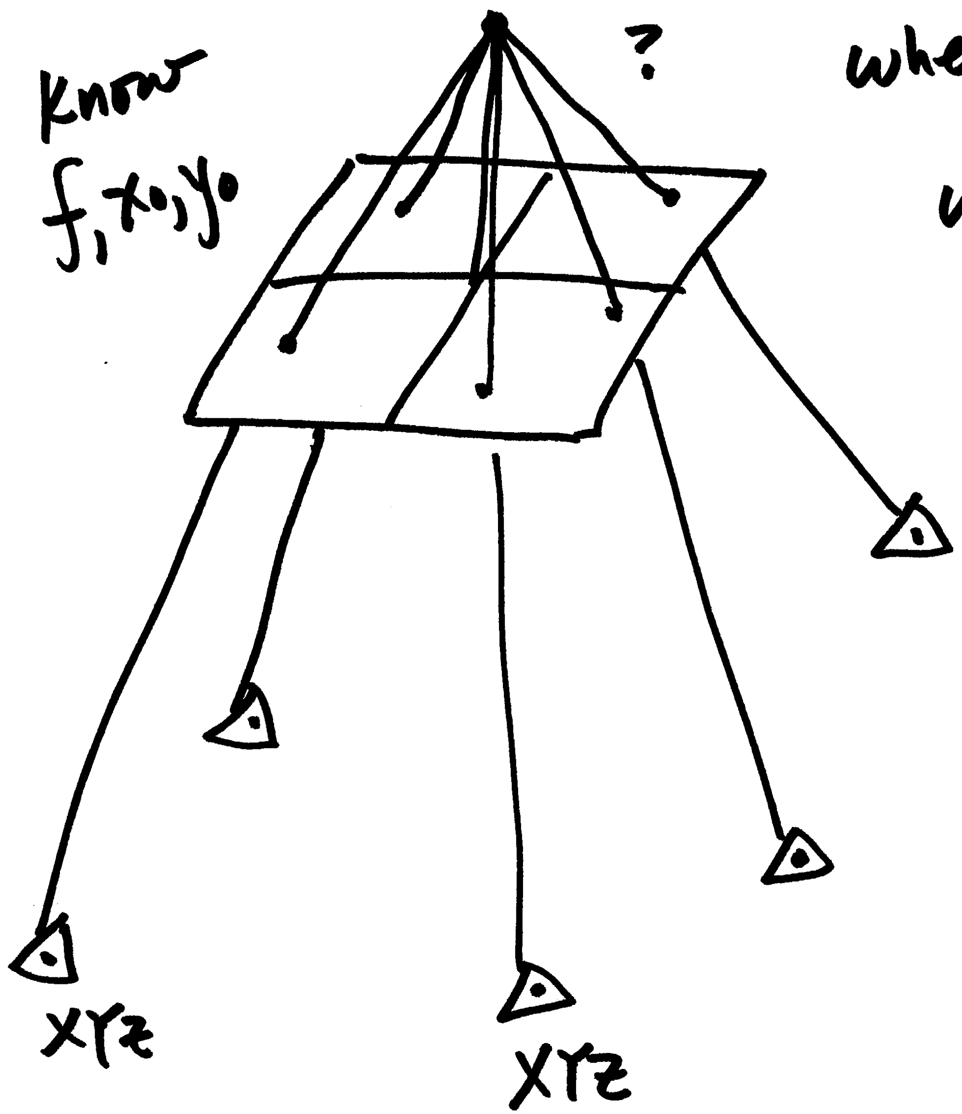


Know ground points, GCP

where is exposure station $X_L Y_L Z_L$?

what is attitude, orientation ?

<u>Space Resection</u>

unknowns $X_L Y_L Z_L \omega \phi \kappa$: 6

5 points : each 2 collin eqn's

10 equations

Motivation for studying Least Squares:
Need to do estimation where you have <u>overdetermined</u> the solution, i.e. you have <u>redundant</u> observations.

math model
- functional model
- stochastic model

counting exercise

$$
\begin{array}{rll}
 & n: & \# \text{ of observations (measurements)} \quad (l) \\
- & n_0: & \text{minimum } \# \text{ of obs. needed to fix model} \\
\hline
 & r: & \text{redundancy (excess } \# \text{ obs) (d.o.f.)}
\end{array}
$$

$$l + v = \hat{l}$$

obs — correction — adjusted obs.

$$\sum_i v_i^2 \rightarrow \text{minimized}$$

$$\text{length} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} \rightarrow \text{min.}$$

$$\sum_i w_i v_i^2 \rightarrow \text{minimum}$$

$$v^T W v$$

$$w_i = \frac{\sigma_0^2}{\sigma_i^2}$$

σ_0^2 ← arbitrary constant

σ_i^2 ← variance of obs.

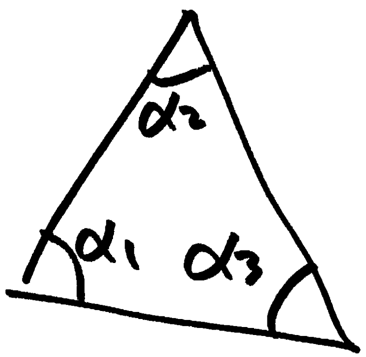
$$W = \begin{bmatrix} w_1 & & & & \\ & w_2 & & & \\ & & w_3 & & \\ & & & \ddots & \\ & & & & w_n \end{bmatrix}$$

you get to choose it's arbitrary → σ_0^2: 1, or var. of a commonly occuring observation

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = v^T W v$$

← expression to be minimized for <u>weighted</u> least squares.

model: shape of plane Δ



$n = 3$ ← # of eqn incl. obs.

$n_0 = 2$

$r = 1$ ← # of eqn obs. only

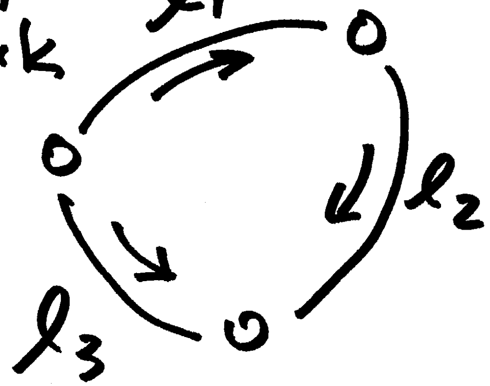
$n = 2$

$n_0 = 2$

$r = 0$

$$\alpha_1 + v_1 + \alpha_2 + v_2 + \alpha_3 + v_3 = 180^\circ$$

level network



l's positive → uphill

$$l_1 + v_1 + l_2 + v_2 + l_3 + v_3 = 0$$

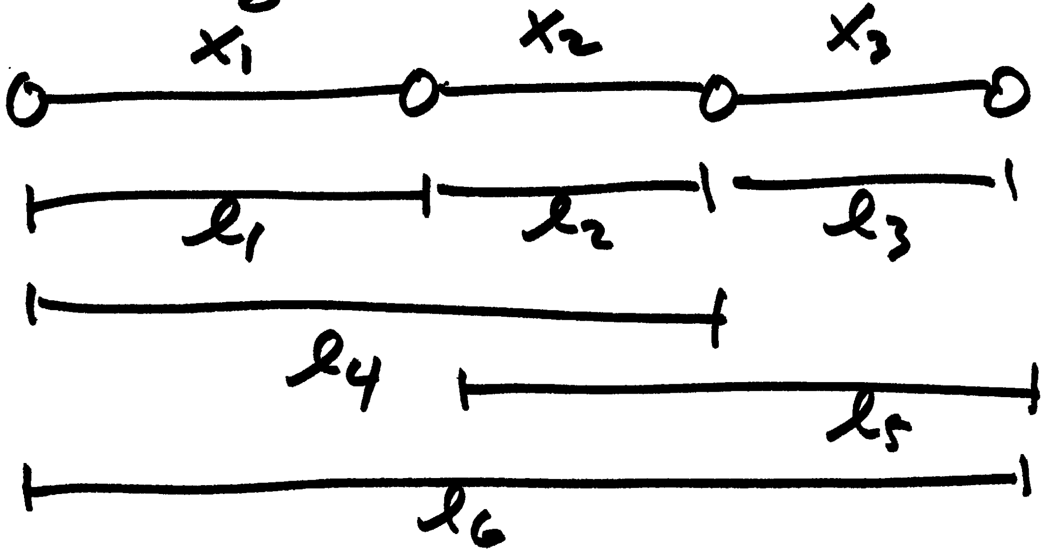
$n = 3$

$n_0 = 2$

$r = 1$ ← 1 cond. eqn

counting & writing condition equations for simple models

3 collinear lengths



$n = 6$ ← # eqn ind. obs.

$n_0 = 3$

$r = 3$

indirect observation method

$$\hat{l}_1 = x_1$$
$$\hat{l}_2 = x_2$$
$$\hat{l}_3 = x_3$$
$$\hat{l}_4 = x_1 + x_2$$
$$\hat{l}_5 = x_2 + x_3$$
$$\hat{l}_6 = x_1 + x_2 + x_3$$

$\underline{c = 6}$

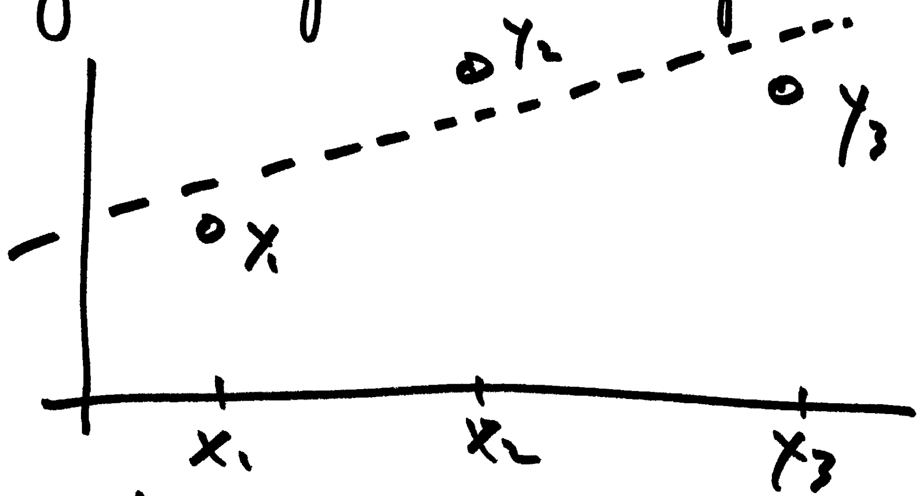
observation only method

$$\hat{l}_4 = \hat{l}_1 + \hat{l}_2$$
$$\hat{l}_5 = \hat{l}_2 + \hat{l}_3$$
$$\hat{l}_6 = \hat{l}_1 + \underline{\hat{l}_2 + \hat{l}_3}$$

$\hat{l}_6 = \hat{l}_1 + \hat{l}_5$ alternate way to write eqn #3

$\underline{c = 3}$

regression problem : 3 points on a line



X constant
y observed

$n = 3$
$n_0 = 2$
$r = 1$

$$\frac{\hat{y}_2 - \hat{y}_1}{x_2 - x_1} = \frac{\hat{y}_3 - \hat{y}_1}{x_3 - x_1}$$ ← observation only condition equations

parameters : m : slope
b : intercept

u = number of parameters
c = # of cond. eqns

$$y_1 + v_1 = m x_1 + b$$
$$y_2 + v_2 = m x_2 + b$$
$$y_3 + v_3 = m x_3 + b$$
← indirect observations condition equations

linear regression problem

linear in a's, x: constant

$$\overset{obs}{y} = \overset{par}{a_0} + a_1 x + a_2 x^2$$

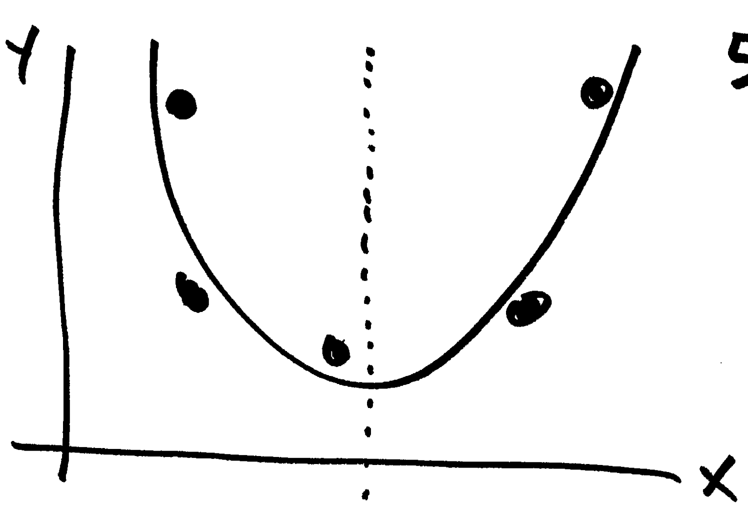
linear in a's because, they appear alone or with constant multiplier

on the other hand if x: observed

$$y = a_0 + \underset{par.}{a_1}\,\underset{obs.}{x} + \underset{par}{a_2}\,\underset{obs}{x^2}$$

product of $a_1 \cdot x$ makes it nonlinear
$a_2 \cdot x^2$

if condition equations are linear in observations and parameters then you can use linear methods otherwise nonlinear



5 points on parabola

$n = 5$

$n_0 = 3$

$r = 2$

$Y = a_0 + a_1 X + a_2 X^2$

indirect obs method

$u = 3$ parameters a_0, a_1, a_2

$n = 5$ cond. eqn's

$$\left.\begin{aligned} Y_1 + V_1 &= a_0 + a_1 X_1 + a_2 X_1^2 \\ Y_2 + V_2 &= a_0 + a_1 X_2 + a_2 X_2^2 \\ &\vdots \\ Y_5 + V_5 &= a_0 + a_1 X_5 + a_2 X_5^2 \end{aligned}\right\}$$

condition equations

$$v_1 - a_0 - a_1 X_1 - a_2 X_1^2 = -Y_1$$

$$\vdots$$

$$v_5 - a_0 - a_1 X_5 - a_2 X_5^2 = -Y_5$$

rearrange with v's and parameters on left, observations on the right.

$$v_1 - a_0 - a_1 x_1 - a_2 x_1^2 = -y_1$$
$$\vdots$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_5 \end{bmatrix} + \begin{bmatrix} -1 & -x_1 & -x_1^2 \\ -1 & -x_2 & -x_2^2 \\ & \vdots & \\ -1 & -x_5 & -x_5^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -y_1 \\ -y_2 \\ \vdots \\ -y_5 \end{bmatrix}$$

$$V \quad + \quad B \quad x \quad = \quad f$$

$$v + Bx = f$$

$$\boxed{v + B\Delta = f}$$

vector/matrix form of condition eqn's for indirect obs model

Ind. Obs

$$v + B\Delta = f \;, \quad W$$

$$v = f - B\Delta$$

$$v^T W v \rightarrow \text{minimum}$$

$$(f - B\Delta)^T W (f - B\Delta)$$

$$(f^T - \Delta^T B^T) W (f - B\Delta)$$

$$(f^T W - \Delta^T B^T W)(f - B\Delta)$$

here we derive the solution to the least squares problem (continued next page)

$$(f^T W - \Delta^T B^T W)(f - B\Delta) \qquad \underset{1,n}{V^T}\,\underset{n,n}{W}\,\underset{n,1}{V} \quad (1,1)$$

$$f^T W f + \Delta^T B^T W B \Delta - f^T W B \Delta - \Delta^T B^T W f$$

$$- f^T W B \Delta$$

$$\boxed{f^T W f + \Delta^T \underline{B^T W B} \Delta - 2 f^T W B \Delta} \rightarrow \text{minimize}$$

$$\frac{d}{d\Delta}(\cdot) = 0, \quad \frac{d}{dx} \underline{Ax = A}, \quad \frac{d}{dx} x^T A x = 2x^T A, \quad A \text{ symmetric}$$

special rules for derivatives with respect to a vector

$$\frac{d}{d\Delta}\square = 0 + \cancel{2}\Delta^T B^T W B - \cancel{2} f^T W B = 0$$

$$B^T W B \Delta - B^T W f = 0$$

$$\underbrace{B^T W B}_{N} \Delta = \underbrace{B^T W f}_{t}, \quad N\Delta = t$$

normal equations

$$\Delta = N^{-1} t, \quad v = f - B\Delta$$

Final Result:

$$\boxed{\begin{aligned} \Delta &= N^{-1} t \\ v &= f - B\Delta \end{aligned}}$$

LS 1. estimation
2. error propagation

$y = Ax$, know Σ_{xx}

$$\boxed{\Sigma_{yy} = A \Sigma_{xx} A^T}$$

$W_i = \sigma_0^2 / \sigma_i^2$, $W = \sigma_0^2 \Sigma^{-1}$

$\Sigma = \sigma_0^2 Q$

$(Q = W^{-1})$

$Q_{\hat{x}\hat{x}} = N^{-1}$