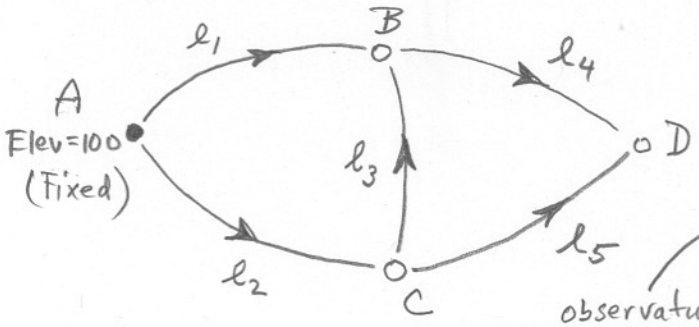
Example: Level Network by Indirect Observations CE506, Fall 2004

Model: Absolute elevation of all points in network



observations

ℓ_1	10.1
ℓ_2	7.9
ℓ_3	2.2
ℓ_4	1.9
ℓ_5	3.8

$n = 5$

$- \; n_0 = 3$

$r = 2$

B, C, D

With indirect observation method, we carry $n_0 = 3$ unknown parameters, and we write $n = 5$ condition equations, with one observation per equation

1. $\ell_1 + v_1 = B - 100$
2. $\ell_2 + v_2 = C - 100$
3. $\ell_3 + v_3 = B - C$
4. $\ell_4 + v_4 = D - B$
5. $\ell_5 + v_5 = D - C$

$\Rightarrow$

$v_1 = B - 100 - 10.1$

$v_2 = C - 100 - 7.9$

$v_3 = B - C - 2.2$

$v_4 = D - B - 1.9$

$v_5 = D - C - 3.8$

$\Rightarrow$

$v_1 = B - 110.1$

$v_2 = C - 107.9$

$v_3 = B - C - 2.2$ (*)

$v_4 = D - B - 1.9$

$v_5 = D - C - 3.8$

Objective function to Minimize $\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2$, differentiate with respect to 3 unknowns B, C, D and set equal to zero.

$$\Phi = (B - 110.1)^2 + (C - 107.9)^2 + (B - C - 2.2)^2 + (D - B - 1.9)^2 + (D - C - 3.8)^2$$

$$\partial\Phi/\partial B = \tfrac{1}{2}(B - 110.1) + \tfrac{1}{2}(B - C - 2.2) + \tfrac{1}{2}(D - B - 1.9)(-1) = 0$$

$$\partial\Phi/\partial C = \tfrac{1}{2}(C - 107.9) + \tfrac{1}{2}(B - C - 2.2)(-1) + \tfrac{1}{2}(D - C - 3.8)(-1) = 0$$

$$\partial\Phi/\partial D = \tfrac{1}{2}(D - B - 1.9) + \tfrac{1}{2}(D - C - 3.8) = 0$$

$$3B - C - D = 110.1 + 2.2 - 1.9$$
$$-B + 3C - D = 107.9 - 2.2 - 3.8$$
$$-B - C + 2D = 1.9 + 3.8$$

$\Rightarrow$

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 110.4 \\ 101.9 \\ 5.7 \end{bmatrix}$$

Normal Equations (symmetric)

Solve system with matlab:

```
N = [3 -1 -1; -1 3 -1; -1 -1 2];
t = [110.4 ; 101.9 ; 5.7];
X = inv(N) * t;
B = X(1);
C = X(2);
D = X(3);
```

$$\begin{bmatrix} B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 110.0625 \\ 107.9375 \\ 111.8500 \end{bmatrix}$$

plug into (*) ↑

adjusted observations

residuals → $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} -0.0375 \\ 0.0375 \\ -0.0750 \\ -0.1125 \\ 0.1125 \end{bmatrix}$, $\begin{bmatrix} \hat{\ell}_1 \\ \hat{\ell}_2 \\ \hat{\ell}_3 \\ \hat{\ell}_4 \\ \hat{\ell}_5 \end{bmatrix} = \begin{bmatrix} \ell_1 + v_1 \\ \ell_2 + v_2 \\ \ell_3 + v_3 \\ \ell_4 + v_4 \\ \ell_5 + v_5 \end{bmatrix} = \begin{bmatrix} 10.0625 \\ 7.9375 \\ 2.1250 \\ 1.7875 \\ 3.9125 \end{bmatrix}$