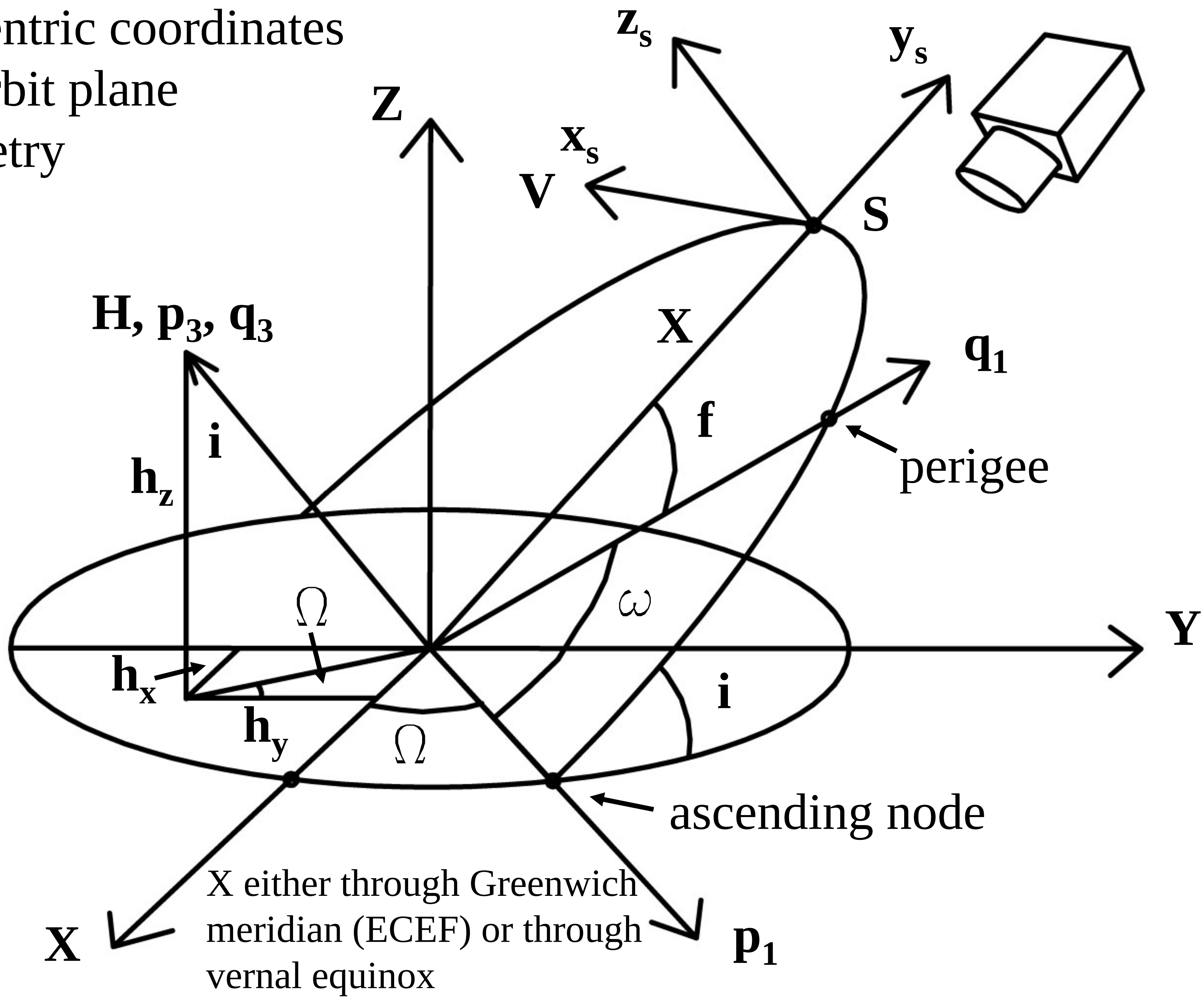
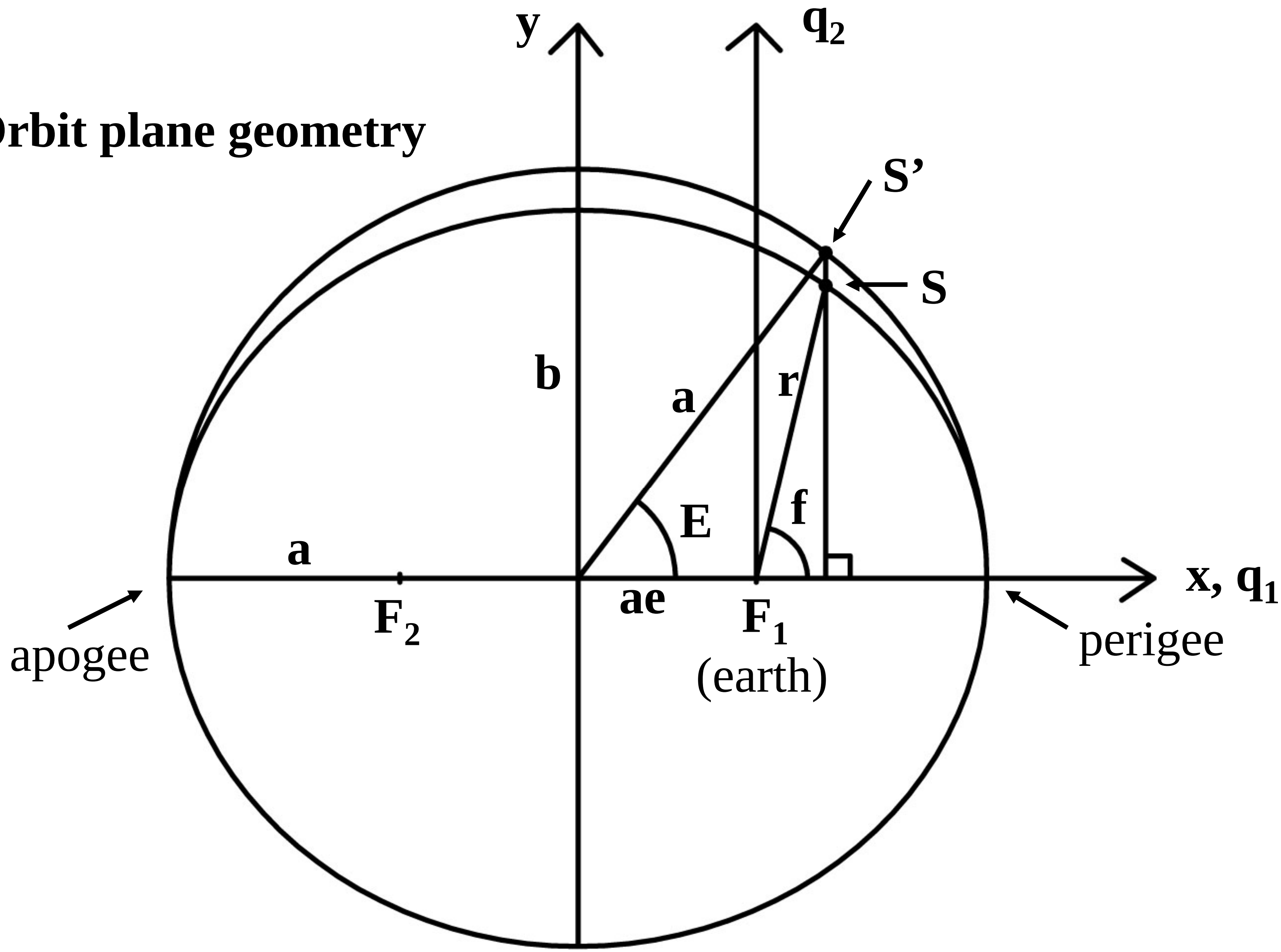
CE 603 Photogrammetry II

Geocentric coordinates and orbit plane geometry



Orbit plane geometry



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Conversion from Position & Velocity to Kepler Elements (Ref: See A. Leick, GPS Satellite Surveying)

$$\mu = 3.986005E+05$$

Given $\mathbf{X}$ and $\mathbf{V}$, at a given time

$$r = |\mathbf{X}|$$

$$v = |\mathbf{V}|$$

$$\mathbf{H} = \mathbf{X} \times \mathbf{V}$$

$$\mathbf{H} = \begin{bmatrix} h_x \\ h_y \\ h \end{bmatrix}$$

$$h = |\mathbf{H}|$$

$$\Omega = \tan^{-1}\left(\frac{h_x}{-h_y}\right)$$

$$i = \tan^{-1}\left(\frac{\sqrt{h_x^2 + h_y^2}}{h_z}\right)$$

$$R_1(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

$$R_3(\Omega) = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P} = R_1(i) R_3(\Omega) \mathbf{X} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

$$\omega + f = \tan^{-1}\left(\frac{p_2}{p_1}\right)$$

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$$a = \frac{r}{2 - \left(rv^2 / \mu \right)}$$

$$e = \sqrt{1 - h^2 / \mu a}$$

$$rv_r = \mathbf{X} \cdot \mathbf{V}$$

$$\sin E = \frac{rv_r}{e\sqrt{\mu a}}$$

$$\cos E = \frac{(a - r)}{ae}$$

$$f = \tan^{-1}\left(\frac{\sqrt{1-e^2}\sin E}{\cos E - e} \right)$$

$$E = \tan^{-1}\left(\frac{\sin E}{\cos E} \right)$$

$$M = E - e\sin E$$

$$\omega = (\omega + f) - f$$

so now we have,

Ω, i, ω, f, a, e

Based on the time associated with each ephemeris point, we can estimate/interpolate the values at the frame center.

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Conversion from kepler elements to **X, V**

$$b^2 = a^2\left(1 - e^2\right)$$

$$b = a\sqrt{1 - e^2}$$

$$x = a\cos E$$

$$q_1 = x - ae$$

$$q_1 = a\cos E - ae = a\left(\cos E - e\right)$$