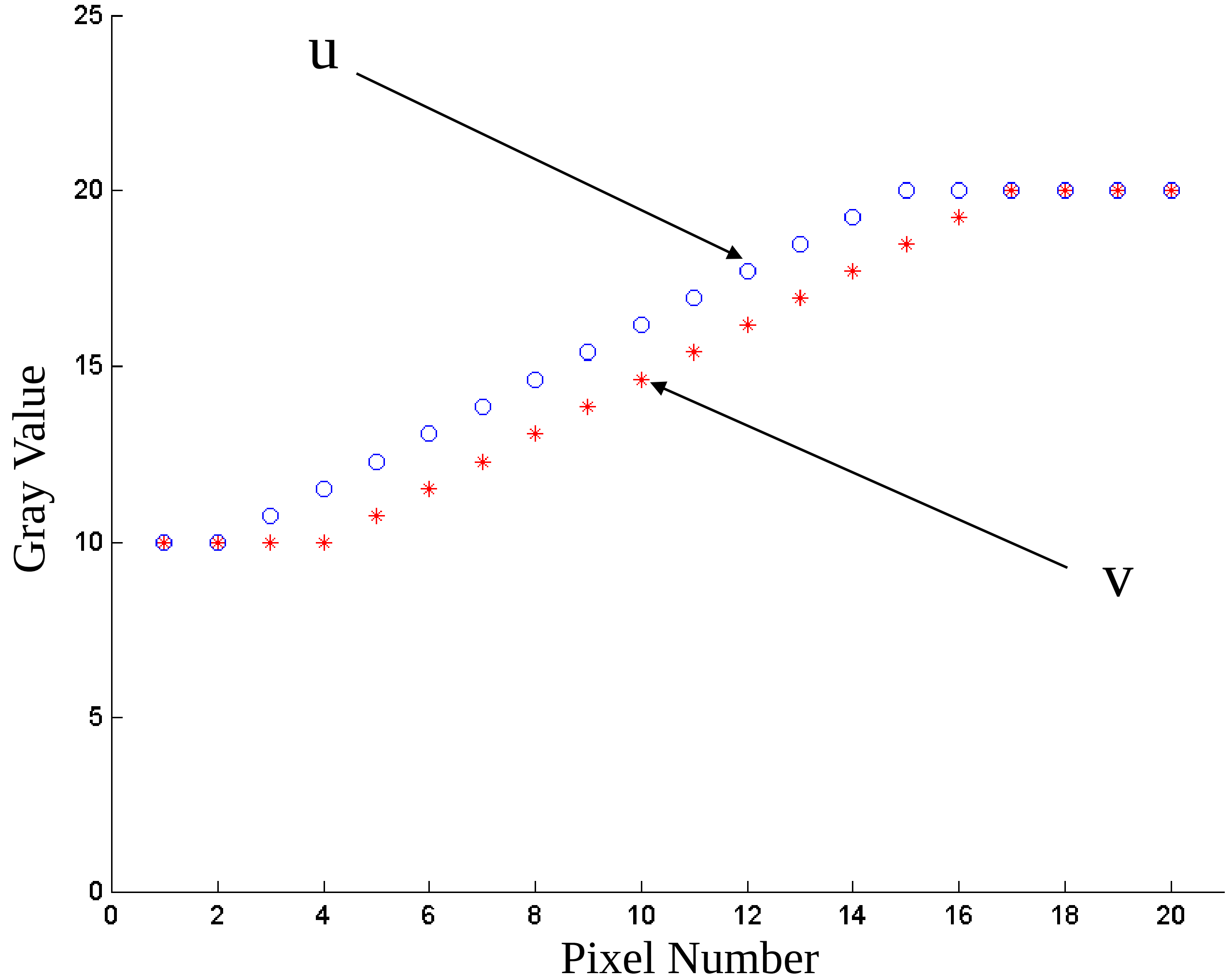
Least Squares Matching - 1D Tutorial

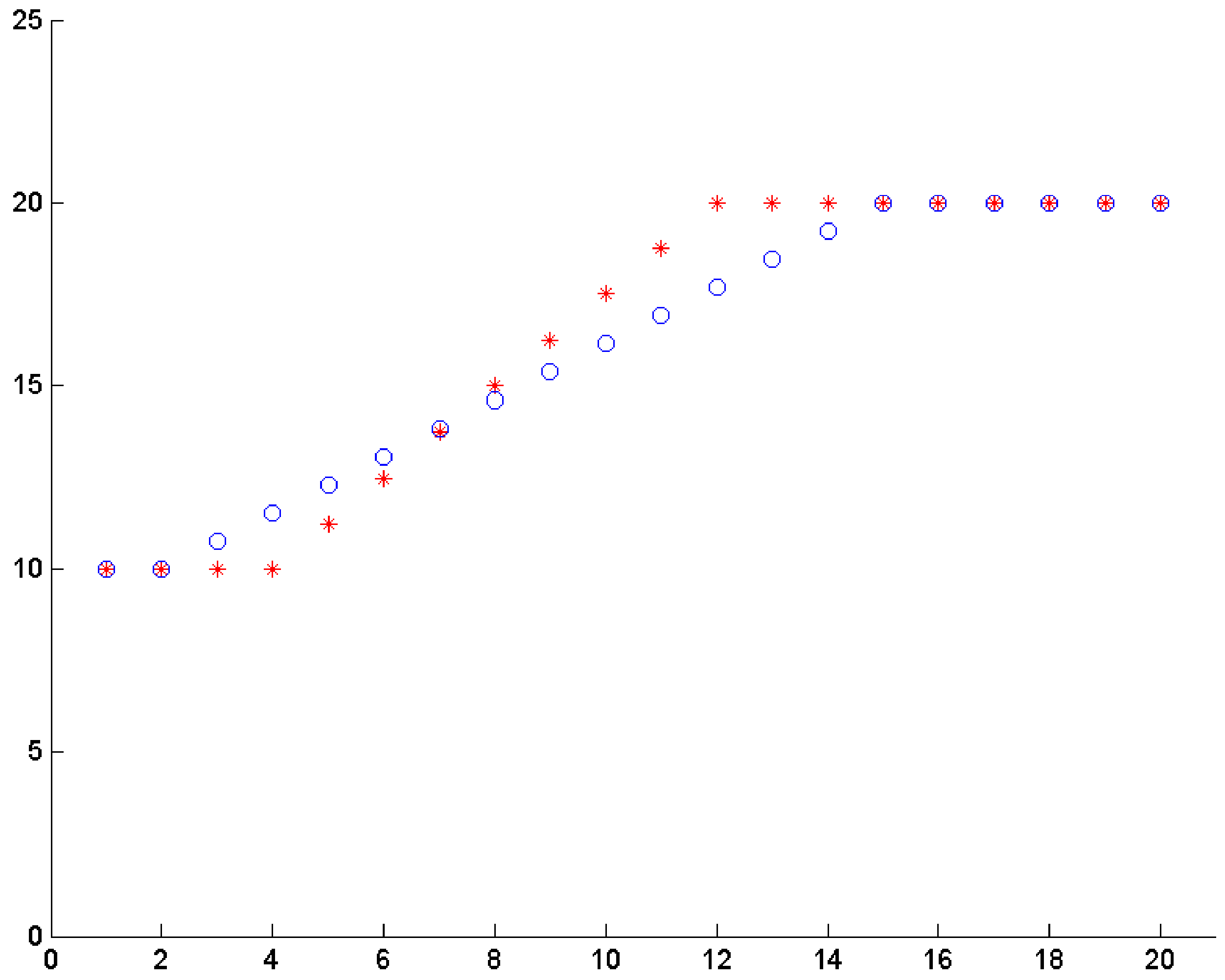


Carry one (horizontal) shift parameter, and estimate this shift by minimizing the corrections to the v-sequence. It is nonlinear so linearize in the usual way and make iterative solution.

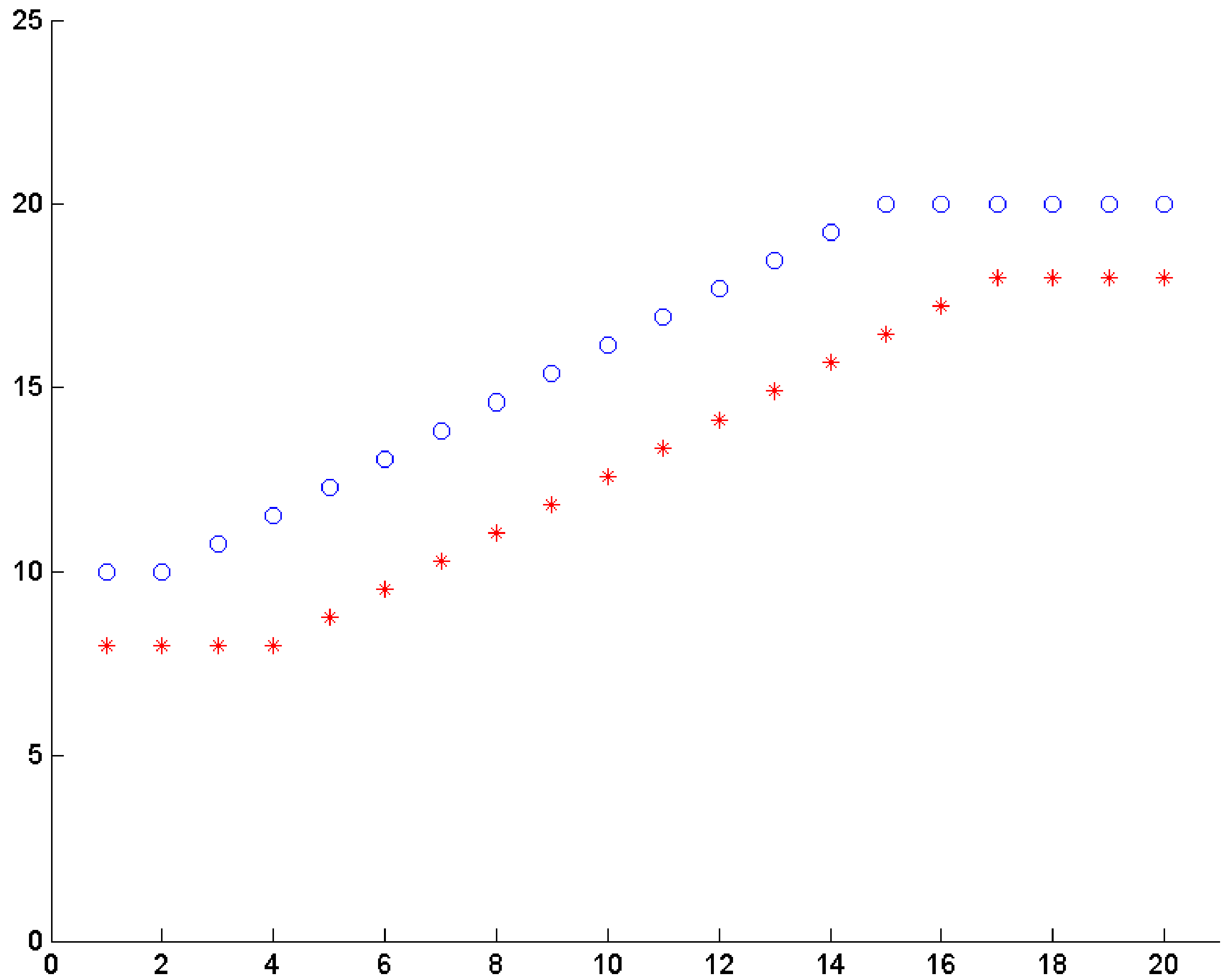
Appproach

- We will carry one horizontal shift parameter
- Could carry also a horizontal scale parameter, a vertical shift parameter, or a vertical scale parameter (see following sketches)
- Will write one condition (observation) equation per pixel
- Think of the horizontal axis as pixel number , and the vertical axis as gray level
- The condition equation will be a nonlinear function of the shift parameter
- Linearize by the usual taylor series approach
- This will require solving for a *correction* rather than the actual *parameter value*
- Two important consequences: (a) it becomes iterative, (b) you need a good approximation, in this case you need to know the correct alignment (shift) to within a "few" pixels
- The 2D case is the exact analog of this 1D tutorial, if you understand this, then the 2D case follows
- Because of the above necessity of good *a priori* knowledge of the alignment (shift), this procedure only works for refinement - not coarse matching
- Good news: the precision of final result can be small fraction (0.01) of a pixel

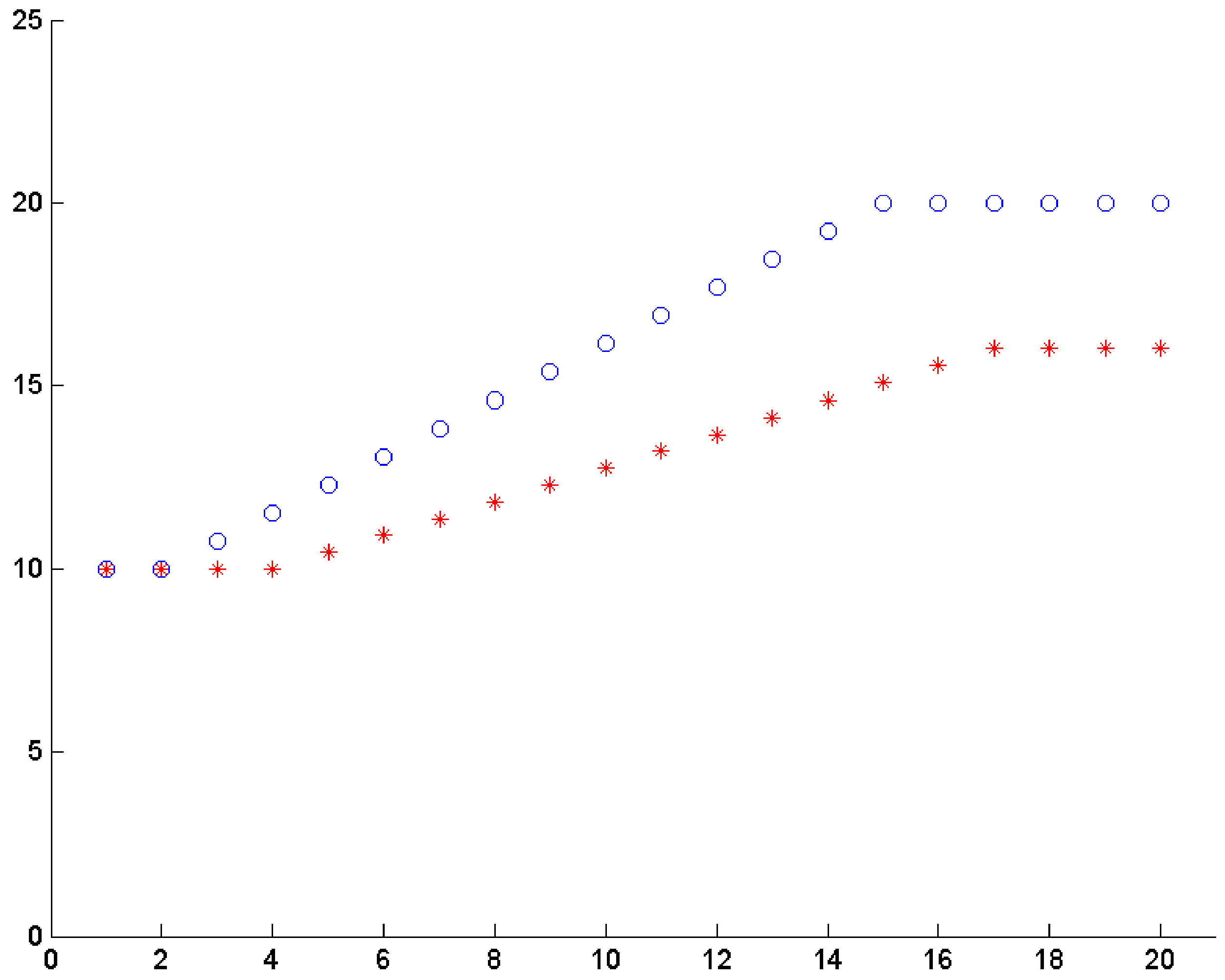
Show Potential Need for Horizontal Scale Parameter



Show Potential Need for Vertical Shift Parameter



Show Potential Need for Vertical Scale Parameter



Condition (Observation) Equation – First Only with Horizontal (Parallax) Shift Parameter

The condition says that, at same location, the left and right intensities are the same

$$u_i = v_i$$

$$u_i - v_i = 0$$

$$F = u_i - v_i = 0$$

Now, introduce a shift parameter

$$F = u_{i+x} - v_i = 0$$

LSM Condition Equation

Linearize the equation using taylor series, at x^0

$$F \approx F^0 + \frac{\partial F}{\partial x}\Delta x = 0$$

$$F \approx \left[u_{i+x^0} - v_i\right] + \frac{\partial F}{\partial x}\Delta x = 0$$

But we always linearize at x = 0, then after estimation we resample so that the alignment gets progressively better and better, and this assumption gets progressively more and more justified

$$F \approx \left[u_i - v_i\right] + \frac{\partial F}{\partial x}\Delta x = 0$$

How de we get derivative? Get it numerically!

LSM Condition Equation

$$\frac{\partial F}{\partial x} \approx \frac{\Delta F}{\Delta x} = \frac{(u_{i+\Delta x} - v_i) - (u_i - v_i)}{\Delta x}$$

$$\frac{\partial F}{\partial x} \approx \frac{u_{i+\Delta x} - u_i}{\Delta x}$$

$$\frac{\partial F}{\partial x} \approx \frac{\Delta u}{\Delta x}$$

call it U_x just the slope of u !

LSM Condition Equation

$$F \approx u_i - v_i + U_x \Delta x = 0$$

rearrange,

$$v_i = U_x \Delta x + u_i$$

add a correction, r_i

$$v_i + r_i = U_{x_i} \Delta x + u_i$$

observation

correction

coefficient

unknown

constant

Least Squares Model

$$r_i - U_{x_i}\Delta x = u_i - v_i$$

same form as LS: $v + B\Delta = f$

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} = \begin{bmatrix} -U_{x_1} \\ -U_{x_2} \\ \vdots \\ -U_{x_n} \end{bmatrix} [\Delta x] = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \\ \vdots \\ u_n - v_n \end{bmatrix}$$

LS solution for indirect observations

$$\Delta = (\mathbf{B}^{\mathbf{T}}\mathbf{W}\mathbf{B})^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{W}\mathbf{f}$$

W is a weight matrix which may be identity