Point Correspondence Determination by Hough Transform and 4-Parameter Transformation

- Take all combinations of point pairs between the two data sets
- For each point pair, determine the possible 4-parameter coefficients (a,b,c,d) that are consistent with this point pair
- For each such set of coefficients, vote in the 4-dimensional parameter space
- You need to specify a range and a cell width
- Should be able to do hierarchical approach: coarse -> fine
- When done all point pairs, find cell with largest vote count

•For each point pair, you can write two equations

•There are 4 unkowns (a,b,c,d) in these two equations.

•For each such pair of equations, exhaustively enumerate the parameter values for 2 of the parameters (I chose b,c).

•Then for each of the enumerated b,c pairs, compute (2 equations, 2 unkowns) the remaining two parameters (in my case a,d)

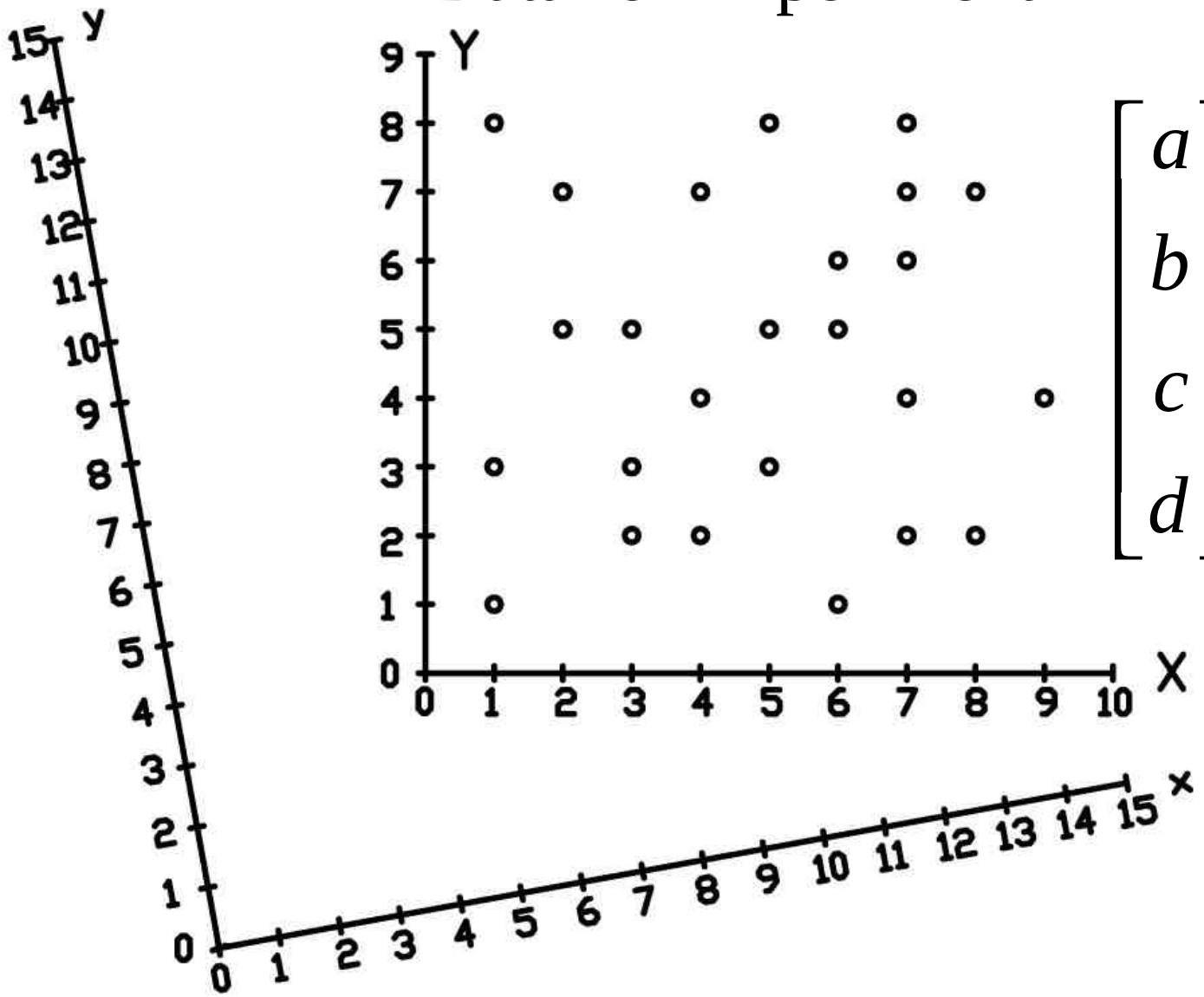
•So, one point pair can vote many times, all clustered in adjacent cells

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

given that we "know" b, c solve for a, d

$$\begin{bmatrix} X & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} x - bY - c \\ y + bX \end{bmatrix}$$

Data for Experiment



$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0.2 \\ 3.0 \\ 4.0 \end{bmatrix}$$

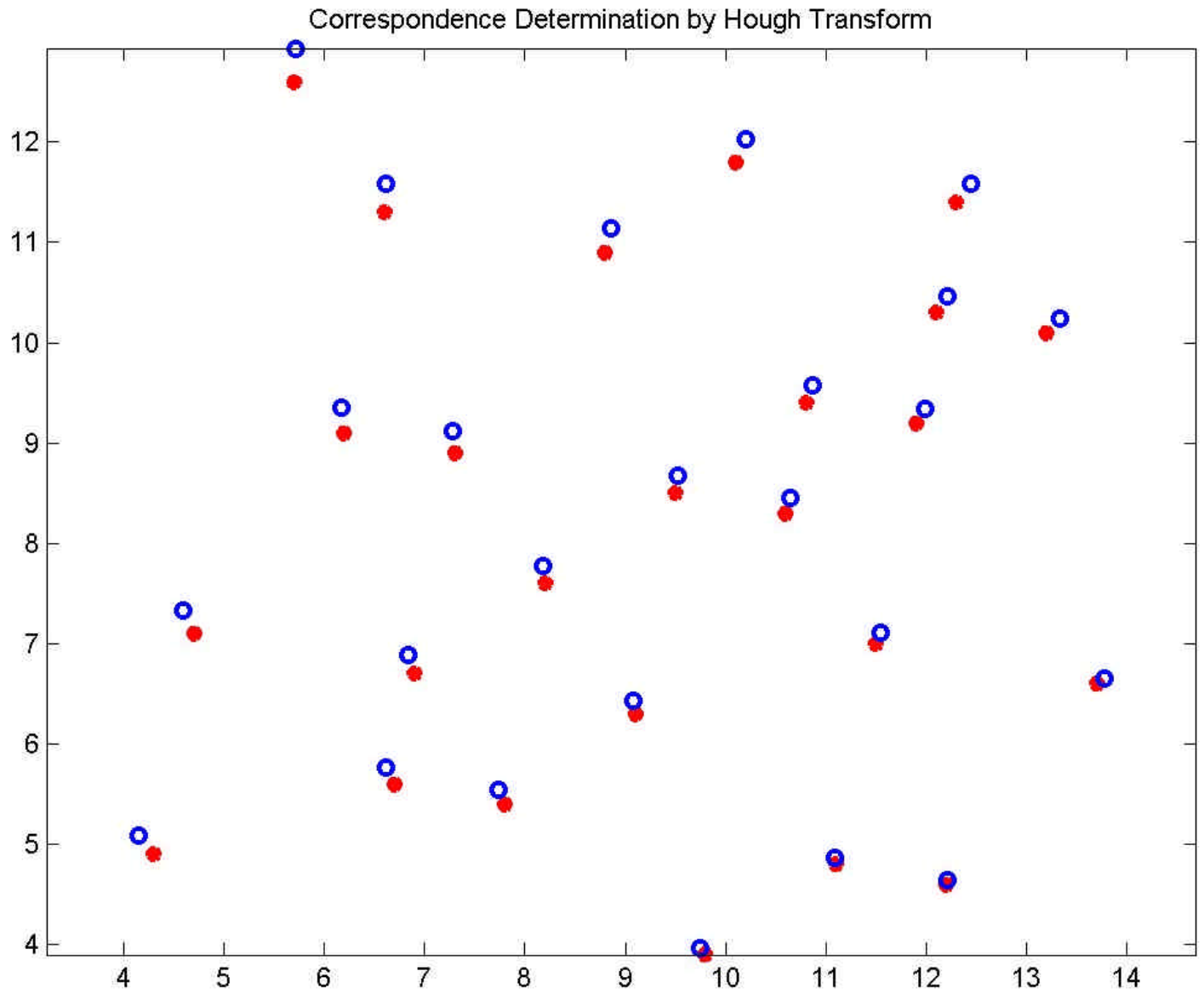
```
ds =
   18    7    5    5    6
   13    7    4    6    5
   10    6    5    6    7
    9    6    5    6    6
    9    7    3    6    4
    9    8    5    4    5
    8    6    3    7    5
    8    6    4    7    6
    8    7    5    2    4
    8    7    6    4    6
    7    5    4    8    7
    7    6    6    5    7
    6    4    3   10    5
    6    5    5    7    8
    6    6    3    8    5
    6    6    5    9    9
    6    7    3    6    5
    6    7    4    2    2
    6    7    4    5    4
    6    7    4    5    5
estimated transformation parameters
ans =
   1.1200
   0.2250
   2.8000
   4.2000
ans =
   1.1200
   0.1750
   3.2000
   3.8000
```

Coarse Discretization – 10x10x10x10 cells

This listing shows top 20 cell counts with the corresponding array indices

Numbers below show the parameters a,b,c,d that go with the top two cells

See the graphs next slide


Correspondence Determination by Hough Transform
12
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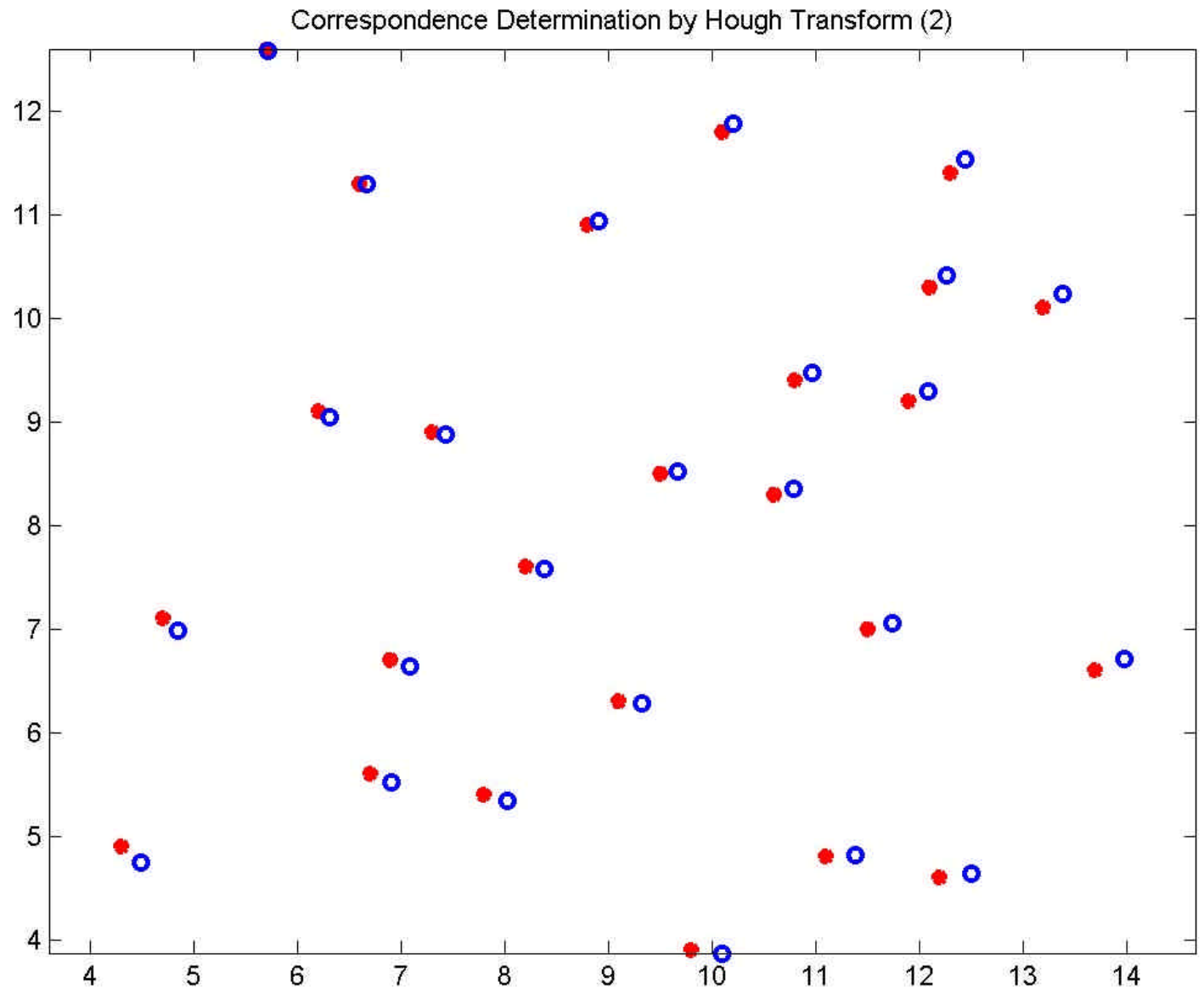
14


Correspondence Determination by Hough Transform (2)
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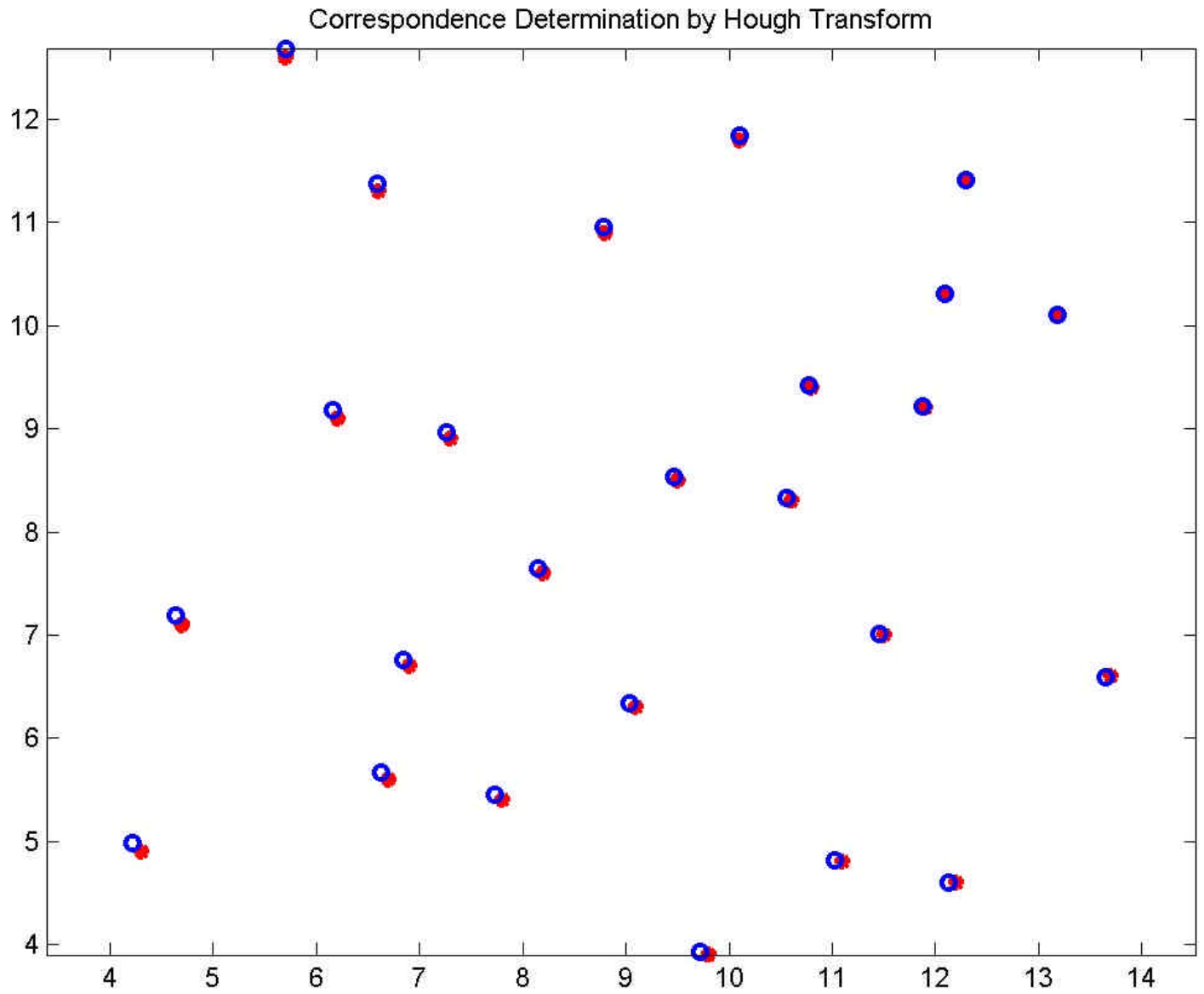
```
ds =
    18    13     9    10    11
    17    13     8    11    10
    11    12     9    11    12
    10    14     8    10     9
     9    12     8    12    11
     9    13    10    10    12
     9    14     9     9    10
     8    13     7    11     9
     7    12     7    13    10
     7    13     9     4     7
     7    14    10     8    11
     7    14    10     9    11
     6    12     7    12    10
     6    12    10    11    12
     6    12    10    11    13
     6    13     7    11    10
     6    13     7    12    10
     6    13     8     1    18
     6    13     9    20     3
     6    13    10     3     7
estimated transformation parameters
ans =
    1.1000
    0.2125
    2.9000
    4.1000
ans =
    1.1000
    0.1875
    3.1000
    3.9000
```

Fine Discretization 20x20x20x20 cells

Listing shows top twenty cell counts along with cell array indices

Numbers below are a,b,c,d corresponding to the top two cells

See plots that follow


Correspondence Determination by Hough Transform
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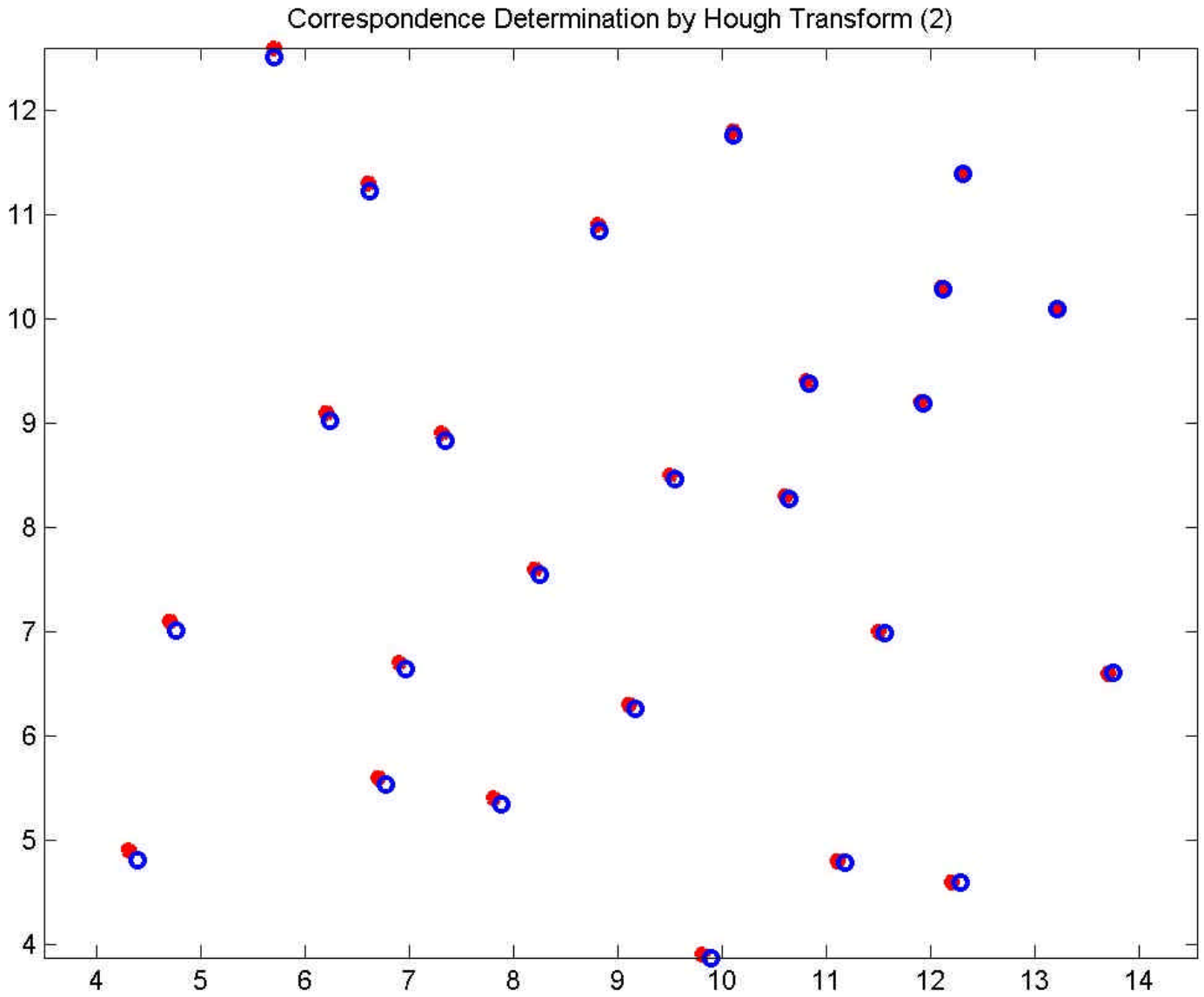
Correspondence Determination by Hough Transform (2)
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Conclusions

- It seems that we can recover the point correspondences and, as a by product, obtain a good approximation for the transformation parameters
- There were no “non-matched” points in this experiment, I don’t believe that a small percentage of non-matched points would seriously degrade the performance
- How to handle the hierarchical approach: setting the parameter ranges and cell sizes? Big question. That was an initial problem, very small “a” magnitude, need some kind of approximation to the parameters.
- Interesting approach – anyone like to pursue and do a conference paper? More simulations, more math models, maybe stereo pairs?