

Observations Only

$$\mu = 0$$

$$c = r$$

$$\text{NL: } F(\hat{\ell}) = 0$$

$$\text{L: } Av = f$$

$$A = \frac{\partial F}{\partial \ell}$$

$$f = -F(\ell^0) - A(\ell - \ell^0)$$

$$Q = W^{-1}$$

$$Q_e = AQA^T$$

$$W_e = Q_e^{-1}$$

$$k = W_e f$$

$$v = QA^T k$$

$$\text{update } \ell^0_{\text{new}} = \ell + v$$

$$\text{compute } \Delta \ell = \ell^0_{\text{new}} - \ell^0_{\text{old}}$$

re-evaluate A, f

test convergence $\Delta \ell$ small
or Φ stable

iteration loop

General Least Squares or Mixed Model

$$0 < \mu < n_0$$

$$c = r + \mu$$

$$\text{NL: } F(\hat{\ell}, x) = 0$$

$$\text{L: } Av + B\Delta = f$$

$$A = \frac{\partial F}{\partial \ell}$$

$$B = \frac{\partial F}{\partial x}$$

$$f = -F(\ell^0, x^0) - A(\ell - \ell^0)$$

$$Q = W^{-1}$$

$$Q_e = AQA^T$$

$$W_e = Q_e^{-1}$$

$$\Delta = (B^T W_e B)^{-1} B^T W_e f$$

$$k = W_e (f - B\Delta)$$

$$v = QA^T k$$

$$\text{update } x^0_{\text{new}} = x^0 + \Delta$$

$$\text{update } \ell^0_{\text{new}} = \ell + v$$

$$\text{compute } \Delta \ell = \ell^0_{\text{new}} - \ell^0_{\text{old}}$$

re-evaluate A, B, f

test convergence $\Delta, \Delta \ell$ small
or Φ stable

iteration loop

Indirect Observations

$$\mu = n_0$$

$$c = n$$

$$\text{NL: } F(\hat{\ell}, x) = \hat{\ell} - G(x) = 0$$

$$\text{L: } v + B\Delta = f$$

$$B = \frac{\partial F}{\partial x}$$

$$f = -F(\ell, x^0)$$

$$\Delta = (B^T W B)^{-1} B^T W f$$

$$v = f - B\Delta$$

$$\text{update } x^0_{\text{new}} = x^0 + \Delta$$

re-evaluate B, f

test convergence Δ small
or Φ stable

iteration loop

initial $\ell^0 = \ell$
so initial $\ell - \ell^0 = 0$

If linear just do once through loop
 f : "misclosure vector" should be small

$$\Phi = V^T W V$$

$$w_i = \frac{\sigma_{\Delta}^2}{\sigma_i^2}$$

choose σ_0^2 to be typical σ^2 , or 1, or anything

Least Squares Estimation

(left out Error Propagation!)