$$a_1^T K_b a_2 = 0$$

$$a_2 = \frac{1}{k} M^T C \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\frac{1}{k_1}\frac{1}{k_2}(x_1\ y_1\ 1)\underbrace{C_1^T \underbrace{M_1 K_b M_2^T}_{E} C_2}_{F}\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = 0$$

E: Essential Matrix

F: Fundamental Matrix

$$(x_1 \; y_1 \; 1)\, F \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = 0$$

$$(x_1 - x_0 \quad y_1 - y_0 \quad -f)\, \underbrace{E}_{3\times 3} \begin{pmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{pmatrix} = 0$$

$$(x_1 \; y_1 \; 1)\, C_1^T$$

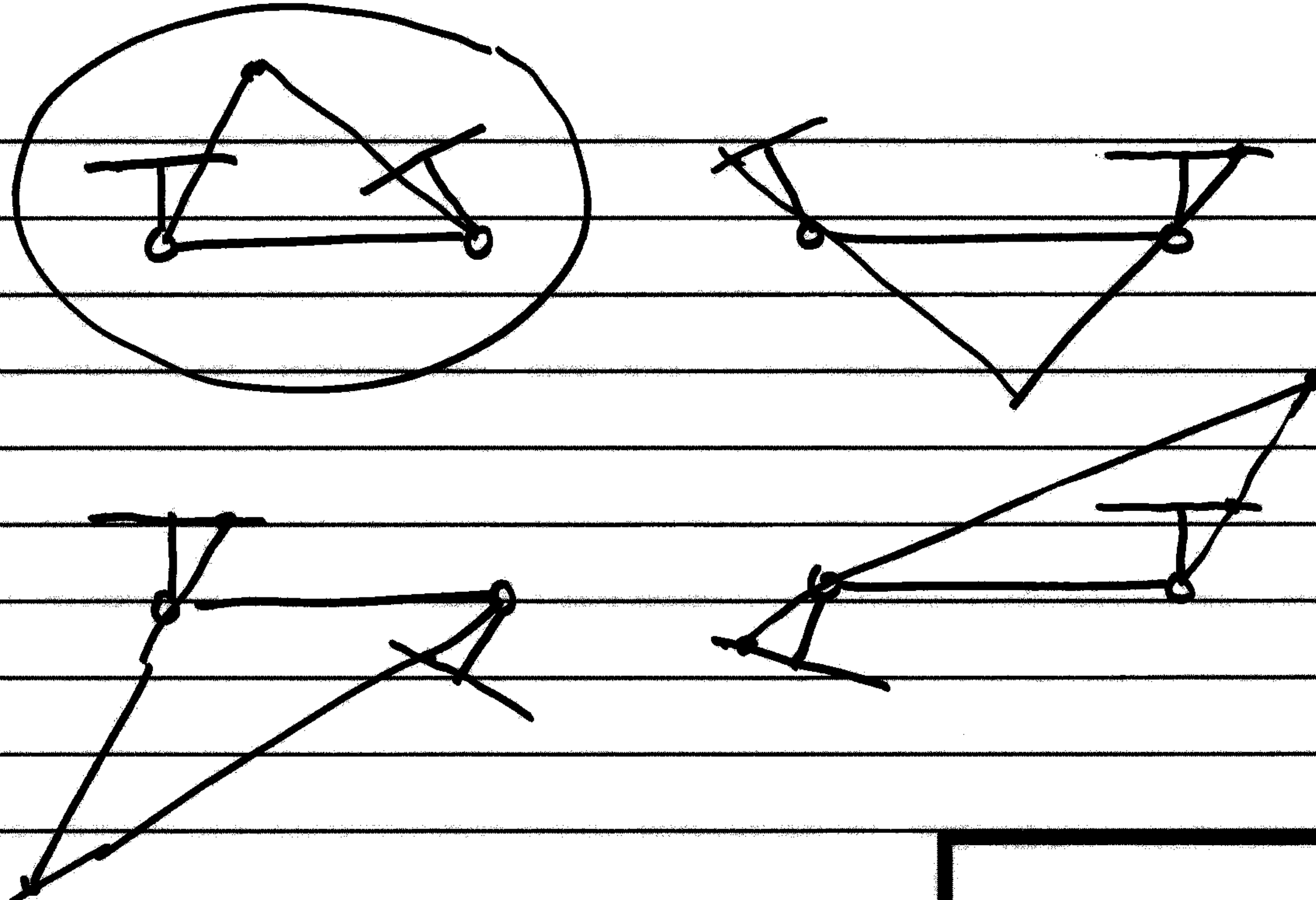
5 parameters of relative orientation

$$\begin{pmatrix} x_1 & y_1 & -f \end{pmatrix} \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ -f \end{pmatrix} = 0$$

$$x_1 x_2 e_{11} + x_2 y_1 e_{21} - x_2 f e_{31} + y_2 x_1 e_{12} + y_2 y_1 e_{22} - y_2 f e_{32} - f x_1 e_{13} - f y_1 e_{23} + f^2 \underset{\substack{\shortparallel \\ 1}}{e_{33}} = 0$$

$$\begin{bmatrix} x_1 x_2 & x_2 y_1 & -x_2 f & y_2 x_1 & y_2 y_1 & -y_2 f & -f x_1 & -f y_1 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \\ \vdots \\ e_{23} \end{bmatrix} = -f^2$$

21-6



Extract physical parameters from E

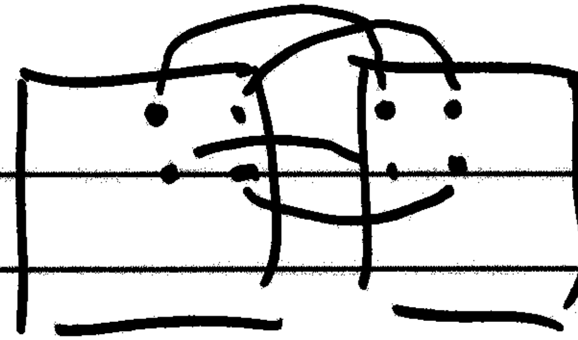
$$E = USV^T$$

$$\left.\begin{aligned} M_2 &= UWV^T \text{ or } UW^TV^T \\ b &= u_3 \text{ or } -u_3 \end{aligned}\right\}$$

where u last column of U

$$W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

8-point algorithm



with 8 points

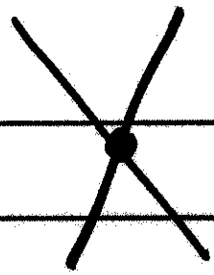
$\Rightarrow$ unique LINEAR solution

with more points

$\Rightarrow$ unique LINEAR LS solution

$$E = \underbrace{M_1}\,\underbrace{K_b}\,\underbrace{M_2^T}$$

$$\underbrace{I}\ \underbrace{\begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix}}\ \underbrace{M_2^T}$$

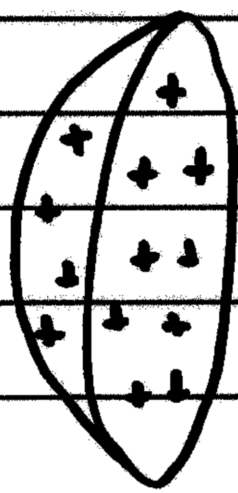


I, M_2, b

$$\begin{pmatrix} x \\ y \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

Select which of 4 cases has intersected point in <u>front</u> of the <u>cameras</u> (both)

Signalized Points or Targets + Automation to locate



close range applications

Sample covariance

$$S_{xy} = \frac{1}{N-1} \sum (x_i - \bar{x})(y_i - \bar{y})$$

ρ_{xy} : correlation coefficient

normalized covariance $-1 \to +1$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \qquad S_x : \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1) S_x S_y}$$

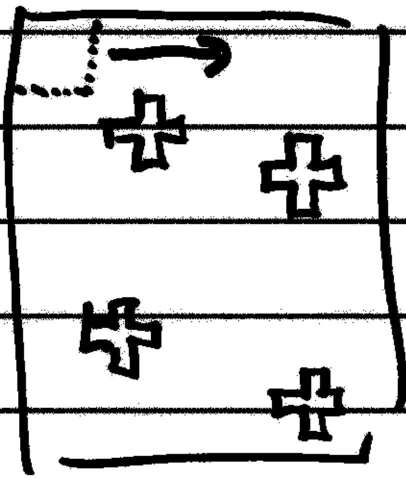
next: simplify it

$$\boxed{r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left[\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2\right]^{1/2}}}$$

template matching

Sample correlation coefficient

crosscorrelation

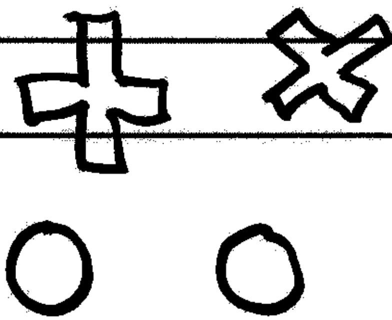


image

template

response function

threshold



famous property of Fourier Transform:
multiplication in Freq. Domain corresponds to convolution in Time/Space Domain

DFT: $X(k) = \sum_{n=0}^{N} x(n)\, e^{-i2\pi kn/N}$

IDFT: $x(n) = \frac{1}{N} \sum_{k=0}^{N} X(k)\, e^{i2\pi kn/N}$

$x(n)$: space domain

$X(k)$: freq. domain

correlation closely related to convolution if data length = 2^n

Fast Fourier Transform : FFT

signal, image   template

1. transform signal & template to Freq. Domain
2. multiply (element by element)
3. inverse transform back to space domain
4. CC response map

$$F_x: \; x - x_0 + d_x + f\frac{u}{w} = 0$$

$$F_y: \; y - y_0 + d_y + f\, v/w = 0$$

$$d_x = (x - x_0)\frac{dr}{r} = (x - x_0)\frac{k_1 r^3 + k_2 r^5 + k_3 r^7}{r}$$

$$d_y = (y - y_0)\frac{dr}{r} = \ldots$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

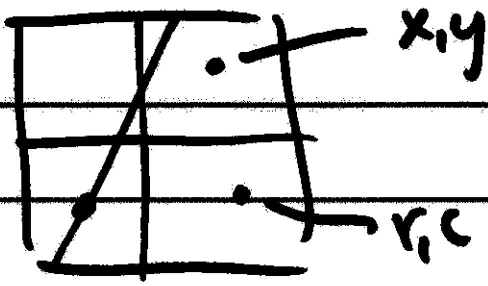
Bundle Block Adj
w/ self calibration

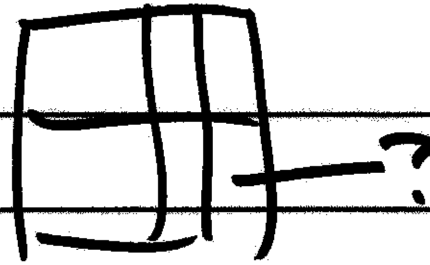
unknowns: $X_c Y_c Z_c \; \omega \varphi \kappa$

$\longrightarrow f, x_0, y_0, k_1, k_2, k_3$

XYZ object points

Feature Modelling





straight linear feature ✓
circular feature ✓
ellipse
spline