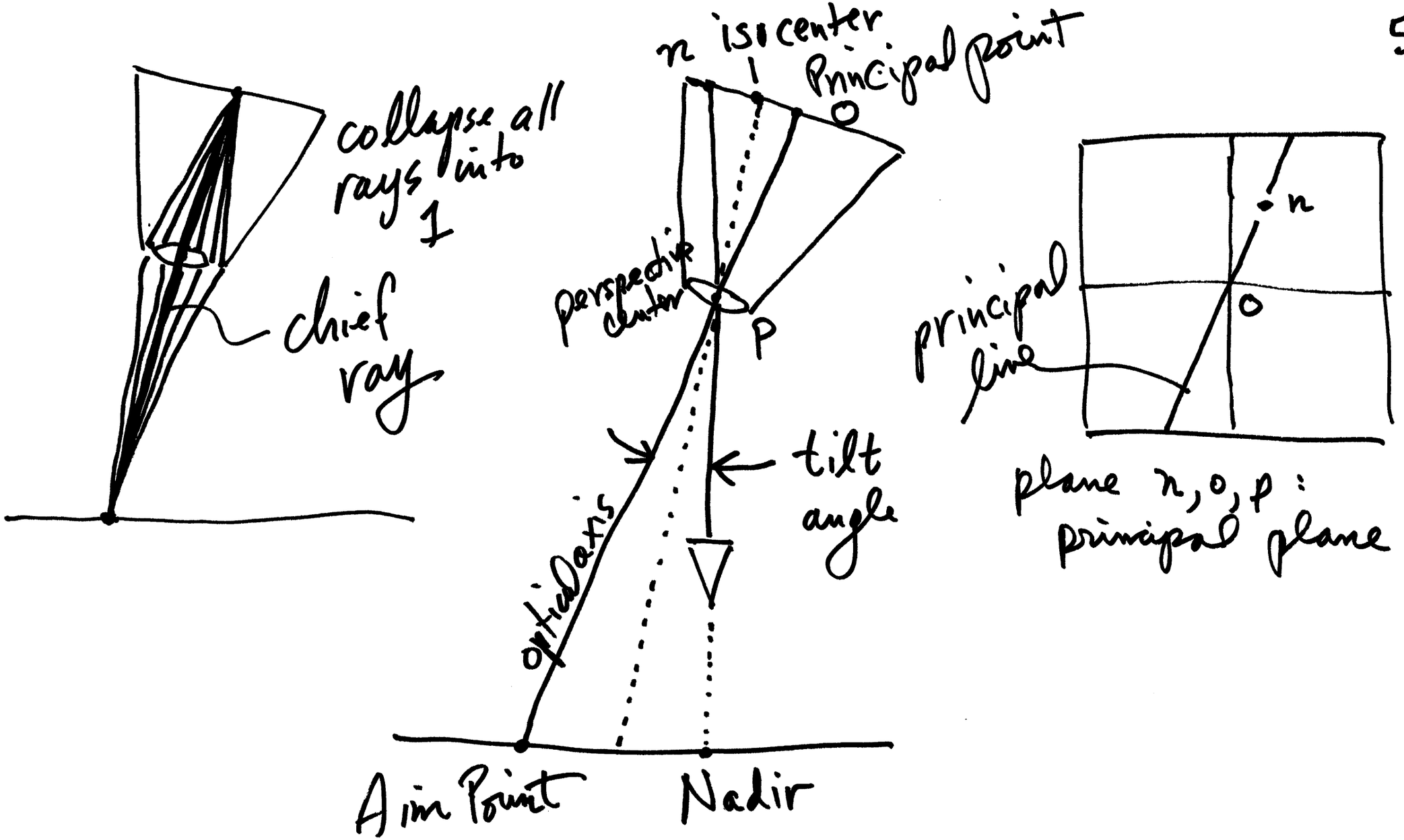
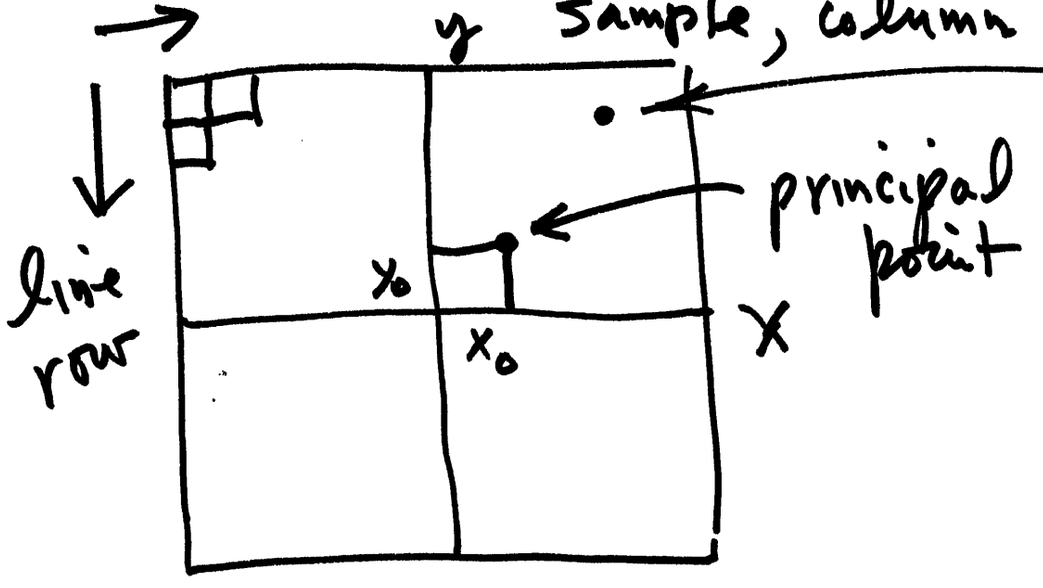
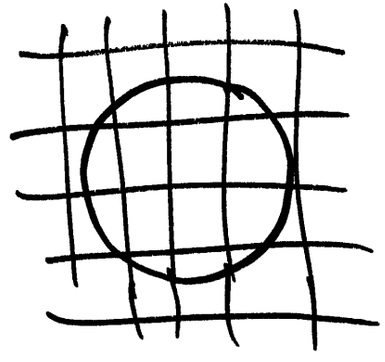

collapse all rays into 1
chief ray
n isocenter
1
Principal point
o
perspective center
p
principal line
n
o
plane n, o, p : principal plane
optical axis
tilt angle
Aim Point
Nadir



$(x-x_0, y-y_0)$



2D image coordinates

(l, s)

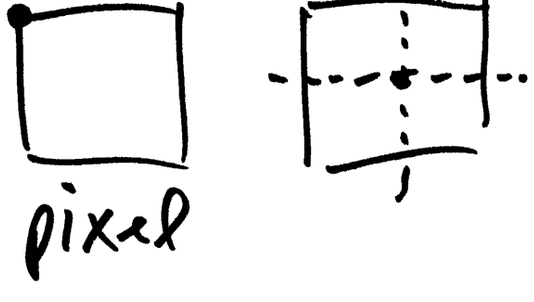
raw image measurements

↓

refined image coordinates (sensor)

digital imagery origin is upper left

0,0 1,1 choose + be consistent



(l, s)

(r, c)

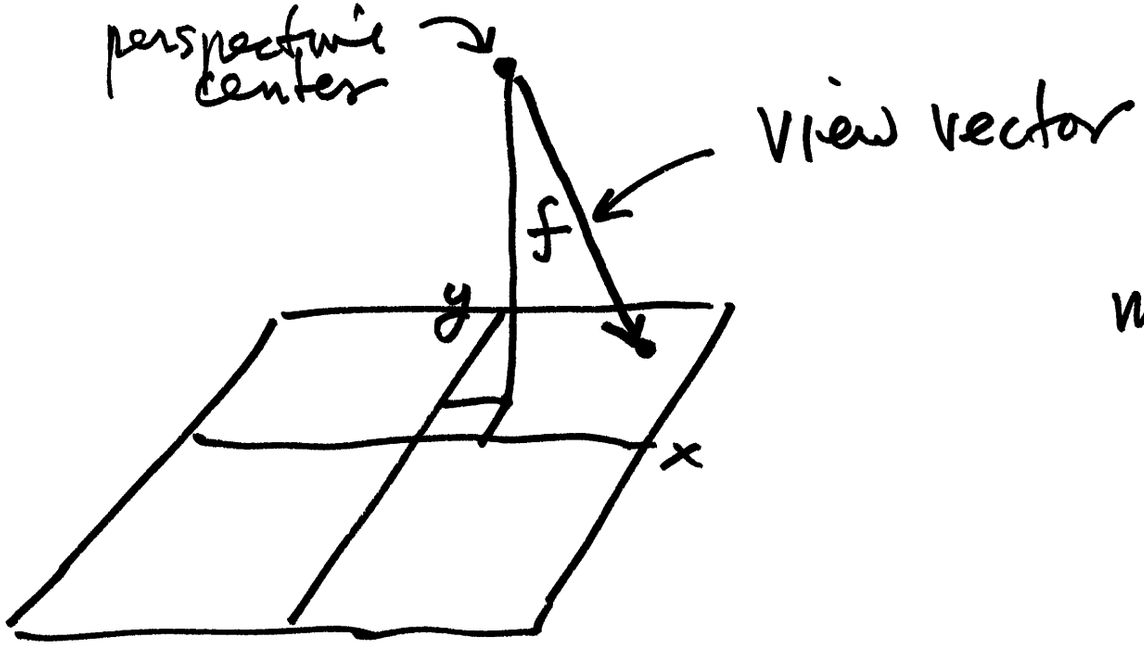
photoshop (x, y)

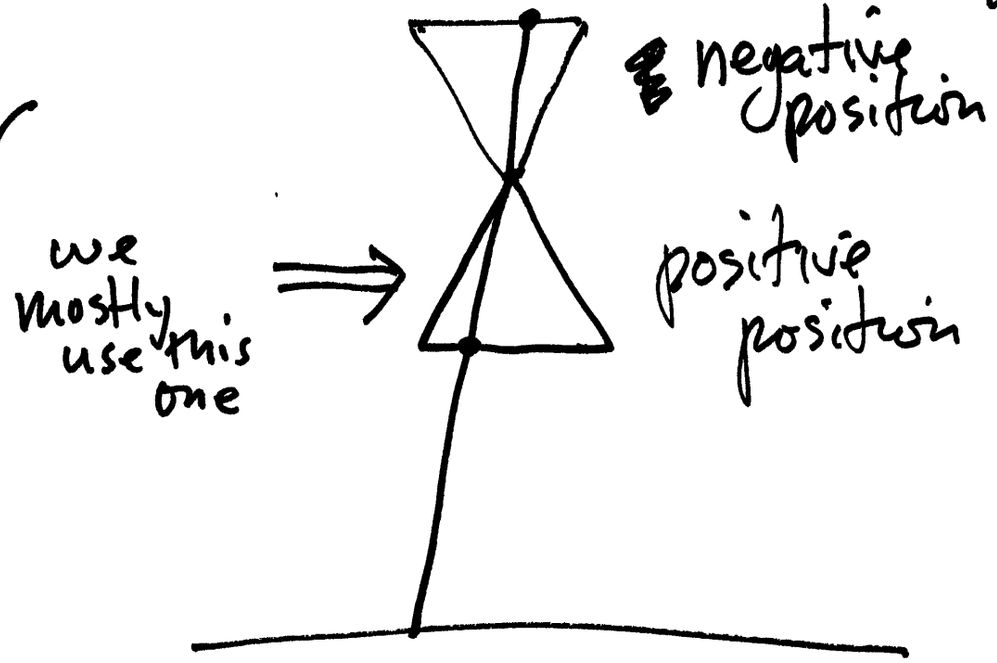
$x \rightarrow$

$y \downarrow$

(l,s) (r,c) (x,y)

are all right-handed coord sys.





$$\begin{bmatrix} X - X_0 \\ Y - Y_0 \\ -f \end{bmatrix}$$: 3D sensor coordinates

better phrase for "focal length" is "principal distance" in case we have to focus close — to satisfy

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

4D vector : computer graphics, computer vision

4×4 transformation matrices

4×1 vectors

$\begin{pmatrix} x \\ y \end{pmatrix}$ choose w, $x' = xw$, $y' = y \cdot w$

$\begin{pmatrix} x' \\ y' \\ w \end{pmatrix}$ homogeneous coordinates $\rightarrow$ $x = x'/w$, $y = y'/w$

shift, rotation, scale change

Shift

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix}$$

$$x'' = x' + t_x \cdot w$$
$$y'' = y' + t_y \cdot w$$
$$z'' = z' + t_z \cdot w$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ w \end{bmatrix} = \begin{bmatrix} M_{3\times3} & & 0 \\ & & 0 \\ & & 0 \\ 0 & 0 & 0 \quad 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix}$$

$$x'' = M_{11}x' + M_{12}y' + M_{13}z'$$

Rotation

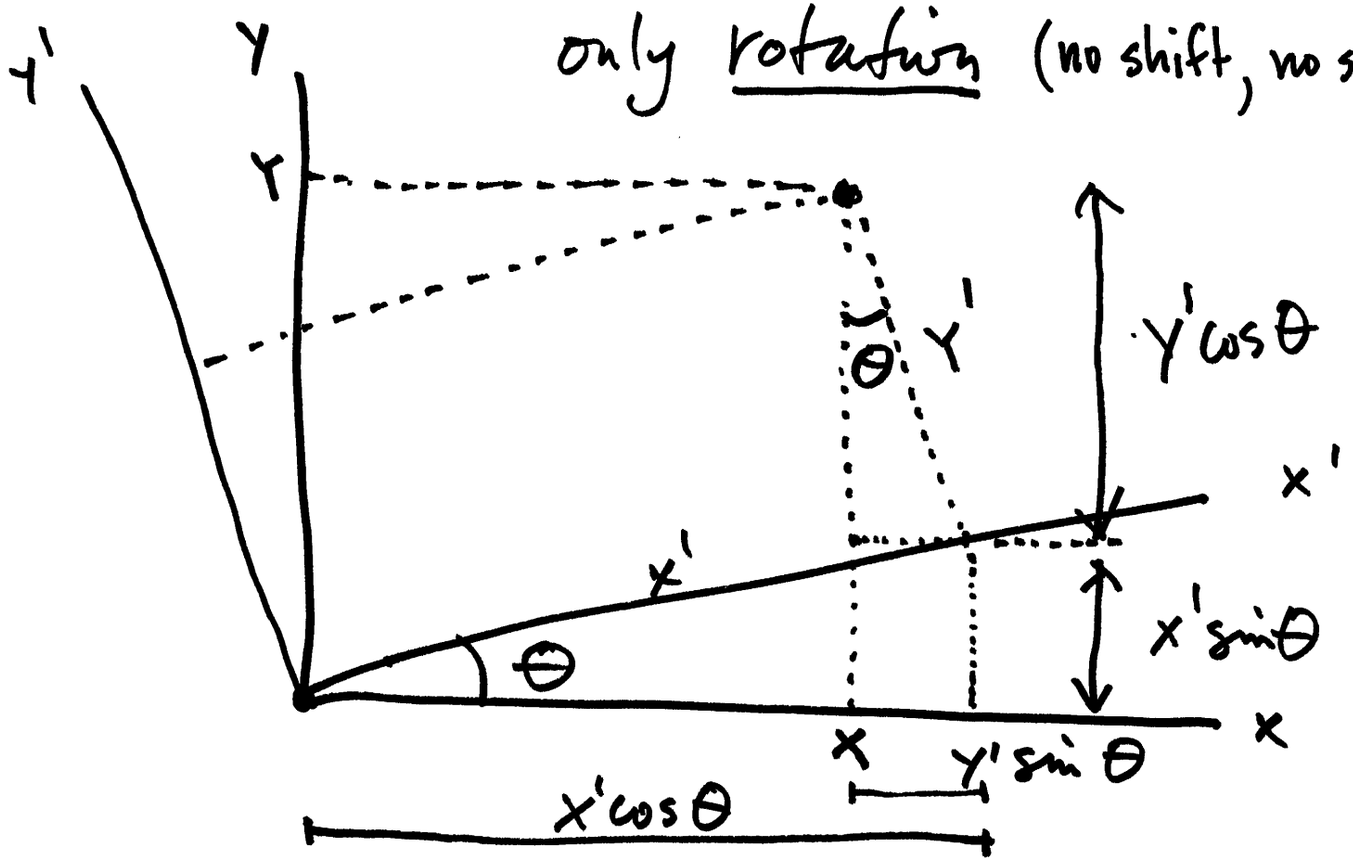
Scale Change

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ w \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix}$$

$$x'' = S_x \cdot x'$$
$$\vdots$$

graphics hardware and function libraries are often optimized for multiple 4x4 matrix multiplications - This is the motivation for using homogeneous coordinates

only rotation (no shift, no scale)



$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}}$$

$X = x'\cos\theta - y'\sin\theta$ fingers curl in

$Y = x'\sin\theta + y'\cos\theta$ ↗ + Dir.

Right Hand Rule: thumb along +z

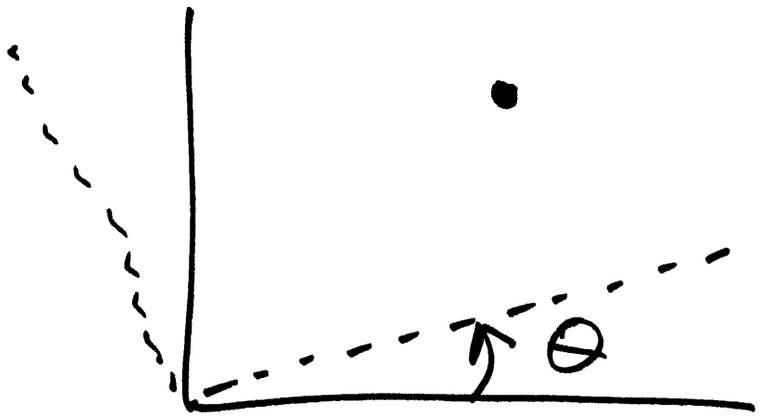
$u = Av$

$A^{-1}u = A^{-1}Av$

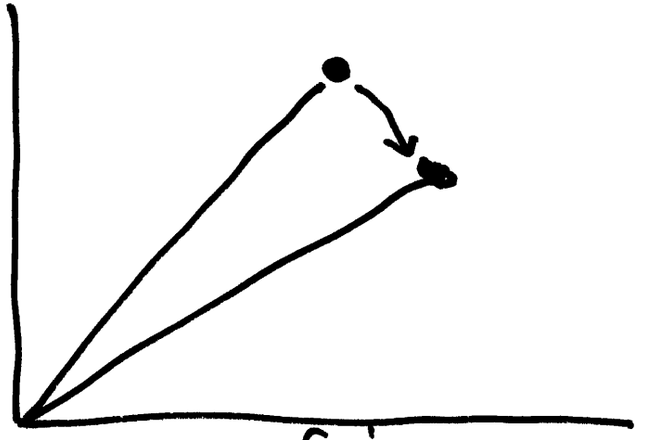
$A^{-1}u = v$, but $A^{-1} = A^T$ if orthonal

$\Rightarrow v = A^T u$

2 interpretations (+θ rotation)



point fixed
axes rotate CCW



axes fixed
point rotates CW

☆☆☆ we mostly use this one.