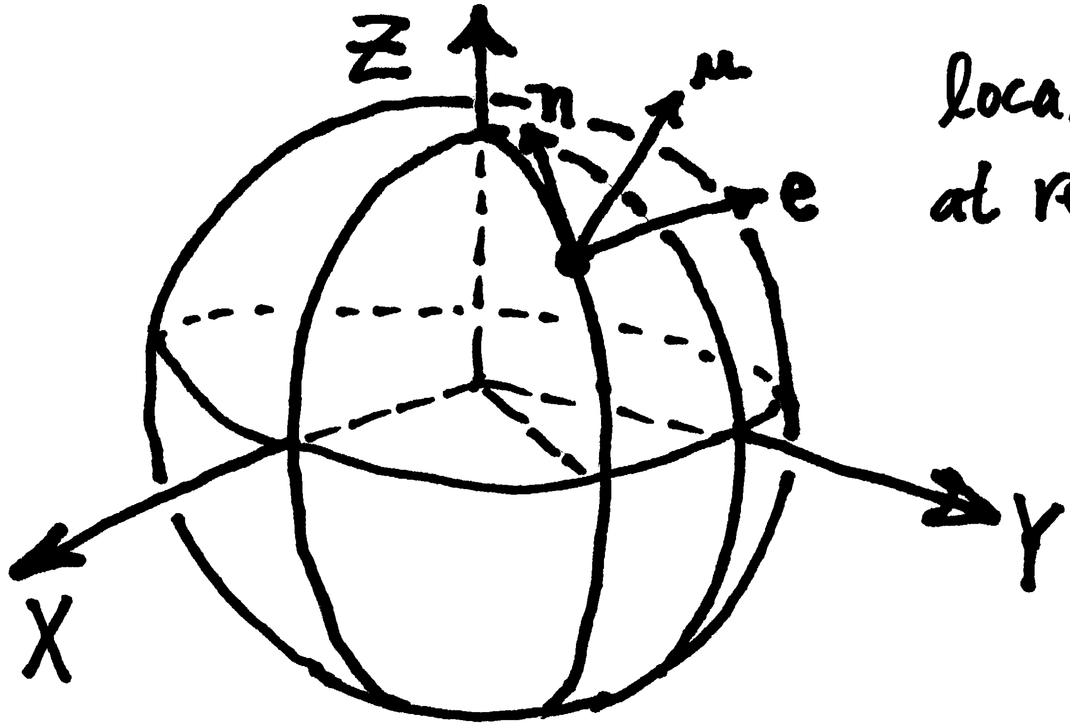


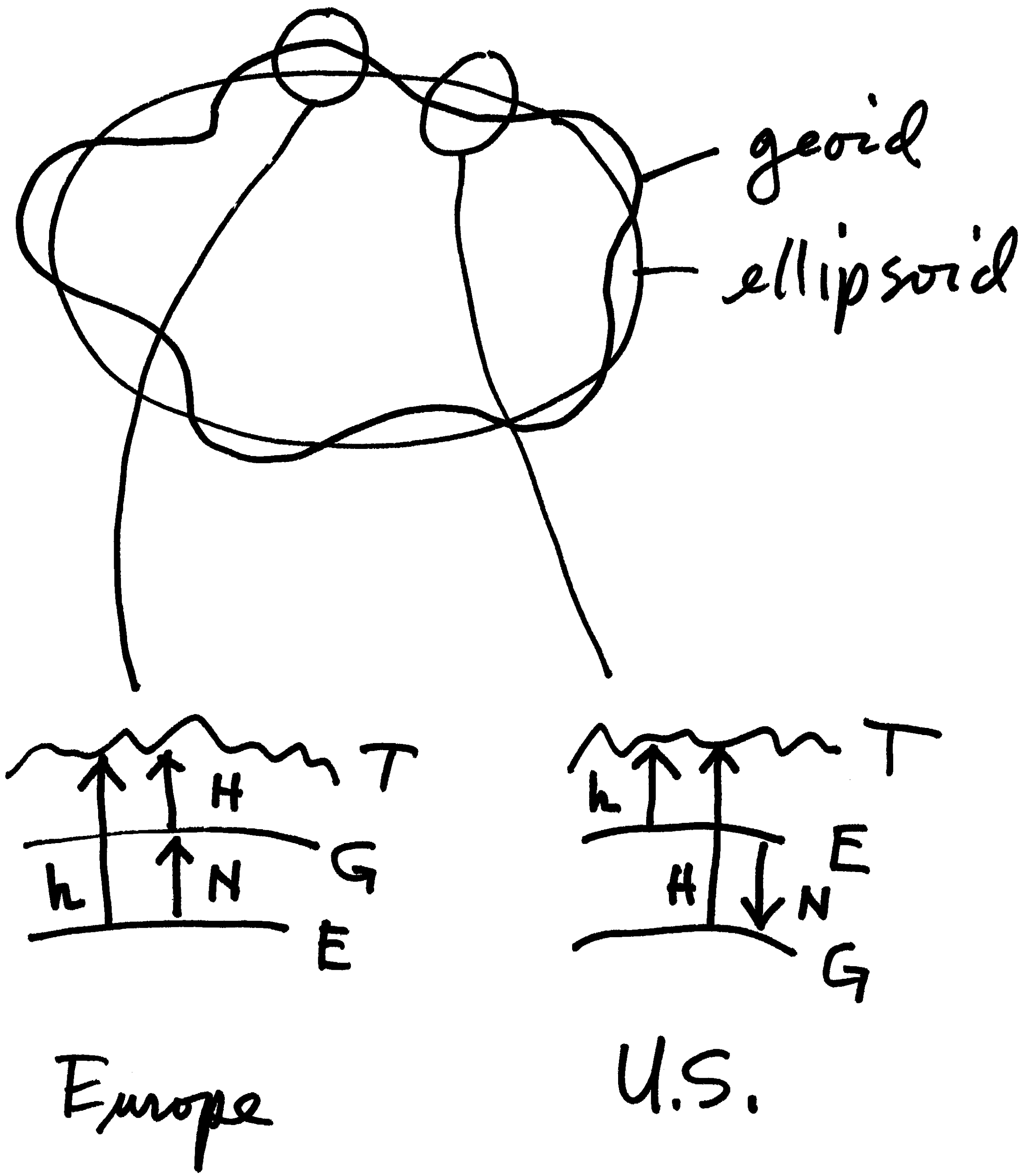
local cartesian = topocentric
at reference point (φ, λ, h)

$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_{ECF}$ of reference point

↓

$$\begin{pmatrix} e \\ n \\ u \end{pmatrix} = M_x(90-\phi)\, M_z(\lambda + 90^\circ) \left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{ECF} - \begin{pmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ ((1-e^2)N+h)\sin\varphi \end{pmatrix} \right]$$

useful to produce a local cartesian system, usually project specific
publish results in a conventional system



h: ellipsoid height

H: orthometric height
(Sea level height)

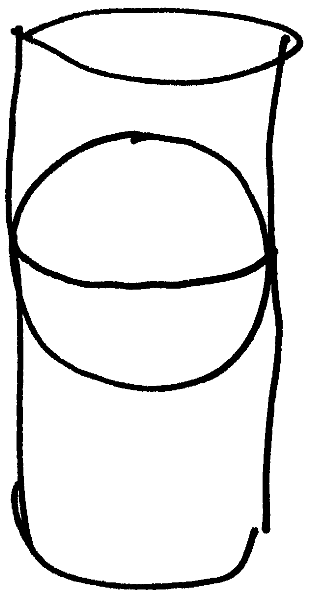
N: Geoid Undulation
Separation

$$h = H + N$$

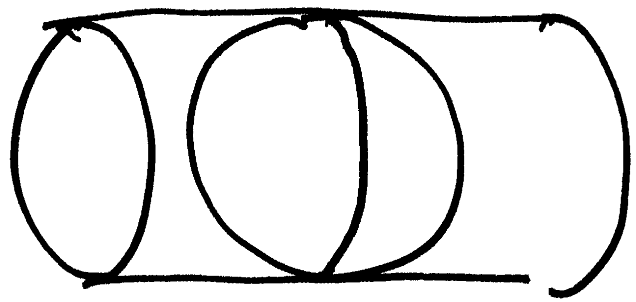
Indiana
$N \approx -30m$

spirit leveling relates heights with respect to equipotential surface such as geoid. geometric analysis uses ellipsoid height, h.

$\phi\lambda \rightarrow XY$ conformal
(shapes are preserved locally)

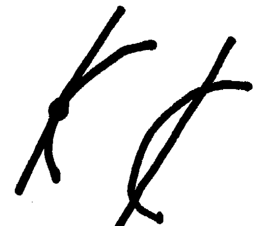


normal
aspect
mercator

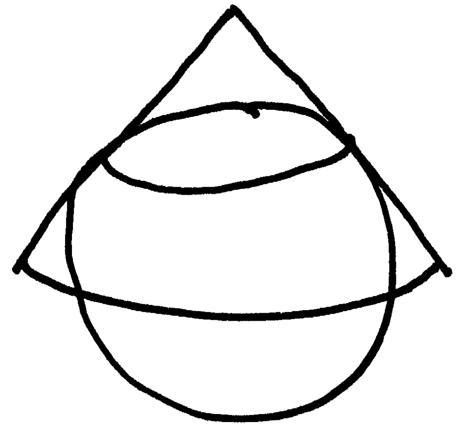


transverse
mercator

zones with large
N-S extent



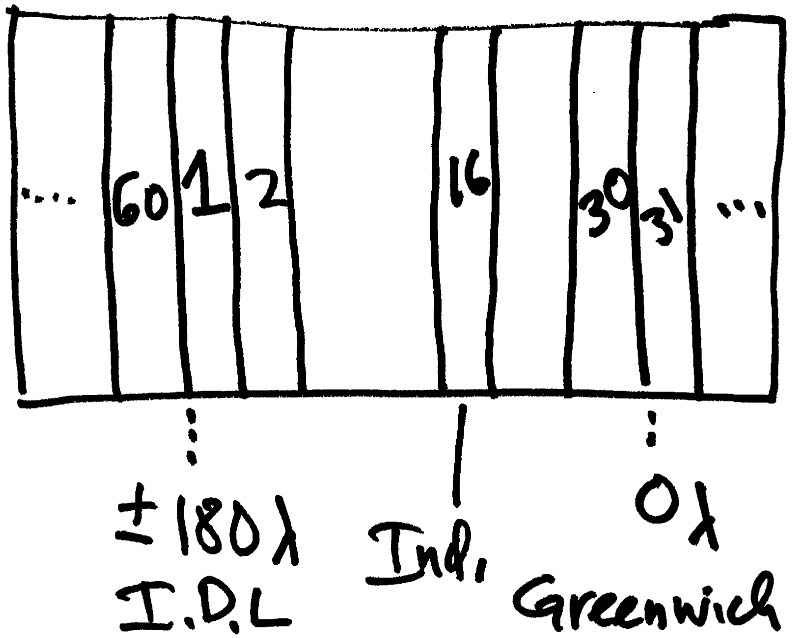
tangent vs.
secant



Lambert
Conic

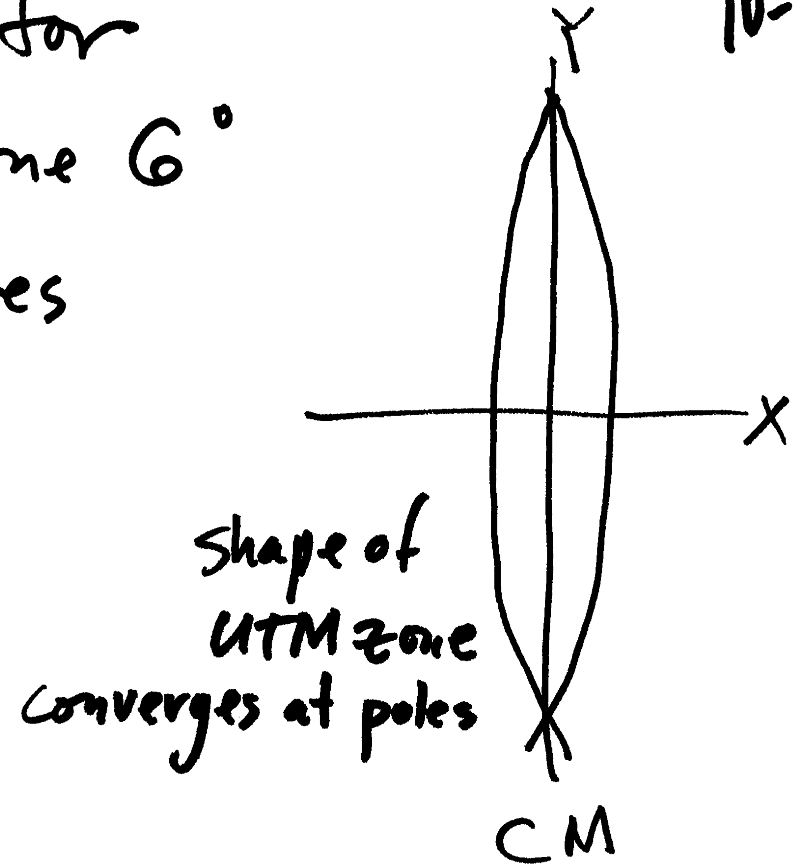
zones with large
E-W extent

UTM: universal transverse merator



each zone 6°

60 zones

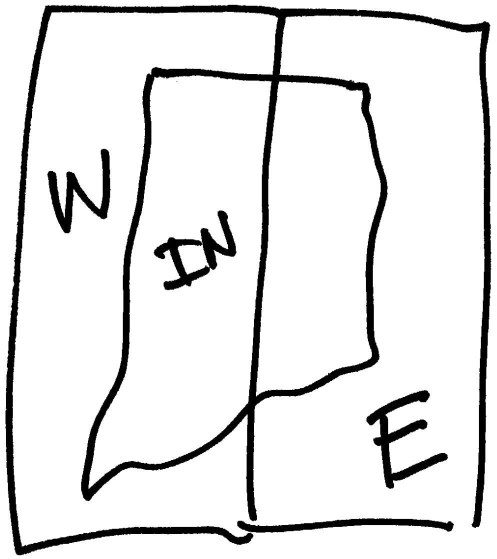


CM zone 1 $-177°$, 177° W

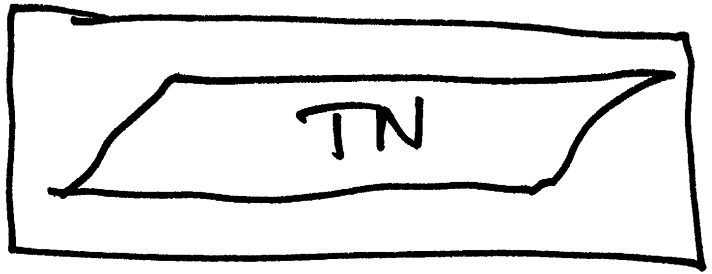
CM zone 16 $-87°$, 87° W

CM zone 31 $+3°$, 3° E

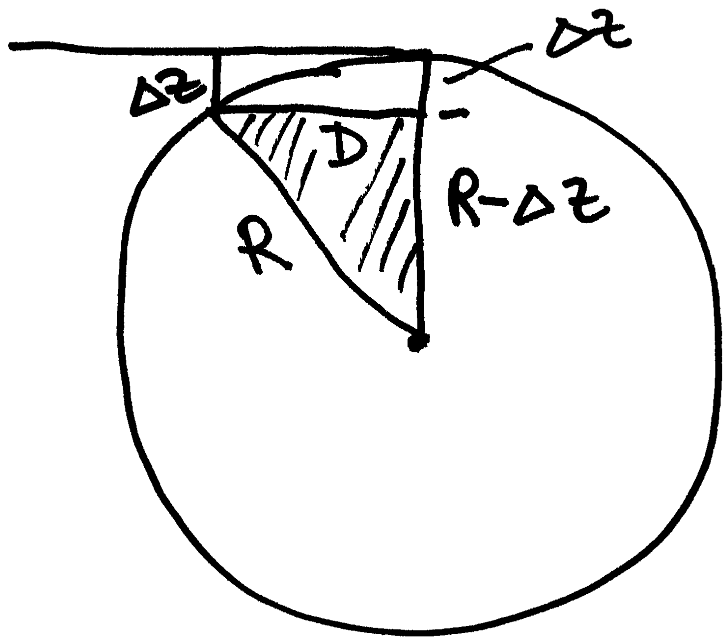
State Plane Coordinates



transverse
mercator



1 Lambert
conic zone



$$R^2 = D^2 + (R - \Delta z)^2$$

$$\cancel{R^2} = D^2 + \cancel{R^2} + (\Delta z)^2 - 2R\Delta z$$

$$D^2 + \cancel{(\Delta z)^2} = 2R\Delta z$$

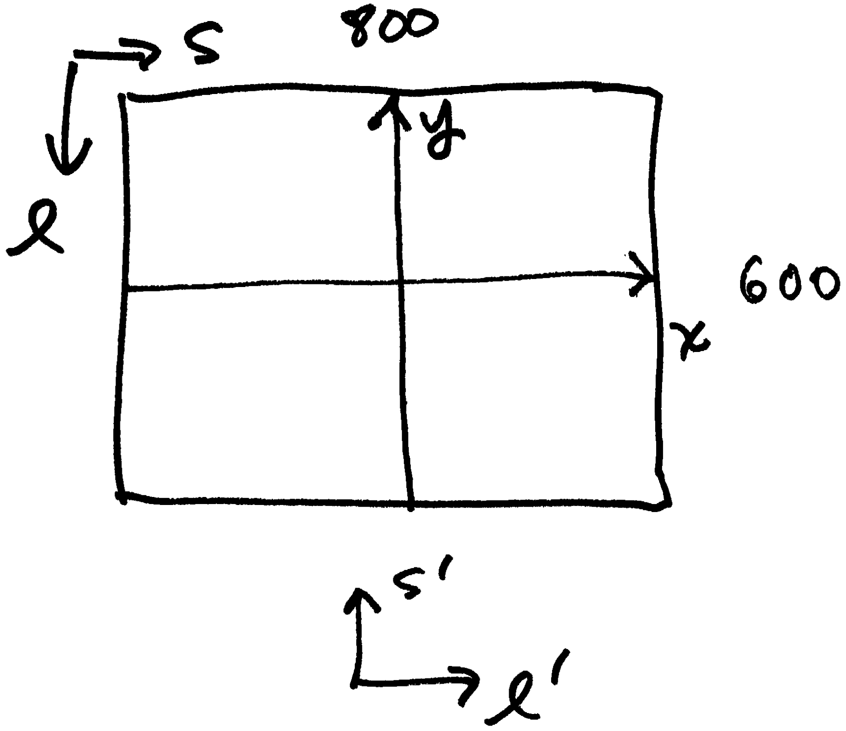
$(\Delta z)^2$ small
negligible

$$D^2 = 2R\,\Delta z$$

$$\boxed{\Delta z = \frac{D^2}{2R}}$$

D (m)	Δz (m)
10	0
100	.0008
500	.02
1000	.08
5000	1.9 m
10 000	7.8 m

Table shows <u>error</u> in assuming that height with respect to curved surface (h, H) is cartesian. Conversely you can "correct" h or H over a project area to approximate a cartesian system together with E, N from a projection



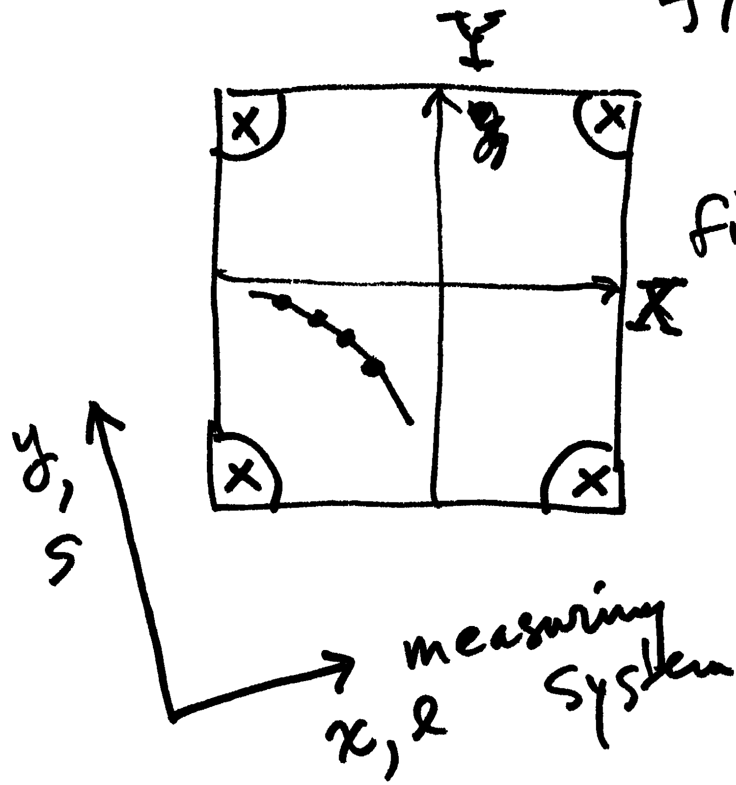
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -400 \\ +300 \end{bmatrix} + \begin{bmatrix} \cos 90 & \sin 90 \\ -\sin 90 & \cos 90 \end{bmatrix} \begin{pmatrix} l \\ s \end{pmatrix}$$

nominal principal point system ← raw pixel coordinates

typical digital camera

convert line, sample to conventional cartesian system with origin at "nominal" principal point.

film camera with fiducial marks



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

calibration

obs, meas.

invert for use

$$x = t + mX$$

$$x - t = mX$$

$$m^{-1}(x-t) = X \,,$$

$$\boxed{X = -m^{-1}t + m^{-1}x}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}^{-1} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$