partial F_{y_1}}{\partial \varphi} & \frac{\partial F_{y_1}}{\partial \kappa} & \frac{\partial F_{y_1}}{\partial X_L} & \frac{\partial F_{y_1}}{\partial Y_L} & \frac{\partial F_{y_1}}{\partial Z_L} \\ - & & & & & \\ - & & & & & \\ - & & & & & \\ - & & & & & \end{bmatrix}$$

$$\frac{\partial F_x}{\partial p} : \frac{f}{w}\left(\frac{du}{\partial p} - \frac{u}{w}\frac{dw}{\partial p}\right) \quad : \frac{\partial F_y}{\partial p}$$

$$\frac{\partial}{\partial \omega}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underline{\underline{\frac{\partial M}{\partial \omega}}}\begin{pmatrix} X-X_c \\ Y-Y_c \\ Z-Z_c \end{pmatrix} \qquad \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M\begin{pmatrix} X-X_c \\ Y-Y_c \\ Z-Z_c \end{pmatrix}$$

$$\frac{\partial}{\partial \phi}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underline{\underline{\frac{\partial M}{\partial \phi}}}\left(\cdot \right)$$

$$\frac{\partial}{\partial \kappa}\begin{pmatrix} u \\ v \\ w \end{pmatrix} : \frac{\partial M}{\partial \kappa}\left(\cdot \right)$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underline{\underline{M}} \begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

$$\omega^\circ, \varphi^\circ, \kappa^\circ$$
$$X_c^\circ \, Y_c^\circ \, Z_c^\circ$$

$$\frac{\partial}{\partial X_c} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -m_{11} \\ -m_{21} \\ -m_{31} \end{pmatrix}$$

$$\frac{\partial}{\partial Y_c} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -m_{12} \\ -m_{22} \\ -m_{32} \end{pmatrix}$$

$$\frac{\partial}{\partial Z_c} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -m_{13} \\ -m_{23} \\ -m_{33} \end{pmatrix}$$

$$f = -F(\ell, x^0)$$

$$\Delta = (B^T W B)^{-1} B^T W \underline{\underline{f}}$$

$$v = f - B\Delta$$

$$\hat{\ell} = \ell + v$$

$$\frac{df}{\partial x} : \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

instead of limit, take Δx small

$$\frac{\partial F}{\partial p} \approx \frac{F(p+\Delta p, q, r) - F(p, q, r)}{\Delta p}$$

$$\frac{\partial F}{\partial q} \approx \frac{F(p, q+\Delta q, r) - F(p, q, r)}{\Delta q}$$

Numerical Derivative
Estimates