Eight Parameter Transformation - Ground XY (plane) to image

$$r = \frac{a_0 + a_1 X + a_2 Y}{1 + c_1 X + c_2 Y}$$

$$c = \frac{b_0 + b_1 X + b_2 Y}{1 + c_1 X + c_2 Y}$$

Equations for application of the transformation, ground XY to r,c or row, column in the image. To insure numerical stability normalize all 4 coordinates before doing matrix computations. See note below.

$$r + c_1 rX + c_2 rY = a_0 + a_1 X + a_2 Y$$
$$c + c_1 cX + c_2 cY = b_0 + b_1 X + b_2 Y$$

Rearrange, multiply by denominator

$$r = a_0 + a_1 X + a_2 Y - c_1 rX - c_2 rY$$
$$c = b_0 + b_1 X + b_2 Y - c_1 cX - c_2 cY$$

make into pseudo-linear equations for ease of computation

Expressed in Matrix-Vector Form

$$\begin{bmatrix} r \\ c \end{bmatrix} = \begin{bmatrix} 1 & X & Y & 0 & 0 & 0 & -rX & -rY \\ 0 & 0 & 0 & 1 & X & Y & -cX & -cY \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix}$$

Matrix-vector equation for 1 point

$$\begin{bmatrix} r_1 \\ c_1 \\ r_2 \\ c_2 \\ \vdots \\ r_n \\ c_n \end{bmatrix} = \begin{bmatrix} 1 & X_1 & Y_1 & 0 & 0 & 0 & -r_1 X_1 & -r_1 Y_1 \\ 0 & 0 & 0 & 1 & X_1 & Y_1 & -c_1 X_1 & -c_1 Y_1 \\ 1 & X_2 & Y_2 & 0 & 0 & 0 & -r_2 X_2 & -r_2 Y_2 \\ 0 & 0 & 0 & 1 & X_2 & Y_2 & -c_2 X_2 & -c_2 Y_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & X_n & Y_n & 0 & 0 & 0 & -r_n X_n & -r_n Y_n \\ 0 & 0 & 0 & 1 & X_n & Y_n & -c_n X_n & -c_n Y_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix}$$

Matrix-vector equation for n points

The ground to image transformation should be setup and solved in the order

(r,c) <= (X,Y), X,Y: Ground, r,c: image

As with prior transformation, the matrix form of the equations and the LS solution can be expressed as,

$$\mathbf{Bx} = \mathbf{f}$$

$$\mathbf{W} \text{ (weight matrix)}$$

$$\mathbf{x} = \left(\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{W}\mathbf{f}$$

$$\mathbf{v} = \mathbf{f} - \mathbf{Bx}$$

Normalize all of the coordinates prior to matrix computations. Scale up the residuals for proper interpretation. When applying the transformation prior to interpolation, be sure to undo the normalization to get actual row and column. Generic normalization computations shown below. Normalization puts into range [-1, +1].

$$X' = \left(X - mean(X)\right) / \left(0.5 \times range(X)\right)$$

$$X = X' \times \left(0.5 \times range(X)\right) + mean(X)$$