Photo 1, Fall 2009 EXAM 1

16 Oct 09, One sheet of notes allowed.

Name ______________________

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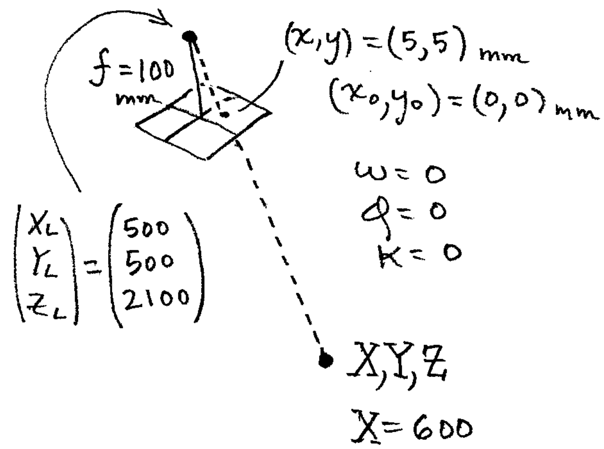
1\.

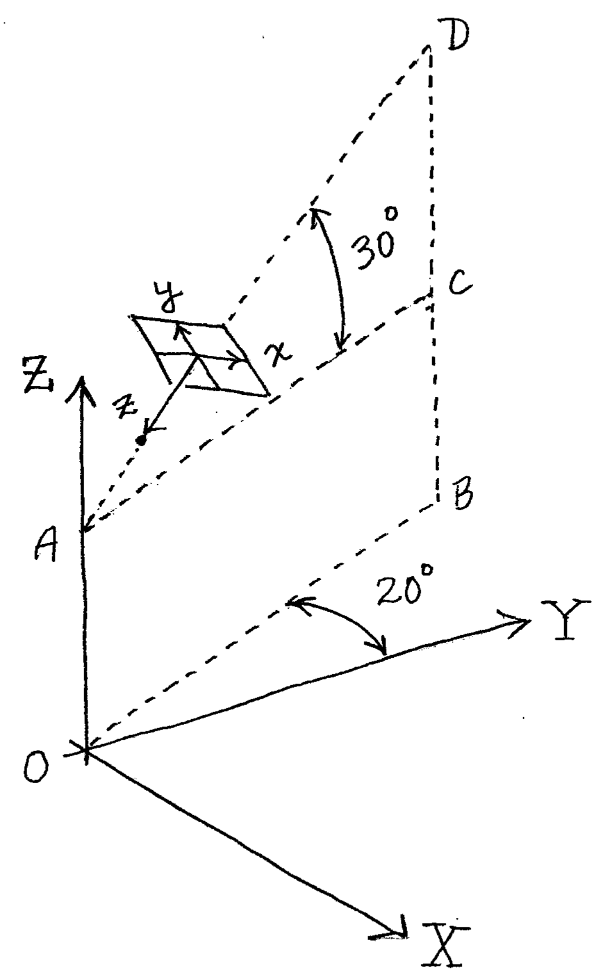


Given the camera interior and exterior orientation, and the image coordinates of a point, and the X coordinate of the point, what are Y and Z ?

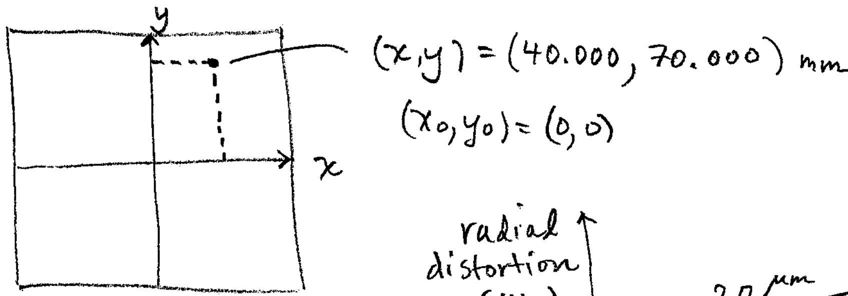
2\. The figure shows the angle relationship between the object space axes XYZ and the image space axes xyz. Show how you would construct M by <u>sequential</u> <u>rotations</u>, where M is defined by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

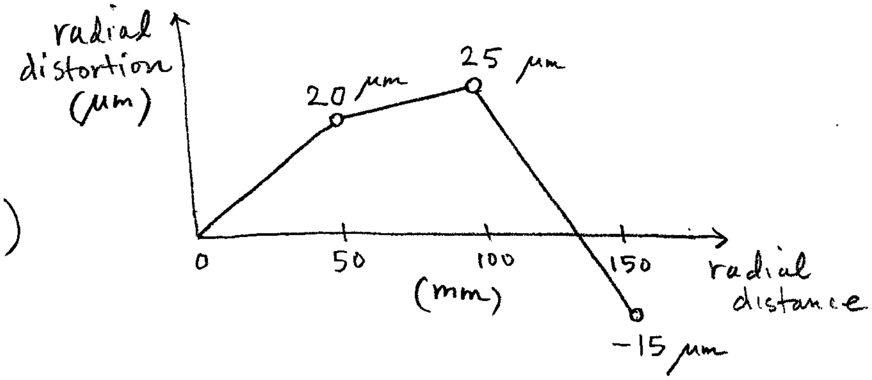
show the numerical values for the elements of M.



3. Correct the given image coordinates for <u>lens</u> <u>distortion</u> as defined by the graph.



$(1\ \mu m = 0.001\ mm)$



4. (a) What <u>condition</u> in <u>object</u> <u>space</u> makes the 8-parameter transformation a valid model for relating object points and image points for a frame camera ?

(b) In Snell's Law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, what do the n_i represent ?

Photo1, Exam 1, Solution   19 Oct 2009

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1. collinearity: $\begin{bmatrix} x-x_0 \\ y-y_0 \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{bmatrix}$ $\omega=0, \varphi=0, \kappa=0 \Rightarrow M = I_3$
3 equations, 3 unknowns

$$\begin{bmatrix} 5 \\ 5 \\ -100 \end{bmatrix} = \lambda \begin{bmatrix} 600-500 \\ Y-500 \\ Z-2100 \end{bmatrix}$$

use 1st equation to solve for λ:
$5 = \lambda \cdot 100$, $\lambda = 0.05$
now solve for Y and Z,

$$5 = 0.05 \cdot (Y-500)$$
$$\frac{5}{0.05} = Y - 500$$
$$Y = 500 + 5/.05 = 500 + 100 = \underline{600}$$

$$-100 = 0.05 \cdot (Z - 2100)$$
$$-\frac{100}{0.05} = Z - 2100$$
$$Z = 2100 + \frac{-100}{0.05} = 2100 - 2000 = \underline{100}$$

OR,

$$M^T \begin{bmatrix} x-x_0 \\ y-y_0 \\ -f \end{bmatrix} = \lambda \begin{bmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{bmatrix} \Rightarrow I \begin{bmatrix} 5 \\ 5 \\ -100 \end{bmatrix} = \lambda \begin{bmatrix} 600-500 \\ Y-500 \\ Z-2100 \end{bmatrix}$$

divide equations 2 & 3 by equation 1:

$$\frac{5}{5} = \frac{Y-500}{100}, \quad 100 = Y - 500, \quad Y = 100 + 500 = \underline{600}$$

$$\frac{-100}{5} = \frac{Z-2100}{100}, \quad -2000 = Z - 2100, \quad Z = -2000 + 2100 = \underline{100}$$

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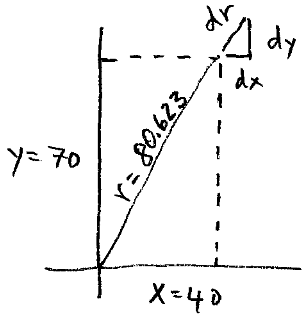
2. 1st rotation: $M_Z(+20°) = \begin{bmatrix} \cos(20) & \sin(20) & 0 \\ -\sin(20) & \cos(20) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .940 & .342 & 0 \\ -.342 & .940 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

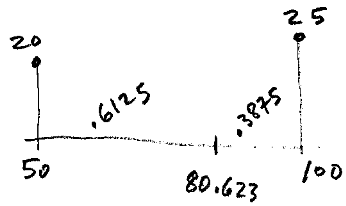
2nd rotation: $M_X(90°+30°) = M_X(120°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(120) & \sin(120) \\ 0 & -\sin(120) & \cos(120) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -.5 & .866 \\ 0 & -.866 & -.5 \end{bmatrix}$

$$M = M_X(120°)\, M_Z(20°)$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -.5 & .866 \\ 0 & -.866 & -.5 \end{bmatrix} \begin{bmatrix} .940 & .342 & 0 \\ -.342 & .940 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} .940 & .342 & 0 \\ .171 & -.470 & .866 \\ .296 & -.814 & -.5 \end{bmatrix}$$

3.



interpolate (linearly) from distortion graph



$.6125 \times 25 + .3875 \times 20 = 23.06$

$\approx 23\,\mu m$

$\frac{x}{r} = \frac{dx}{dr}$, $dx = dr \cdot \frac{x}{r}$, $dx = .023 \times \frac{40}{80.623} = 0.011$

$\frac{y}{r} = \frac{dy}{dr}$, $dy = dr \cdot y/r$, $dy = .023 \times \frac{70}{80.623} = 0.020$

distortion is positive (outward), correction is negative (inward)

$x_{refined} = 40.000 - 0.011 = \underline{39.989 \text{ mm}}$

y refined $= 70.000 - 0.020 = \underline{69.980 \text{ mm}}$

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4. (a) object points must lie in a <u>plane</u>.

(b) n_i are refractive indices of the 2 media on either side of the interface. $n_i = c/v_i$.