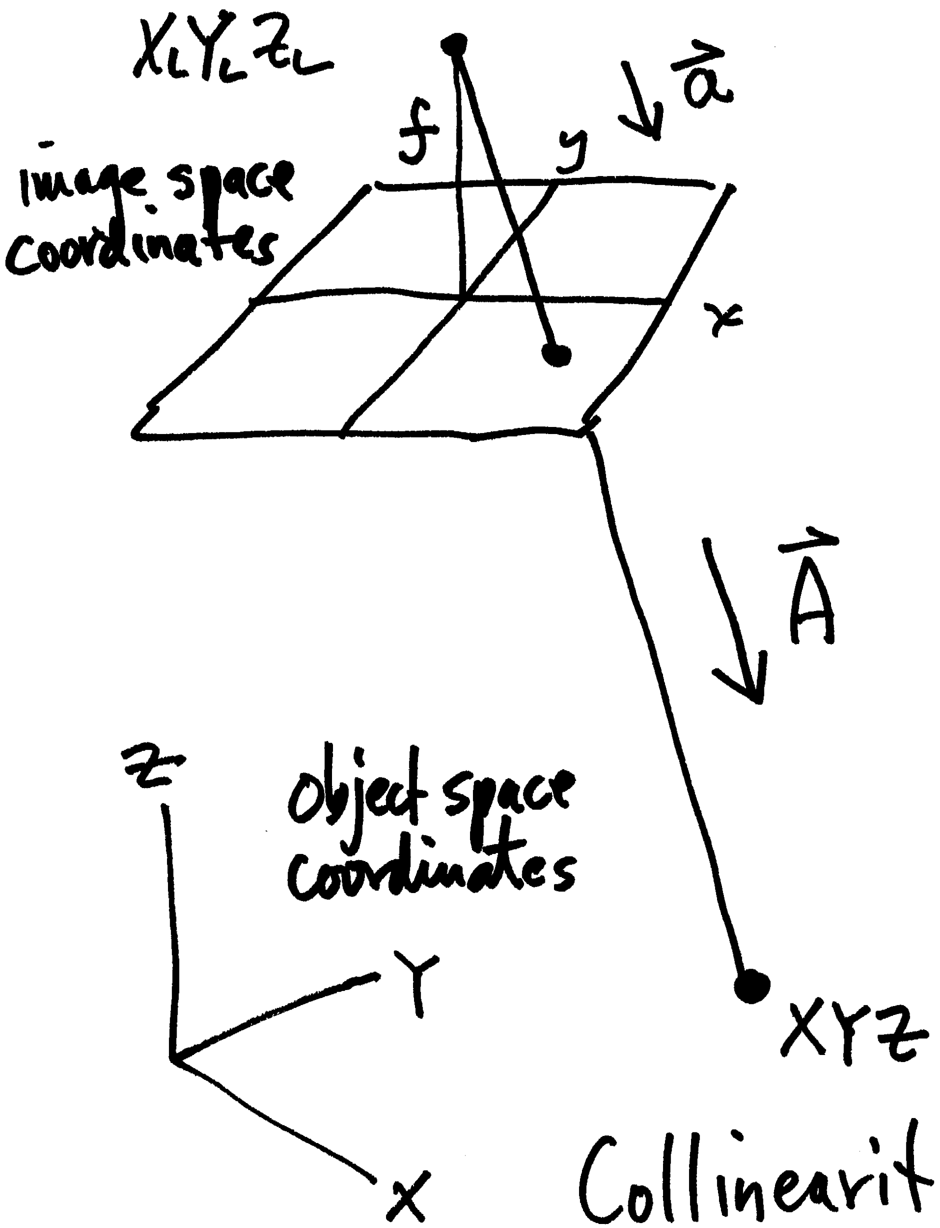


$$\vec{a} = \lambda M \vec{A}$$

$$\underbrace{\begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix}}_{\text{Img.}} = \lambda M \underbrace{\begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}}_{\text{Obj.}}$$

need to eliminate this scale factor "nuisance parameter"

we do it by dividing 2 equations each by the 3rd equation.

Collinearity Equations

$$\begin{bmatrix} x-x_0 \\ y-y_0 \\ -f \end{bmatrix} = \lambda \underbrace{\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{bmatrix}} = \lambda \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

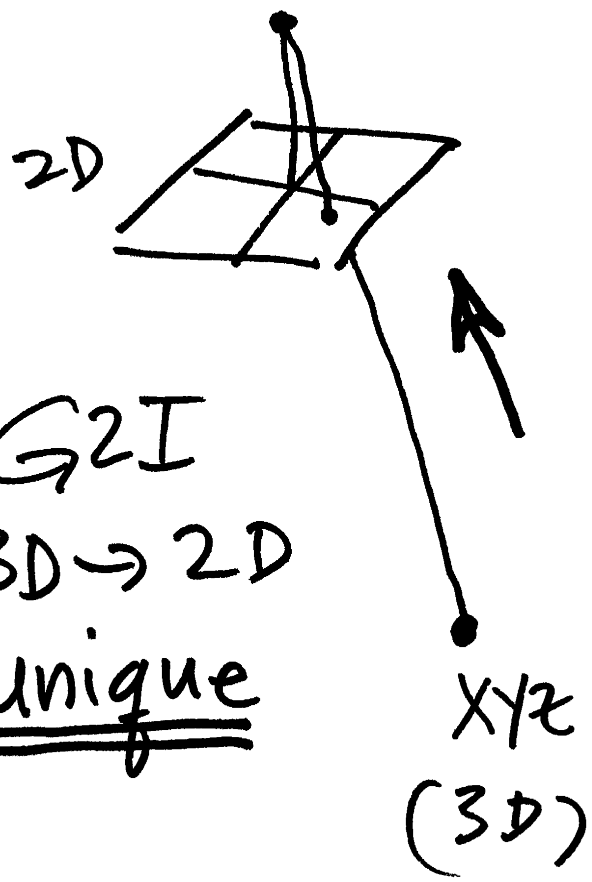
<u>Ground to</u>
<u>Image</u>

G2I

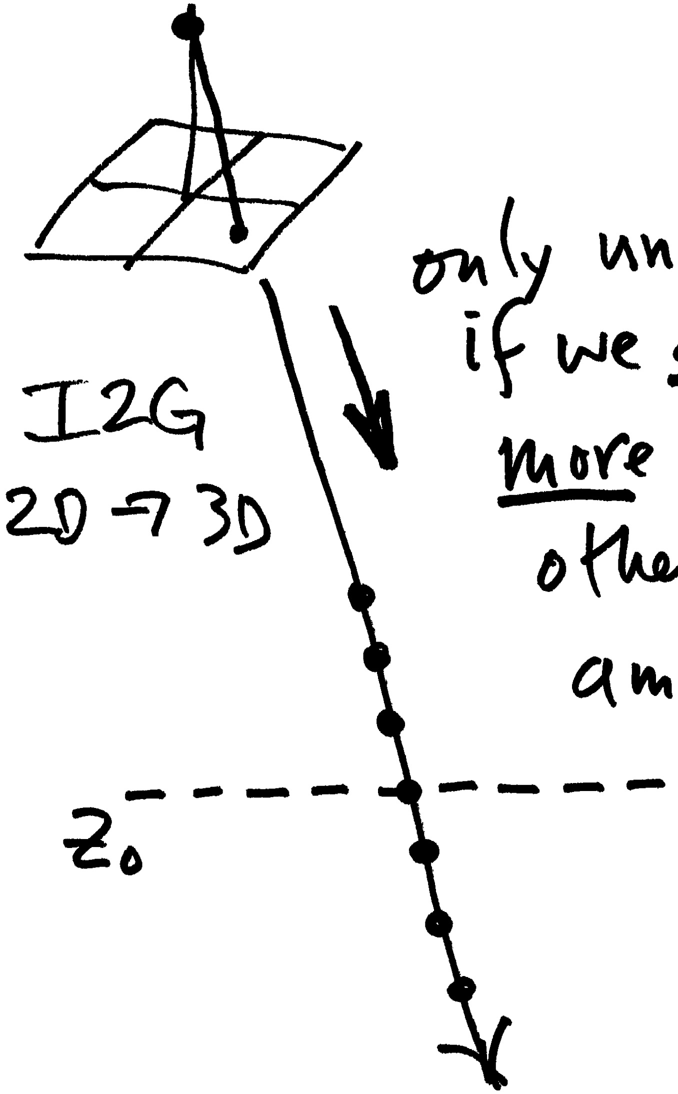
$$M\vec{X} = \begin{bmatrix} m_{11}(X-X_L) + m_{12}(Y-Y_L) + m_{13}(Z-Z_L) \\ m_{21}(X-X_L) + m_{22}(Y-Y_L) + m_{23}(Z-Z_L) \\ m_{31}(X-X_L) + m_{32}(Y-Y_L) + m_{33}(Z-Z_L) \end{bmatrix} = \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$\frac{x-x_0}{-f} = \frac{\cancel{\lambda} U}{\cancel{\lambda} W} = \frac{U}{W} \qquad \frac{y-y_0}{-f} = \frac{\cancel{\lambda} V}{\cancel{\lambda} W} = \frac{V}{W}$$

$$\boxed{x-x_0 = -f\frac{U}{W}} \qquad \boxed{y-y_0 = -f \cdot \frac{V}{W}}$$



G2I
3D → 2D
<u>unique</u>



I2G
2D → 3D

only unique if we <u>add</u> <u>more</u> <u>info</u>. otherwise ambiguous

common strategy to make I2G unique is to fix/specify a Z coordinate

$$\begin{bmatrix} x-x_0 \\ y-y_0 \\ -f \end{bmatrix} = \underbrace{\lambda M}_{} \begin{pmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{pmatrix} \quad , \quad \underbrace{\frac{1}{\lambda} M^T \begin{bmatrix} x-x_0 \\ y-y_0 \\ -f \end{bmatrix}}_{} = \begin{pmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{pmatrix}$$

$$\underset{3,3}{\begin{bmatrix} m_{11} & m_{21} & m_{31} \\ m_{12} & m_{22} & m_{32} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}} \underset{3,1}{\begin{bmatrix} x-x_0 \\ y-y_0 \\ -f \end{bmatrix}} = \begin{bmatrix} m_{11}(x-x_0) + m_{21}(y-y_0) + m_{31}(-f) \\ m_{12}(x-x_0) + m_{22}(y-y_0) + m_{32}(-f) \\ m_{13}(x-x_0) + m_{23}(y-y_0) + m_{33}(-f) \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\frac{1}{\lambda} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{pmatrix}$$

we still have the $1/\lambda$ nuisance scale parameter

$$\left(\frac{1}{\lambda}\right)\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

$$\frac{u \cdot \cancel{\frac{1}{\lambda}}}{w \cdot \cancel{\frac{1}{\lambda}}} = \frac{X - X_L}{Z - Z_L}, \quad \frac{v \cdot \cancel{\frac{1}{\lambda}}}{w \cdot \cancel{\frac{1}{\lambda}}} = \frac{Y - Y_L}{Z - Z_L}$$

$$X - X_L = (Z - Z_L)\frac{u}{w},$$

$$Y - Y_L = (Z - Z_L)\frac{v}{w},$$

$$\boxed{\begin{aligned} X &= X_L + (Z - Z_L)\frac{u}{w} \\ Y &= Y_L + (Z - Z_L)\frac{v}{w} \end{aligned}}$$

I2G
Image to Ground

Z resolves $2D \to 3D$ ambiguity

collinearity equations

$$\begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix} = \lambda R M \underbrace{\begin{bmatrix} (X - X_L)/R \\ (Y - Y_L)/R \\ (Z - Z_L)/R \end{bmatrix}}$$

$$R = \left[(X - X_L)^2 + (Y - Y_L)^2 + (Z - Z_L)^2 \right]^{1/2}$$

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix} = (\lambda R) M \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}$$

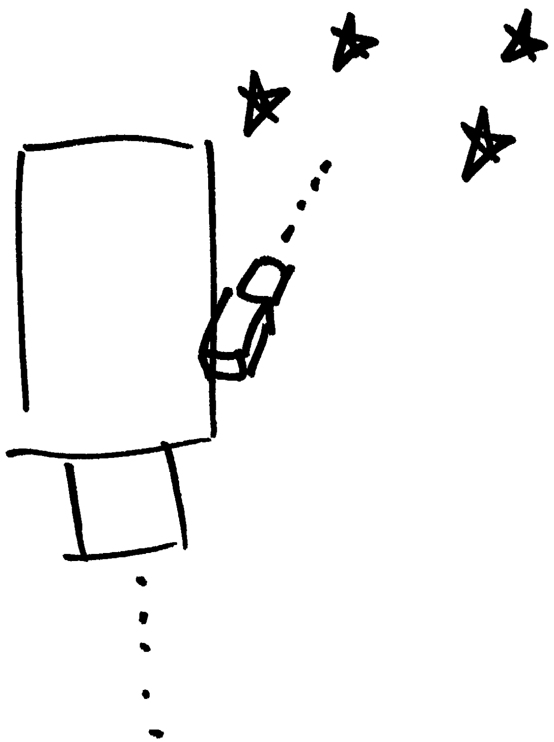
$$\frac{x - x_0}{-f} = \frac{m_{11} C_X + m_{12} C_Y + m_{13} C_Z}{m_{31} C_X + m_{32} C_Y + m_{33} C_Z}$$

$$\frac{y - y_0}{-f} = \frac{m_{21} C_X + m_{22} C_Y + m_{23} C_Z}{(\cdot)}$$

gotten rid of object point coordinates
" " " exposure station

rewrite collinearity for directional control rather than positional control

C_X, C_Y, C_Z : direction cosines



angular control : star directions

star catalog $\Rightarrow$ right ascension + declination

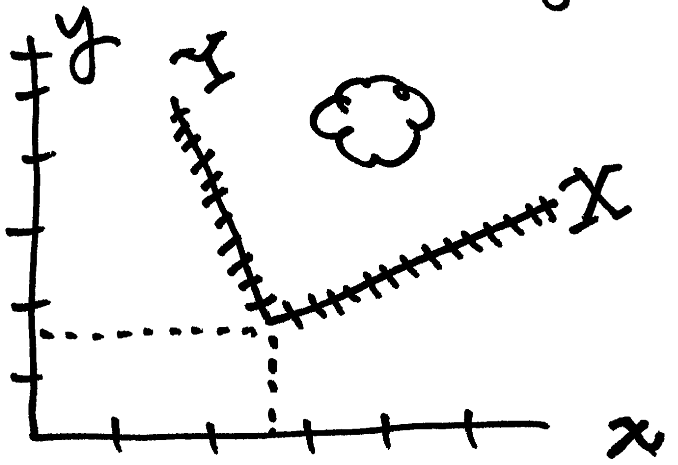
$\Downarrow$

c_x, c_y, c_z

unknowns left
$$\left.\begin{matrix} \omega & \varphi & \kappa \\ q_i & q_j & q_k \\ a, & b, & c \end{matrix}\right\}$$
some set of 3 parameters to define rotation matrix

able to solve for attitude / orientation of star camera using stars as "control points."

Coordinate transformations (2D)



λ: scale

Θ: rotation

T_x: x-shift

T_y: y-shift

$$\vec{x} = \lambda M \vec{X} + \vec{T}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

multiply this out to see where substitutions can be made.

Conformal 2D coord. transf.

$\hookrightarrow$ shape is preserved (at least locally)

Similarity transf.

helmert transf.

$$x = \overbrace{\lambda \cos\theta}^{a}\, X + \overbrace{\lambda \sin\theta}^{b}\, Y + T_x = c$$

$$y = \underbrace{-\lambda \sin\theta}_{-b}\, X + \underbrace{\lambda \cos\theta}_{a}\, Y + T_y = d$$

$$\left.\begin{aligned} x &= aX + bY + c \\ y &= -bX + aY + d \end{aligned}\right] \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \Big[\quad ? \quad \Big] \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$