Steps for Observations Only –Longhand with Lagrange Mult.

1. Analyze problem (n,n_0,r)
2. Write r condition equations among the $\hat{l}_i$
3. Plug in numbers for the l's, so that remaining unknowns are the v's
4. Convert knowledge about observation uncertainty into weight, $w_i = \frac{s_0^2}{s_i^2}$
5. Create the augmented objective funtion,

$$\Phi' = \sum_n w_i v_i^2 + \sum_r k_j (f_j(v_1, v_2, \ldots, v_n))$$

6. Minimize the objective function by differentiating with respect to v's and k's, and setting equal to zero. This yields system of equations (n+r) x (n+r)
    1. Solve above system directly for v's and k's, or
    2. Solve for each v in terms of k's, plug into cond. Eqns., solve for k's, then solve for the v's.
7. You do not actually need the k's (the lagrange multipliers), use v's to get adjusted observations,

$$\hat{l}_i = l_i + v_i$$

Steps for Observations Only – Matrix Method

1. Analyze problem (n, n_0, r)
2. Write r condition equations among the $\hat{l}_i$
3. Extract coefficients for matrix form, $\mathbf{Av} = \mathbf{f}$, or $\mathbf{Av} = \mathbf{d} - \mathbf{Al}$
4. Compute weights and insert into weight matrix **W**
5. Build full normal equations,
$$\begin{bmatrix} -\mathbf{W} & \mathbf{A}^{\mathrm{T}} \\ \mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{k} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{f} \end{bmatrix}$$
6. Solve system for the **v,k** unknown vector
7. Obtain the adusted observations, $\hat{\mathbf{l}} = \mathbf{l} + \mathbf{v}$
8. Confirm that adjusted observations satisfy the condition equations

Steps for Indirect Observations – Longhand Method

1. Analyze problem (n, n_0, r)
2. Obtain weights from uncertainty for each observation
3. Select n_0 parameters, which define the model and are independent (they can be new quantities, or they can be observed quantities, but choose new variable names to avoid confusion)
4. Write n condition equations (one per observation) of the form,

$$\hat{l}_i = f_i(x_1, x_2, \ldots, x_u), \text{ or}$$

$$v_i = f_i(x_1, x_2, \ldots, x_u) - l_i$$

5. Plug each expression for v into the objective function,
6. Differentiate the objective function with respect to each of the x_i and set = 0
7. Solve that system of u x u ($u=n_0$) equations for the x's, then plug into equations of step 4 to obtain the v's, etc.

Steps for Indirect Observations – Matrix Method

1. Analyze problem (n, n_0, r)
2. Obtain weights from observation uncertainty, put into **W**
3. Select n_0 parameters which define the model and are independent
4. Write n condition equations, one per observation, of the form

$$v_i - f_i(x_1, x_2, \ldots, x_u) = -l_i$$

5. Extract coefficients for matrix form, $\mathbf{v} + \mathbf{B}\Delta = \mathbf{f} = \mathbf{d} - \mathbf{l}$
6. Construct and solve normal equations, and compute residuals and adjusted observations

$$\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{B}\Delta = \mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{f}$$

$$\Delta = \left(\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{f}$$

$$\mathbf{v} = \mathbf{f} - \mathbf{B}\Delta$$

$$\hat{\mathbf{l}} = \mathbf{l} + \mathbf{v}$$