1. $\vec{X}, \vec{V} \rightarrow$ Kepler

ECI (earth centered inertial)

$r = |\vec{X}|$

$v = |\vec{V}|$

$\vec{H} = \vec{X} \times \vec{V}$

$\vec{H} : \begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix}$

$h = |\vec{H}|$

$\Omega = \tan^{-1}\left(\frac{h_x}{-h_y}\right)$

$i = \tan^{-1}\left(\frac{\sqrt{h_x^2 + h_y^2}}{h_z}\right)$

$$R_1(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

$$R_3(\Omega) = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = R_1(i)\, R_3(\Omega)\, X = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$\omega + f = \tan^{-1}\left(\frac{P_2}{P_1}\right)$$

$$a = \frac{r}{2 - (rv^2/\mu)}$$

$$E = \tan^{-1}\left(\frac{\sin E}{\cos E}\right)$$

$$e = \sqrt{1 - h^2/\mu a}$$

$$M = E - e\sin E$$

(kepler's equation)

$$\sin E = \frac{\vec{X} \cdot \vec{V}}{e\sqrt{\mu a}}$$

$$\omega = (\omega + f) - f$$

we have Ω, i, ω, f, a, e

$$\cos E = \frac{a - r}{ae}$$

$$f = \tan^{-1}\left(\frac{\sqrt{1-e^2}\sin E}{\cos E - e}\right)$$

2. Kepler $\rightarrow \vec{X}, \vec{V}$

$\Omega, i, \omega, f, a, e$

$$r = \frac{a(1-e^2)}{1+e\cos f}$$

$$\vec{q} = \begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix}$$

$$n = \sqrt{\mu / a^3}$$

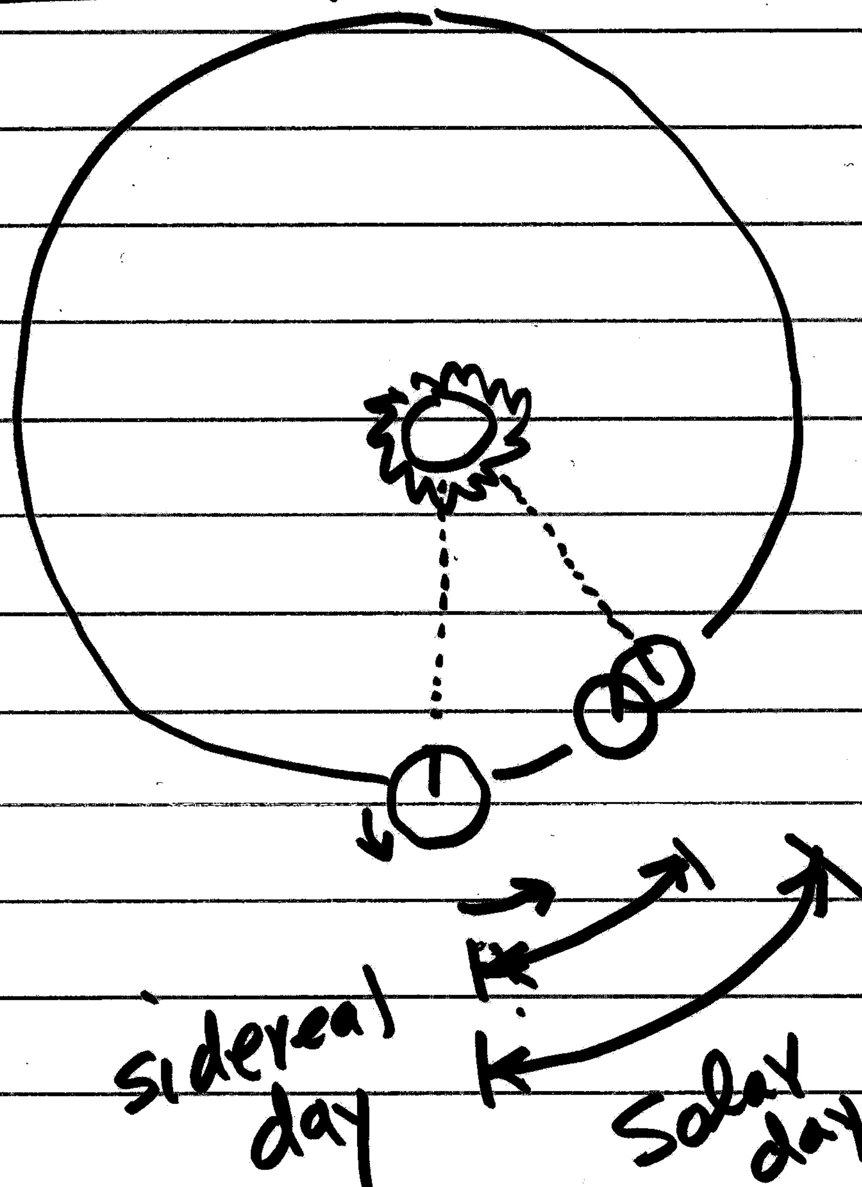
$$\dot{\vec{q}} = \frac{na}{\sqrt{1-e^2}} \begin{bmatrix} -\sin f \\ e+\cos f \\ 0 \end{bmatrix}$$

$$\vec{q} = \underbrace{R_3(\omega)\, R_1(i)\, R_3(\Omega)}_{R_{qx}} \vec{X}$$

$$\vec{q} = R_{qx} \vec{X}$$

$$\boxed{\begin{aligned} \vec{X} &= [R_{qx}]^T \vec{q} \\ \dot{\vec{X}} & \quad [R_{qx}]^T \dot{\vec{q}} \end{aligned}}$$

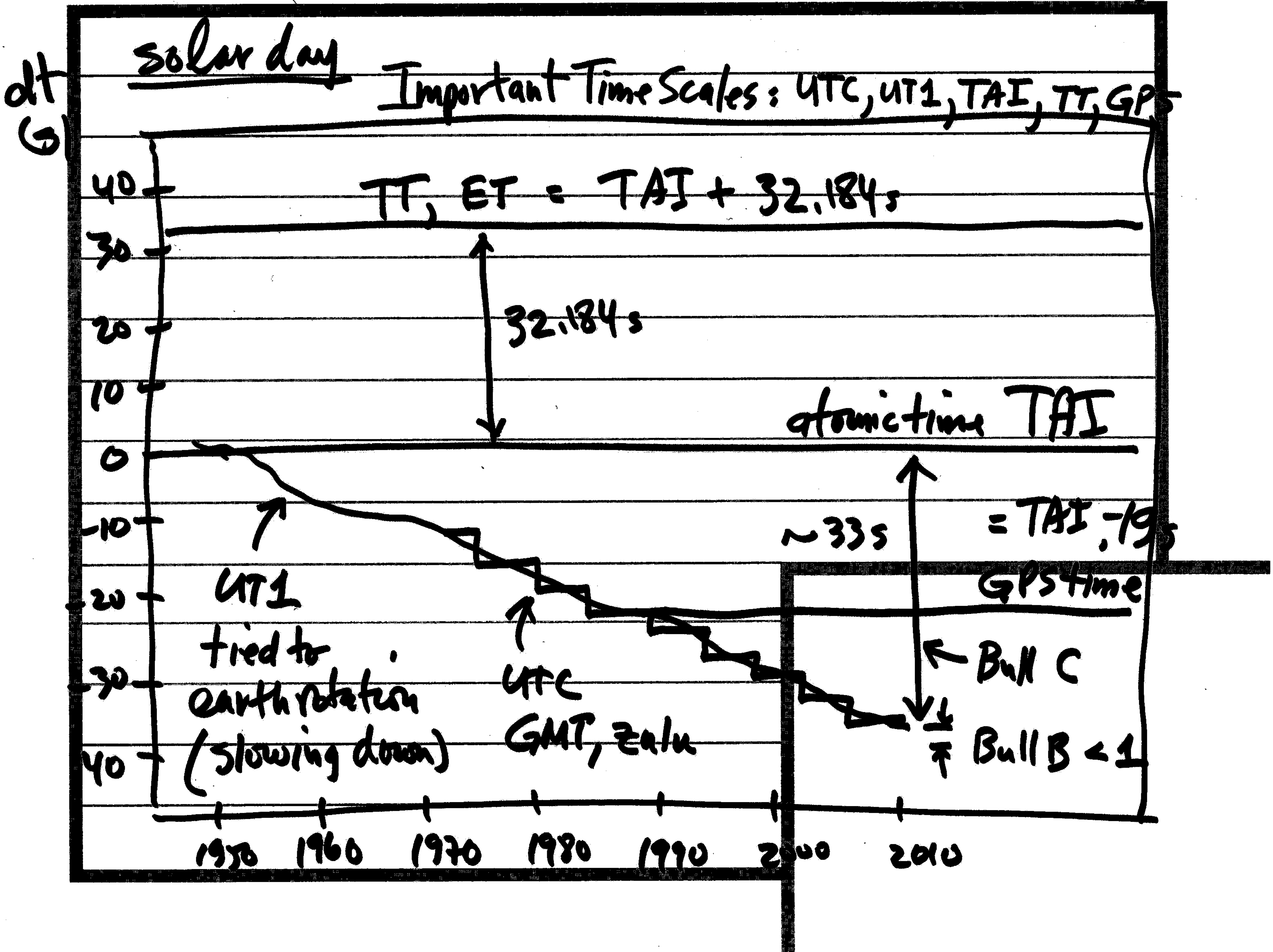
time concepts

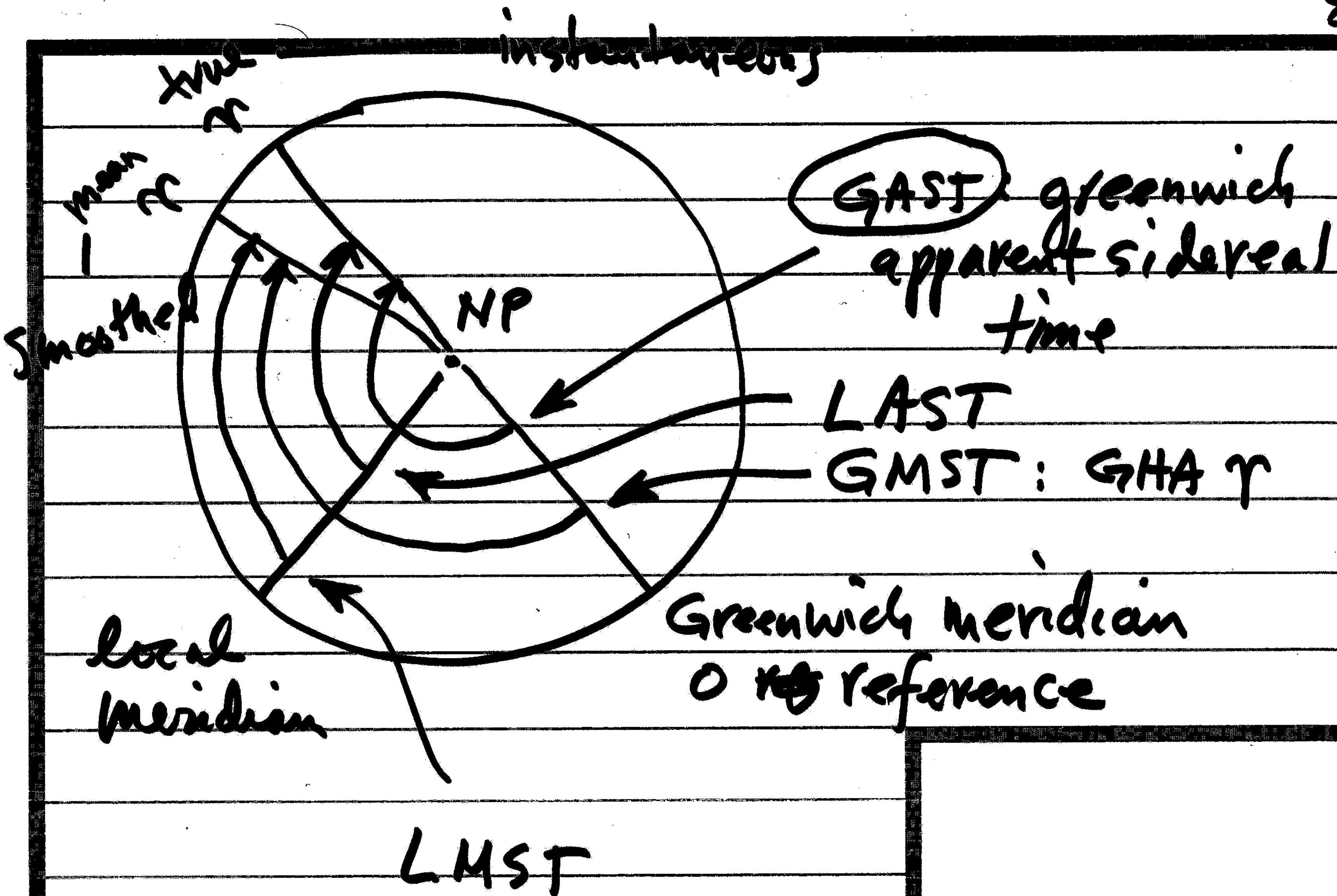


$$\cancel{\frac{365.25}{364.24} = 1.0027}$$

$$\frac{366.25 \text{ sidereal days}}{365.25 \text{ solar days}}$$

$= 1.0027$


solar day
Important Time Scales: UTC, UT1, TAI, TT, GPS
dt (s)
40
30
20
10
0
-10
-20
-30
40
TT, ET = TAI + 32.184 s
32.184 s
atomic time TAI
~33 s
= TAI - 19 s
GPS time
UT1
tried to
earth rotation
(slowing down)
UTC
GMT, Zulu
Bull C
Bull B < 1
1950
1960
1970
1980
1990
2000
2010


instantaneous
true ♈
mean ♈
smoothed
NP
GAST: greenwich apparent sidereal time
LAST
GMST: GHA ♈
Greenwich meridian
0 reference
local meridian
LMST

<u>JD</u> julian day [needed for long time intervals <u>independent</u> of calendar "quirks"]

Y : year

M : month 1, 2, ..., 12

D : day 1..7, with fraction

---

if $M > 2$, Y, M unchanged

Algorithm for JD

if $M = 1, 2$ then $Y = Y - 1$, $M = M + 12$

$A = \text{int}\left(Y/100\right)$

$B = 2 - A + \text{int}\left(A/4\right)$

int : integer truncation ("fix")

$$JD = \mathrm{int}(365.25(Y+4716)) + \mathrm{int}(30.6001(M+1)) + D + B - 1524.5$$

days since 4712 BC

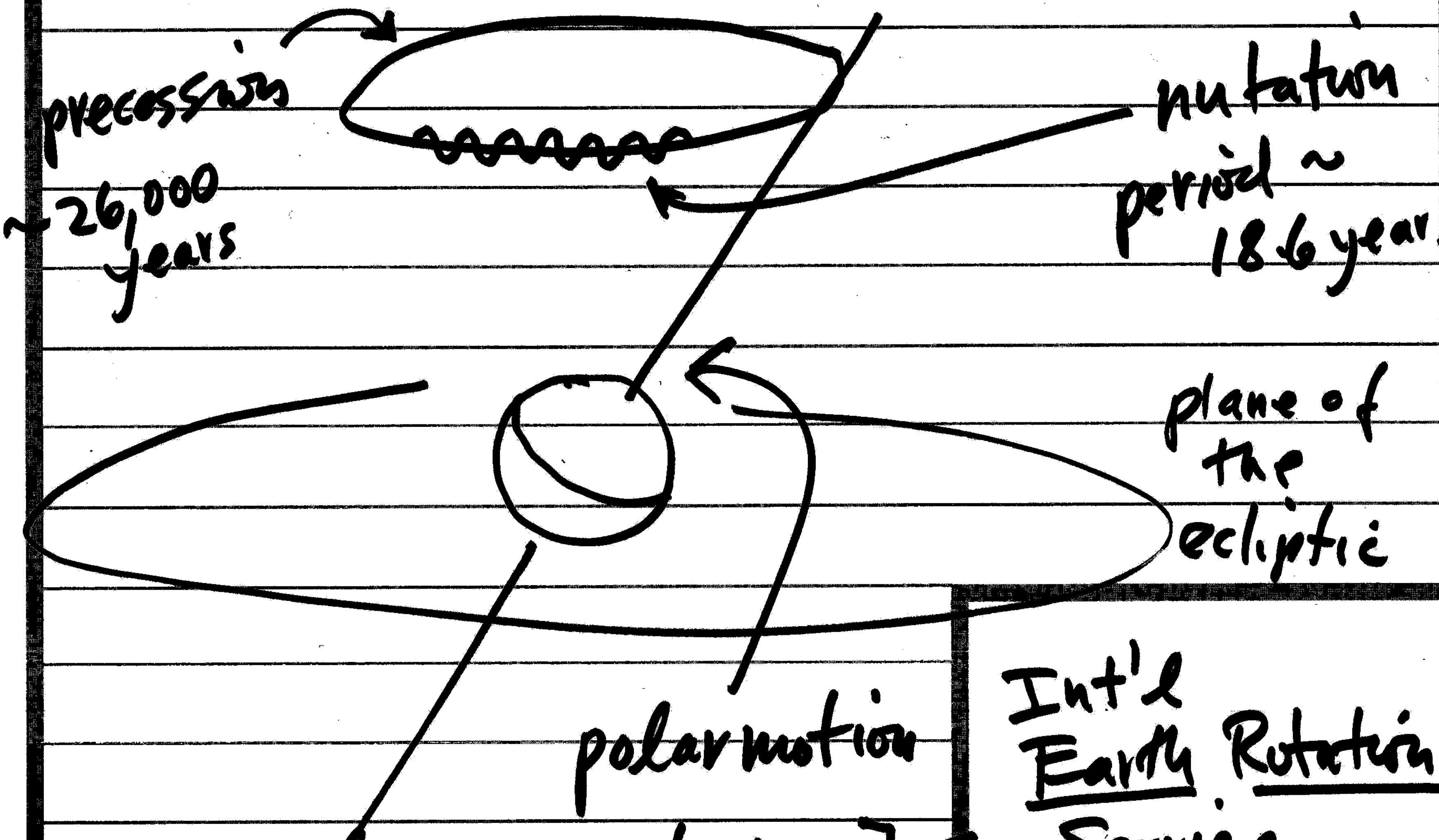
MJD : modified julian day

$$MJD = JD - 2400\,000.5$$

JC : julian century

$$\boxed{JC = \frac{JD - 2451545}{36525}}$$

~~the~~ precession + nutation (we have models)



regular component +
irregular components

Int'l Earth Rotation Service

www.iers.org ....

www.iers.org

└ data

  └ earth orientation data

    └ Bulletin A

      Bulletin B

      Bulletin C

Bulletin B smoothed + interpolated
tabulated for every day

UT1 − UTC (always be < 1 second)
(magnitude)

X_p "
Y_p "
} polar motion, arc seconds

Bulletin C

TAI − UTC