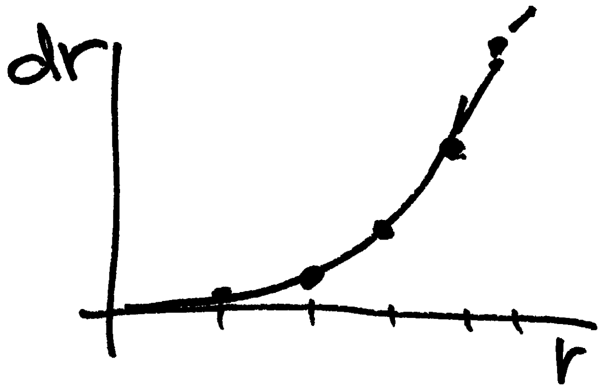
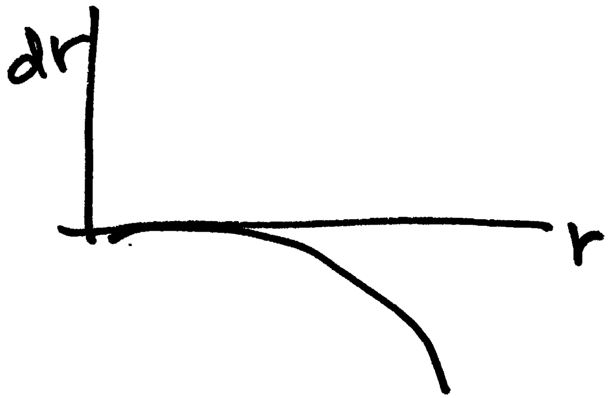
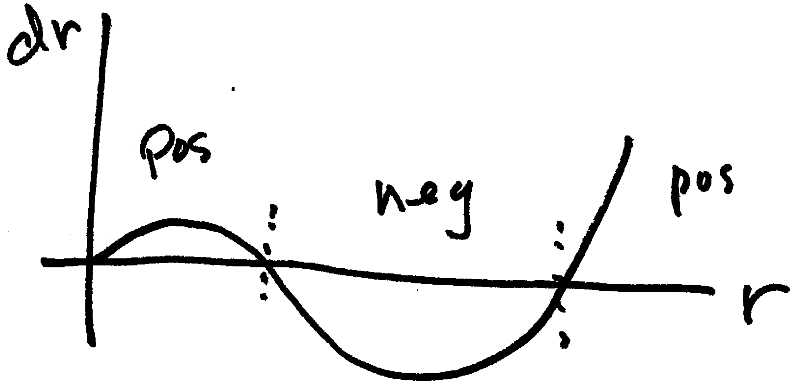
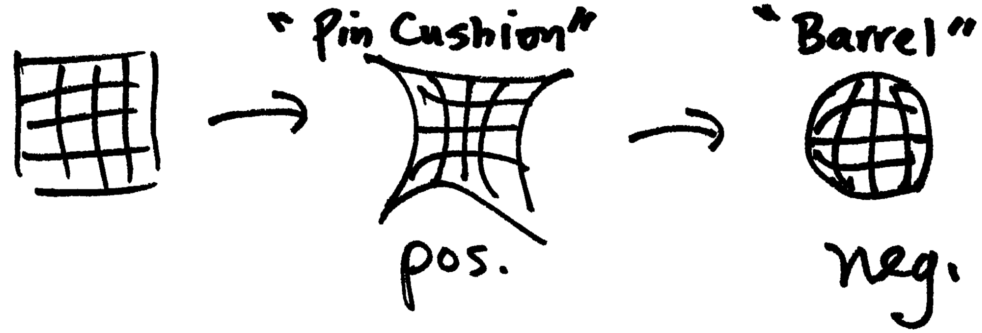
Lens distortion : radial



r = radial distance

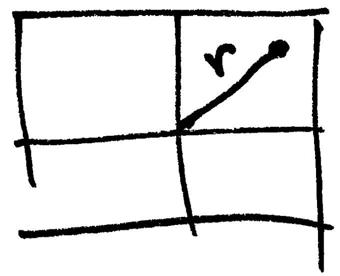






distortion info:

1. function
2. table



function : odd polynomial

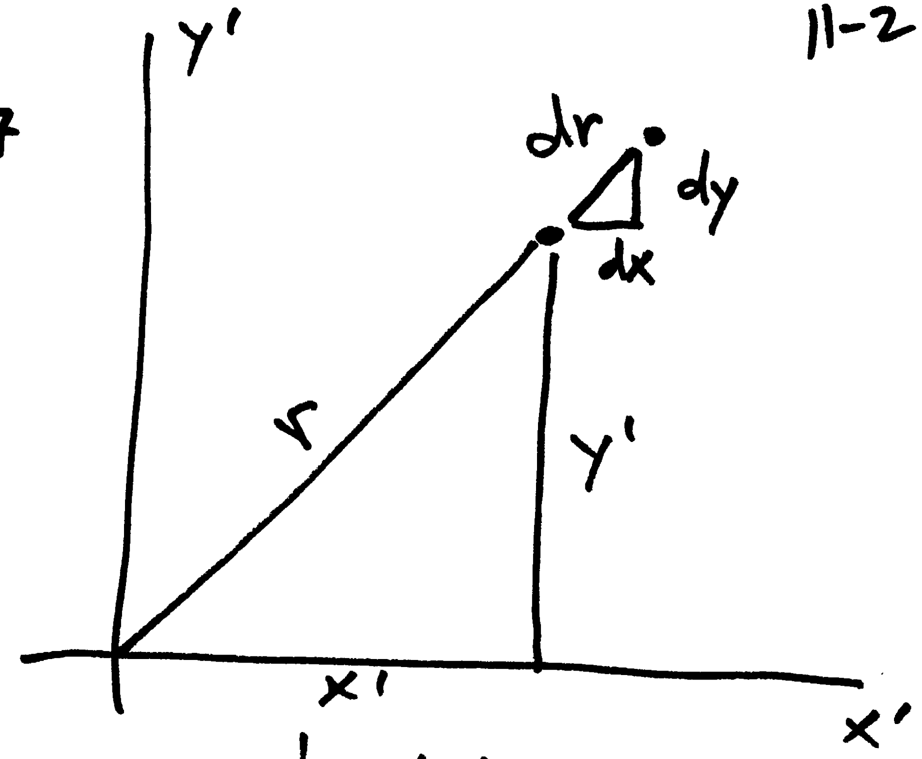
$$dr = \underline{k_1} r^3 + \underline{k_2} r^5 + \underline{k_3} r^7$$

$$\frac{dx}{dr} = \frac{x'}{r}, \quad \boxed{dx = dr \cdot \frac{x'}{r}}$$

$$r = \sqrt{(x')^2 + (y')^2}$$

$$\frac{dy}{dr} = \frac{y'}{r}; \quad \boxed{dy = dr \cdot \frac{y'}{r}}$$

$$dx = \frac{(k_1 r^3 + k_2 r^5 + k_3 r^7)\, x'}{r}$$



$$x' = x - x_0$$
$$y' = y - y_0$$

distortion vs. **correction** (careful about sign!)

$$X_c = x' - dx_r$$

$$Y_c = y' - dy_r$$

$$X_c = x' - dx_r - dx_d$$

$$\underset{\text{corrected}}{y_c} = \underset{\text{raw meas } - x_0, y_0}{y'} - \underset{\text{radial lens dist.}}{dy_r} - \underset{\text{decentering lens dist.}}{dy_d}$$

decentering distortion

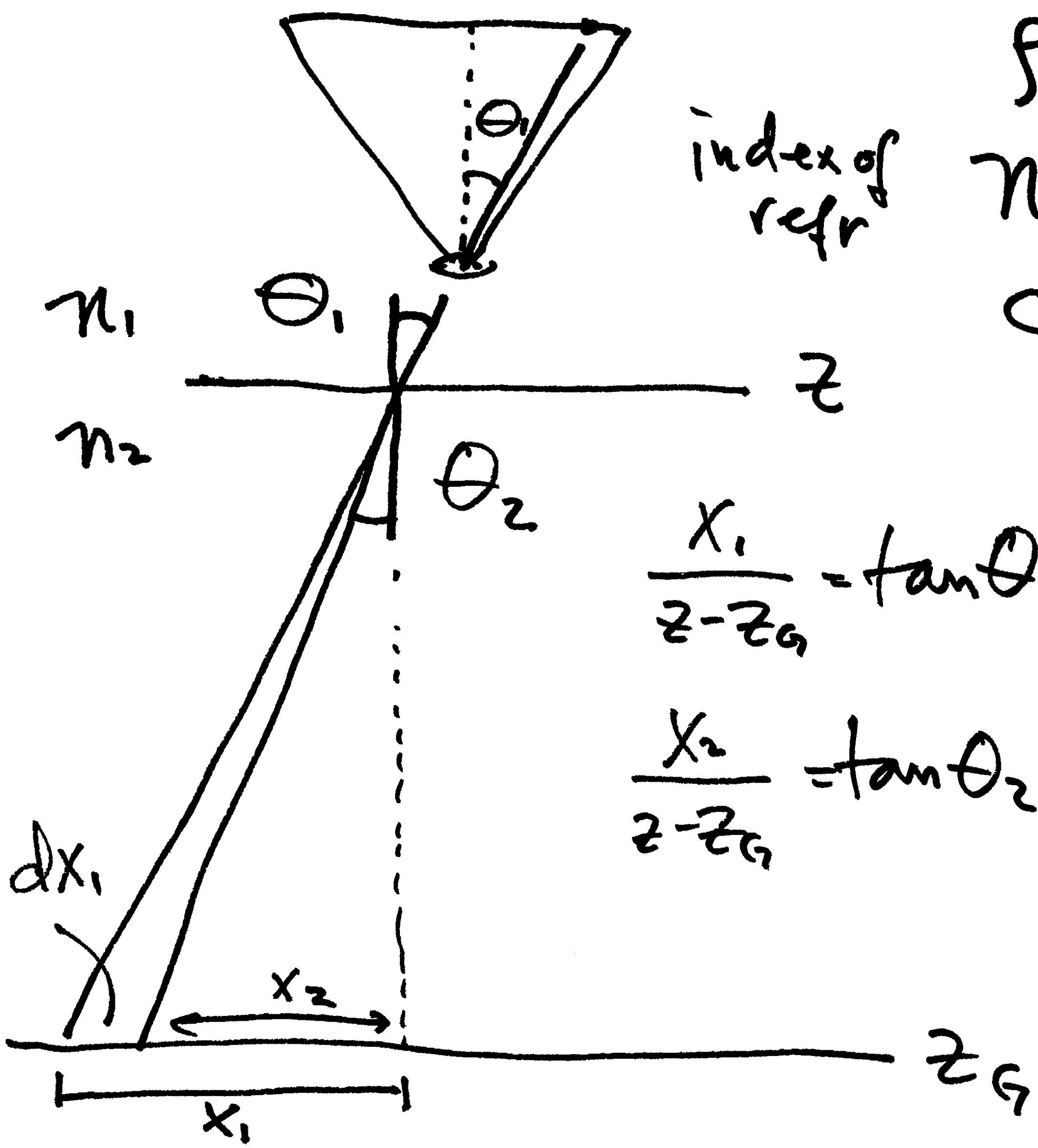
$$dx_d = P_1(r^2 + 2(x')^2) + 2P_2 x'y'$$

$$dy_d = P_2(r^2 + 2(y')^2) + 2P_1 x'y'$$

atmospheric refraction (basic physics + optics)

ρ density   altitude & density

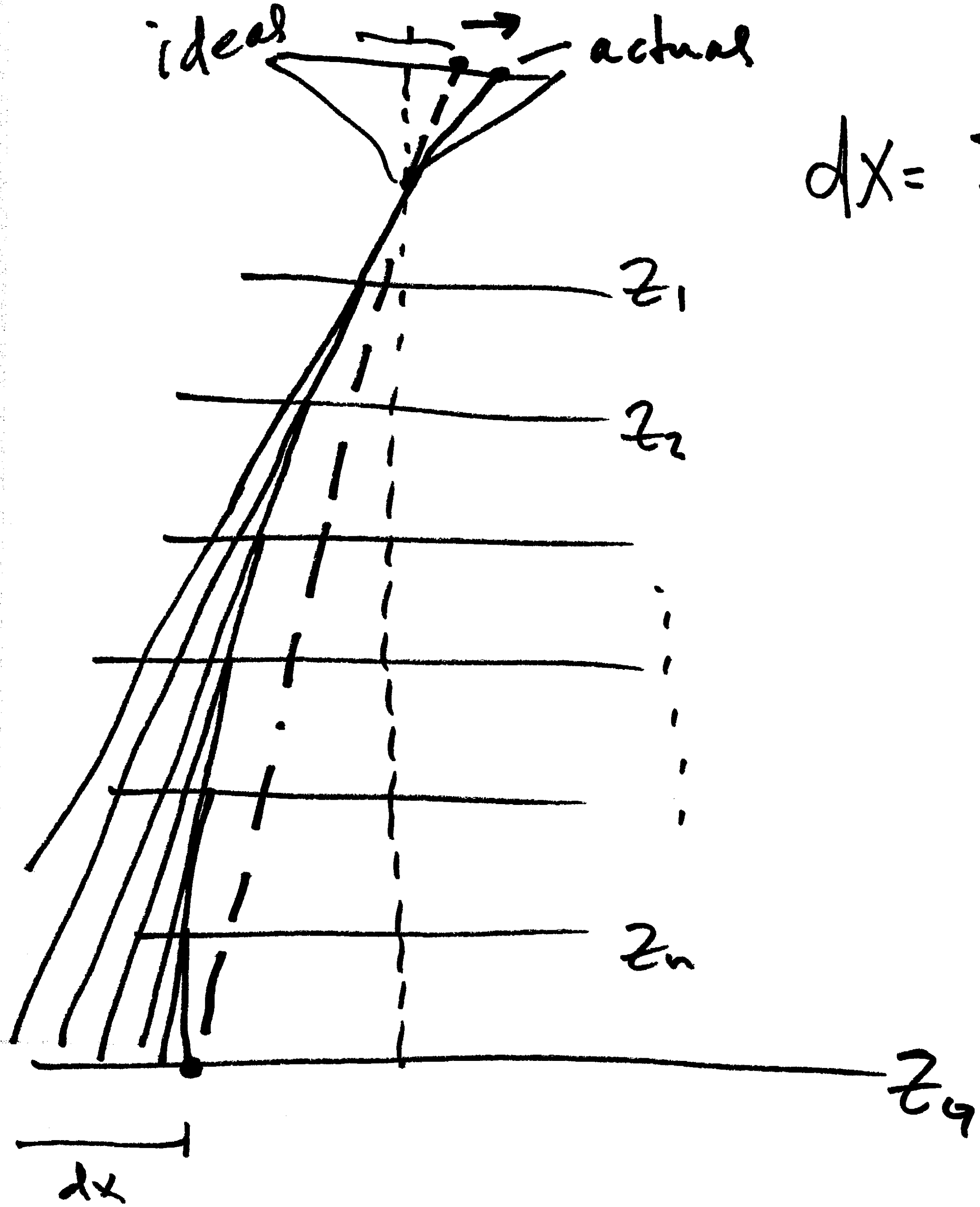
index of refr $\quad n^2 = 1 + 2c\rho$
$c = .0002261$ } atmos. sci.



$$\frac{x_1}{z - z_G} = \tan\theta_1$$

$$\frac{x_2}{z - z_G} = \tan\theta_2$$

$$dx_1 = (z - z_G)\tan\theta_1 - (z - z_G)\tan\theta_2$$

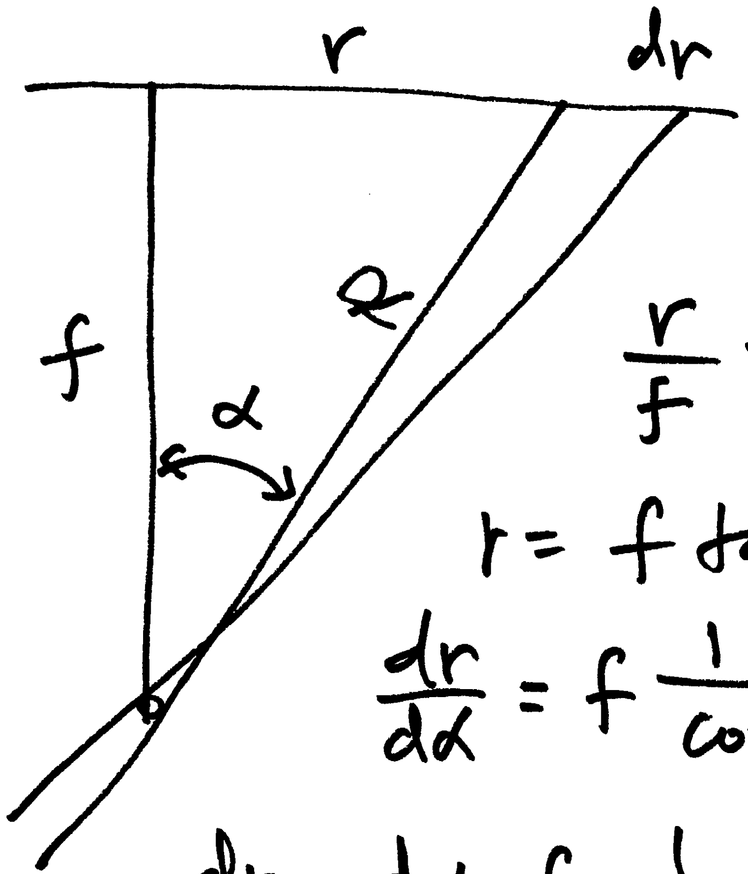


$$dx = \sum_{i=1}^{n} (z_i - z_G)\left[\tan\theta_i - \tan\theta_{i+1}\right]$$

distortion always <u>outward</u>

correction always <u>inward</u>

Discretize the atmosphere below camera into n+1 layers with n boundaries



$$\frac{r}{f} = \tan\alpha$$

$$r = f\tan\alpha$$

$$\frac{dr}{d\alpha} = f\,\frac{1}{\cos^2\alpha}$$

$$dr = d\alpha \cdot f \cdot \frac{1}{\cos^2\alpha}$$

$$\cos\alpha = f/R = f/\sqrt{f^2+r^2}$$

$$\cos^2\alpha = \frac{f^2}{f^2+r^2}$$

$$\frac{1}{\cos^2\alpha} = \frac{f^2+r^2}{f^2} = \boxed{1+\frac{r^2}{f^2}}$$

$$dr = d\alpha \cdot f \cdot \left(1+ r^2/f^2\right)$$

let's implement the correction for A.R. in image space (only for vertical imagery) then the correction will work just like lens distortion

$$\left\{ \begin{array}{l} d\alpha = K \tan\alpha \\ K = \left[\dfrac{2410\,H}{H^2 - 6H + 250} - \dfrac{2410\,h}{h^2 - 6h + 250} \left(\dfrac{h}{H}\right) \right] \times 10^{-6} \quad \text{(radians)} \\ \quad H: \text{flying height (km)} \\ \quad h: \text{elevation of point (km)} \end{array} \right.$$

$$\tan\alpha = r/f \,, \quad d\alpha = K \frac{r}{f}$$

$$dr = K \frac{r}{\cancel{f}} \cdot \cancel{f} \cdot \left(1 + r^2/f^2\right)$$

$$\boxed{dr = K\left(r + \frac{r^3}{f^2}\right)}$$

<u>radial</u> displacement in image space due to atmospheric refraction.

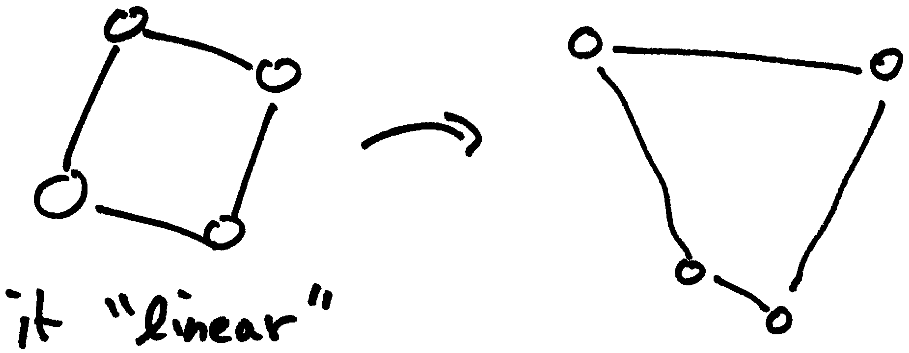
handle radial atmos. refraction like radial lens dist.

$$dx = x' \cdot \frac{dr}{r}, \quad dy = y' \cdot \frac{dr}{r}$$

$$\boxed{\begin{aligned} x_c &= x' - dx_r - dx_d - dx_{AR} \\ y_c &= y' - dy_r - dy_d - dy_{AR} \end{aligned}}$$

may not need A.R. for
low altitude
close-range, terrestrial

8-parameter transformation



$$x = \frac{a_0 + a_1 X + a_2 Y}{c_1 X + c_2 Y + 1}$$

make it "linear"

$$\underline{x c_1} X + \underline{x c_2} Y + x = a_0 + a_1 X + a_2 Y$$

$$y = \frac{b_0 + b_1 X + b_2 Y}{c_1 X + c_2 Y + 1}$$

$$y \underline{c_1} X + y \underline{c_2} Y + y = b_0 + b_1 X + b_2 Y$$

$$x = a_0 + a_1 X + a_2 Y - x c_1 X - x c_2 Y$$

$$y = b_0 + b_1 X + b_2 Y - y c_1 X - y c_2 Y$$

used for inner orientation: measurement to fiducial, but also for ground to image where terrain is <u>very</u> flat. In second case you can use it to approximate EO: $X_L Y_L Z_L\ \omega \varphi \kappa$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & X & Y & 0 & 0 & 0 & -xX & -xY \\ 0 & 0 & 0 & 1 & X & Y & -yX & -yY \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix}$$

XY constant

xy measurements

in this approach the solution is not iterative, no approximations needed

each point generates 2 equations

8 unknowns

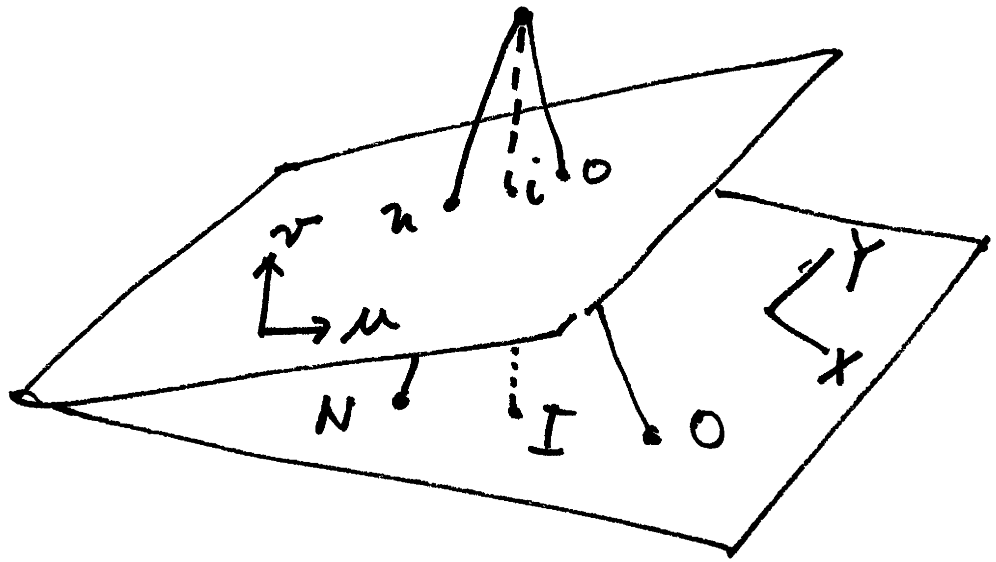
4 pts $\Rightarrow$ unique solution

<u>applications</u> simple rectification (tilt only)

G.C.P. lie in plane

obtain approx for $X_L Y_L Z_L$

$\omega\ \phi\ \kappa$



1. solve $a_0\, a_1\, a_2\, b_0\, b_1\, b_2\, c_1\, c_2$

$\downarrow$

2. transform $\alpha', d', e', \alpha, d, e, f', h'$

3. add either f or x_0, y_0

4. $X_L\, Y_L\, Z_L\, \omega\, \phi\, K$

I will post the algorithm to go from step 1 to step 4

advantages : no approximations, no iterations.

another method for approx Exterior Orientation

D.L.T. direct linear transformation

$$x = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

$$y = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

1. rearrange, make "linear"
2. min 6 pts → 12 eqn's
   solve L's
3. transform L's → X_c ω, Y_c φ, Z_c κ

control should be distributed in 3D
another way to obtain initial approx.

quite similar RPC, RFM, RSM

Least Squares

physics + geometry $\Rightarrow$ mathematical model $\rightarrow$ function model (equations) < condition / constraint

mathematical model $\rightarrow$ Stochastic model

variables: constant — observation $\longrightarrow$ unknown

$\sigma = 0$ $\quad\quad$ σ = finite $\quad\quad$ $\sigma = \infty$