Least Squares Estimation – Progression of Techniques

Linear models

      Method of observations only

            Longhand with elimination

            Inclusion of weights

            Longhand using lagrange multipliers

            Matrix methods

      Method of indirect observations

            Longhand using weights

            Matrix methods

Nonlinear models

      Method of observations only

      Method of indirect observations

      Method of general least squares (mixed model)

      Linearization issues

      iterations and termination

Extras (?) sequential LS (kalman filter), constraints, robust methods

Steps for Observations Only – Longhand with Elimination

1. Analyze problem (n, n_0, r)

2. Write r condition equations among the $\hat{l}_i$

3. Plug in numbers for the l's so that the remaining unknowns are the v's

4. Use each condition equation to solve for a v_i. (n v's – r v's = n_0 v's) The one solved for will be eliminated when substituted in the next step. The n_0 observations, corresponding to the v's that remain, must define the model.

5. Form $\Phi = \sum v_i^2$ substituting as mentioned above, leaving n_0 unknowns

6. By calculus, minimize quadratic form by setting derivatives equal to zero, and solve

7. $d\Phi/dv_i = 0$ should be n_0 equations in n_0 unknowns

8. Solve the n_0 by n_0 system using MATLAB to yield n_0 v's

9. Use the condition equations to solve for the other r v's

10. Compute the adjusted observations $\hat{l}_i = l_i + v_i$

11. Confirm that the adjusted observations satisfy the condition equations.

12. You are done!