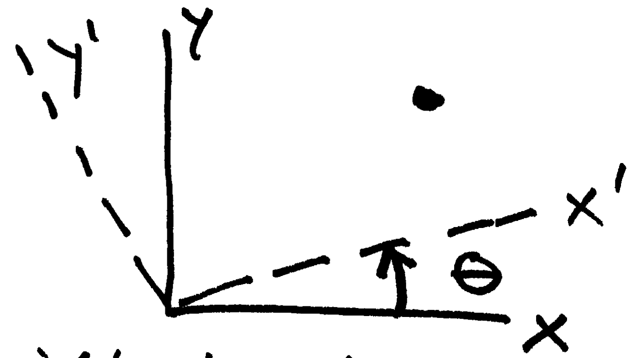
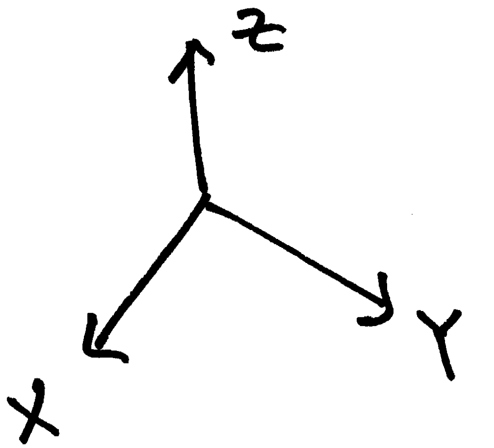
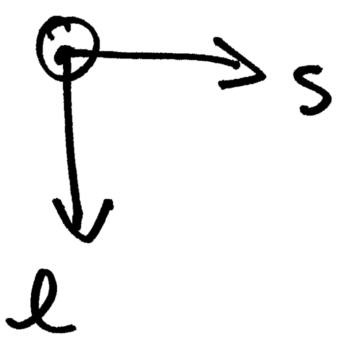
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



preferred interpretation

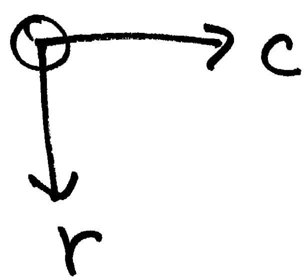


curl fingers of right hand from $+X \to +Y$ then thumb points along $+Z$ if Right Handed



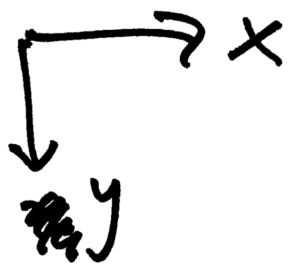
$\begin{pmatrix} l \\ s \end{pmatrix}$ (l, s)

Right Handed



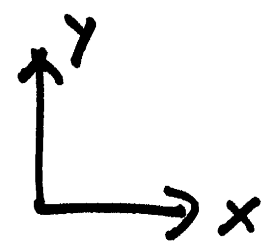
$\begin{pmatrix} r \\ c \end{pmatrix}$ (r, c)

Right Handed



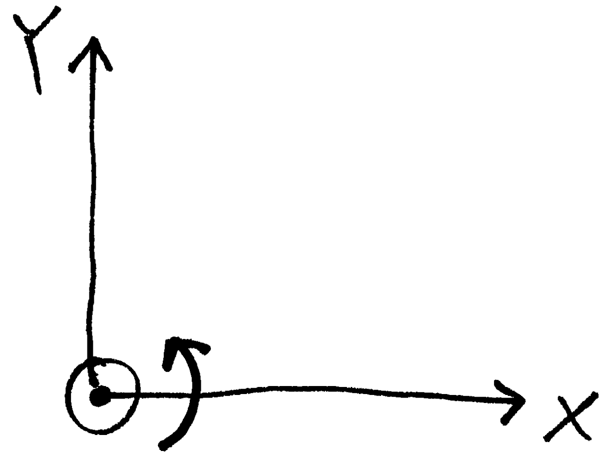
Left Handed

$\begin{pmatrix} x \\ y \end{pmatrix}$ (x, y)



$\begin{pmatrix} x \\ y \end{pmatrix}$ (x, y)

Right Handed



κ, Kappa

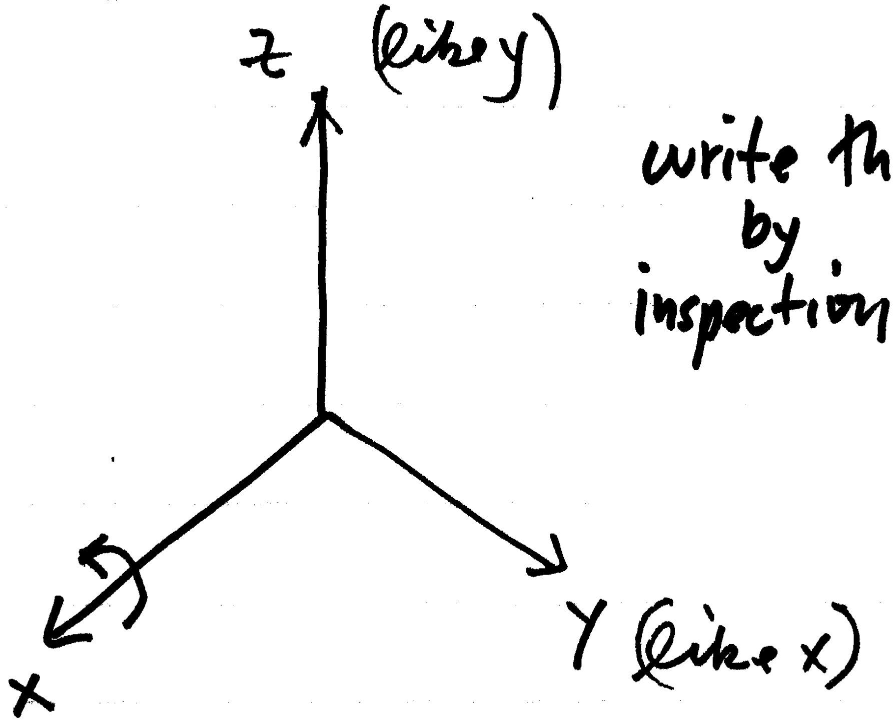
$$\begin{pmatrix}x'\\ y'\\ z'\end{pmatrix} = \underbrace{\begin{bmatrix}\cos\kappa & \sin\kappa & 0\\ -\sin\kappa & \cos\kappa & 0\\ 0 & 0 & 1\end{bmatrix}}_{M_\kappa,\ M_z,\ R_3}\begin{pmatrix}X\\ Y\\ Z\end{pmatrix}$$

Elementary Rotation about Z

ω (x)

φ (y)

κ (z)



ω - omega

write this by inspection

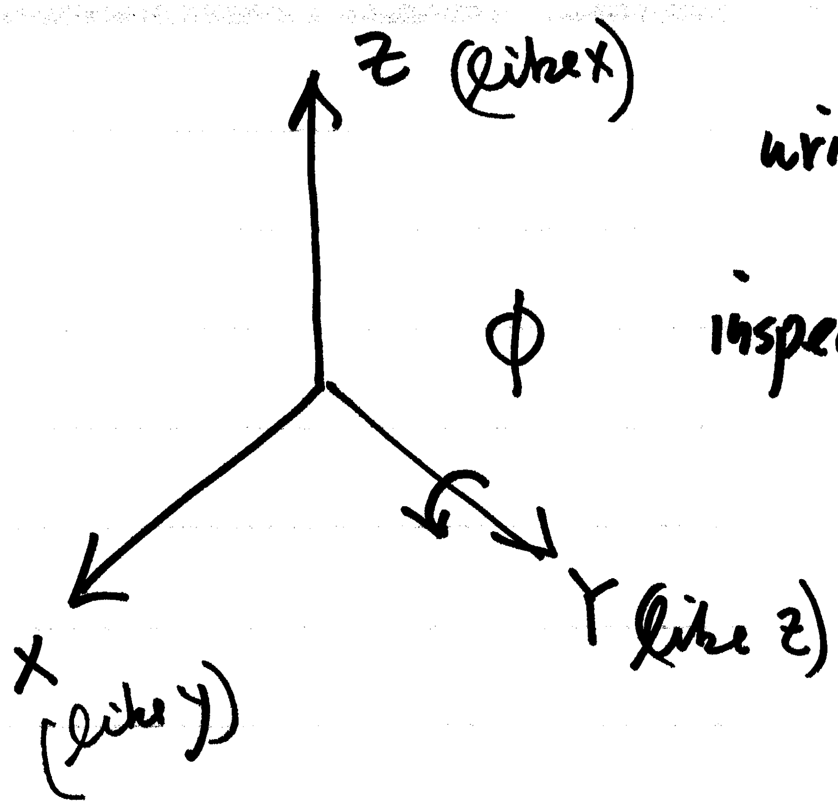
$$\begin{pmatrix} Y' \\ Z' \\ X' \end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Y \\ Z \\ X \end{pmatrix}$$

$$\begin{pmatrix} Y' \\ Z' \\ X' \end{pmatrix} = \begin{bmatrix} 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \\ 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

M_ω, M_x, R_1

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Elementary Rotation about X



write this by inspection

$$\begin{bmatrix} Z' \\ X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ X \\ Y \end{bmatrix}$$

$$\begin{bmatrix} Z' \\ X' \\ Y' \end{bmatrix} = \begin{bmatrix} \sin\phi & 0 & \cos\phi \\ \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

M_ϕ, M_y, R_z

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Elementary Rotation about Y

$$\underset{3,3}{M} = \begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix}$$

$$\underset{3,3}{M} = \underset{3,3}{M_\kappa}\;\underset{3,3}{M_\phi}\;\underset{3,3}{M_\omega}$$

Product (symbollic) of $M_\kappa \cdot M_\phi \cdot M_\omega$

$$M_1 = M_\kappa M_\phi M_\omega$$

$$M_1 \neq M_\phi M_\omega M_\kappa$$

$$M = M_\kappa M_\omega M_\kappa$$

Order is important in sequential rotations. For given ω, ϕ, κ

$$M_\kappa M_\phi M_\omega \neq M_\phi M_\omega M_\kappa$$

etc.

The third expression is a good way to form M for oblique images – example later.

$$M = \underset{\text{Tertiary}}{M_\kappa}\ \underset{\text{Secondary}}{M_\phi}\ \underset{\text{Primary}}{M_\omega}$$

M_ω is primary in this case since, when we do matrix multiplication to a vector on the right, M_ω is the first to be multiplied

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

(to) (from)

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

columns of rotation matrix are coordinates in the "to" system of unit basis vectors in "from" system*

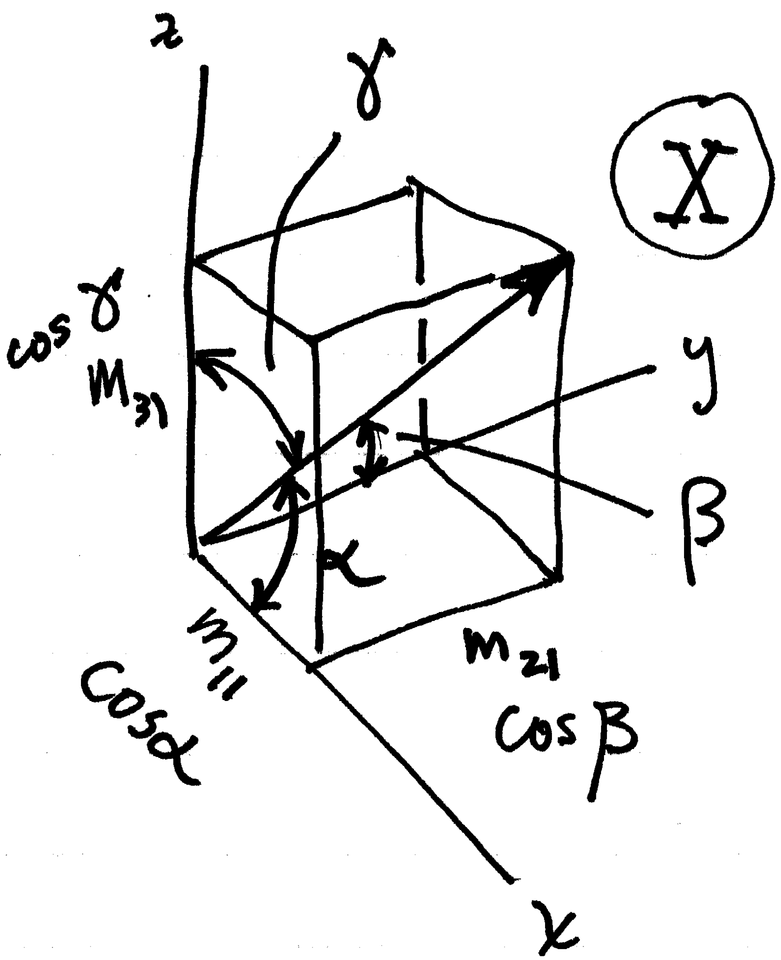
$$\begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \end{bmatrix} = M \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} m_{12} \\ m_{22} \\ m_{32} \end{bmatrix} = M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \dots \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

coordinates in "to" system

unit basis vectors in "from" system

* long-winded, pedantic statement of the obvious.

direction cosines



$$\begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix} : \begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix}$$

$$\begin{bmatrix} \cos X_x & \cos Y_x & \cos Z_x \\ \cos X_y & \cos Y_y & \cos Z_y \\ \cos X_z & \cos Y_z & \cos Z_z \end{bmatrix}$$

Y, Z axes not shown for clarity of figure
principle is same

algebraic parameters a, b, c, d

$$M = \begin{bmatrix} d^2+a^2-b^2-c^2 & 2(ab+cd) & 2(ac-bd) \\ 2(ab-cd) & d^2-a^2+b^2-c^2 & 2(bc+ad) \\ 2(ac+bd) & 2(bc-ad) & d^2-a^2-b^2+c^2 \end{bmatrix}$$

no physical interpretation for a,b,c,d

$\Rightarrow$ difficult to find initial approximations

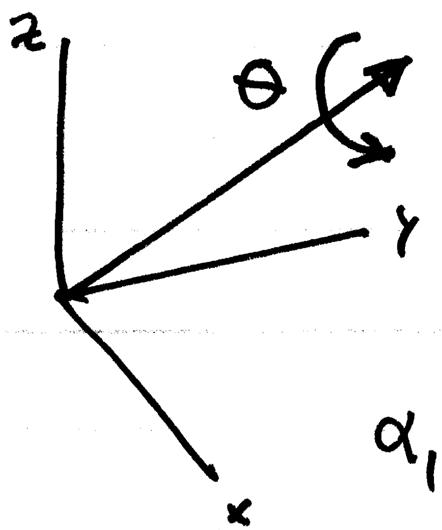
$$a^2+b^2+c^2+d^2=1$$

advantage: no singularities as you <u>always</u> have with sequential rotations (= euler angles)

we come back to this

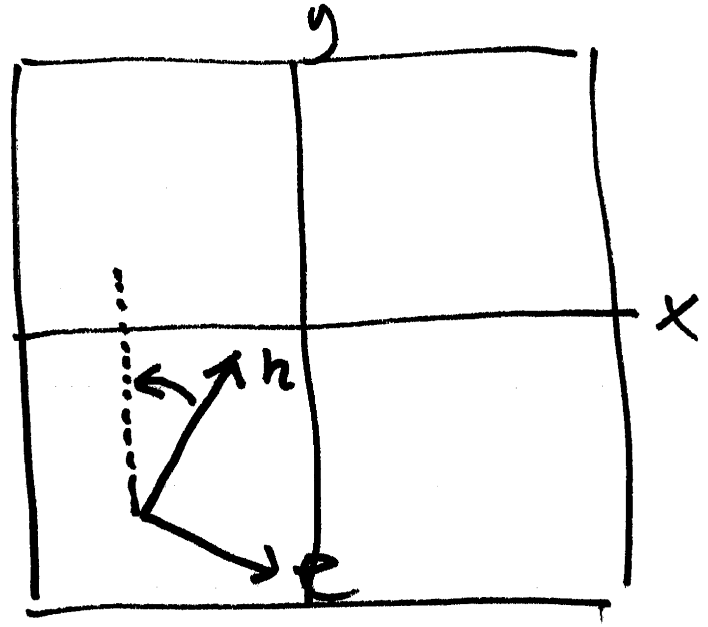
Rotation about a directed line, Axis / Angle

$$M_{3,3} = \begin{bmatrix} \alpha^2(1-\cos\theta)+\cos\theta & \alpha\beta(1-\cos\theta)-\gamma\sin\theta & \alpha\gamma(1-\cos\theta)+\beta\sin\theta \\ \alpha\beta(1-\cos\theta)+\gamma\sin\theta & \beta^2(1-\cos\theta)+\cos\theta & \beta\gamma(1-\cos\theta)-\alpha\sin\theta \\ \alpha\gamma(1-\cos\theta)-\beta\sin\theta & \beta\gamma(1-\cos\theta)+\alpha\sin\theta & \gamma^2(1-\cos\theta)+\cos\theta \end{bmatrix}$$



$\left.\begin{array}{l} \alpha, \beta, \gamma : \text{components of } \underline{\text{unit}} \text{ vector} \\ \theta : \text{rotation about that vector} \end{array}\right\}$ any rotation can be expressed in this way

α, β, γ, $\alpha^2+\beta^2+\gamma^2 = \underline{1}$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{bmatrix} e \\ n \\ u \end{bmatrix} \qquad \Theta_z = K \approx \underline{+30}$$

$$M = \begin{bmatrix} \cos 30^\circ & \sin 30^\circ & 0 \\ -\sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ω, φ, K vertical $\quad \omega \approx 0$, $\varphi \approx 0$

$\omega = 0, \ \varphi = 0, \ K = 30^\circ$

$$M = M_{K=30} M_{(\varphi=0)} M_{(\omega=0)} = M_{K=30}$$

example to find initial approximations for orientation / attitude

<u>orthogonal matrix</u>, $M^{-1} = M^T$

inner product (dot product) of any row with itself = 1

" " " " column " " = 1

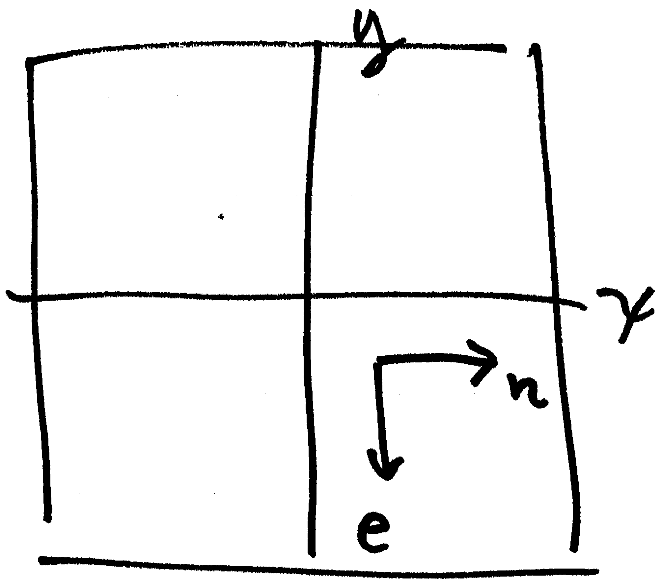
" " of any row with another row = 0

" " " " col with another col = 0

determinant = $\underbrace{+1}_{\substack{\text{rotation}\\ \text{matrix}}}$ or $\underbrace{-1}$ reflection matrix

LH ← RH

RH ← RH



vertical $\omega \approx 0$, $\varphi \approx 0$, $\kappa = ?$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} e \\ n \\ u \end{pmatrix}$$

image object

$\kappa \approx +90°$

$$M = \begin{pmatrix} \cos 90 & \sin 90 & 0 \\ -\sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

another example to find initial approximations for orientation / attitude