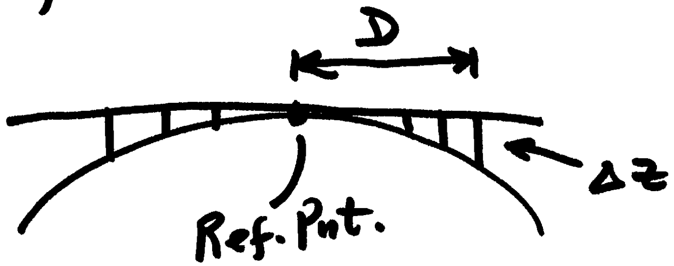
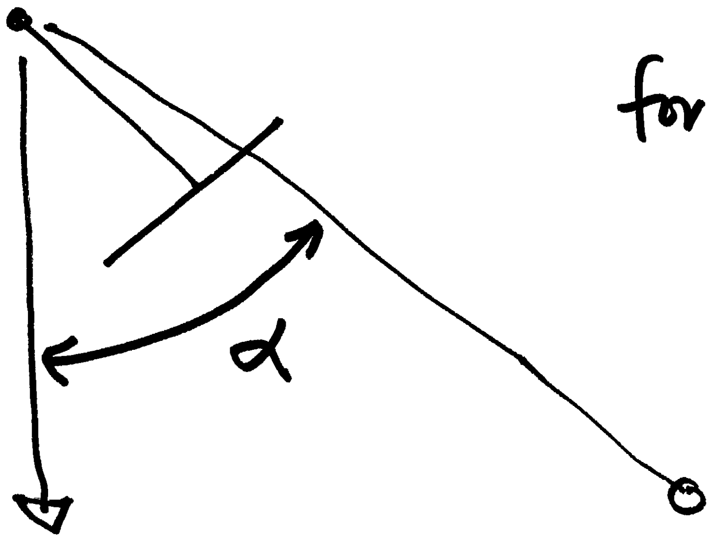
HW3 correct h's, use $R = 6371$ km, 6371000 m



correction: subtract Δz



for atmospheric refraction question – remember α is angle of ray with respect to nadir direction (not the same as optical axis for obliques)

Exam : Tues. 19th

- material covered Thursday 14th
- excludes rectification
- short answer, vocabulary, problems (calculator)
- look @ old exams

Taylor series

$$F_1(x_1, x_2) \overset{(\approx)}{=} 0 = F_1(x_1^0, x_2^0) + \frac{\partial F_1}{\partial x_1}\Delta x_1 + \frac{\partial F_1}{\partial x_2}\Delta x_2 + \cdots$$

$$F_2(x_1, x_2) = 0 = F_2(x_1^0, x_2^0) + \frac{\partial F_2}{\partial x_1}\Delta x_1 + \frac{\partial F_2}{\partial x_2}\Delta x_2 + \cdots$$

$$\begin{bmatrix} -F_1(x_1^0, x_2^0) \\ -F_2(x_1^0, x_2^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

matrix of partial derivatives

Jacobian

$$-F^0 = J\Delta$$

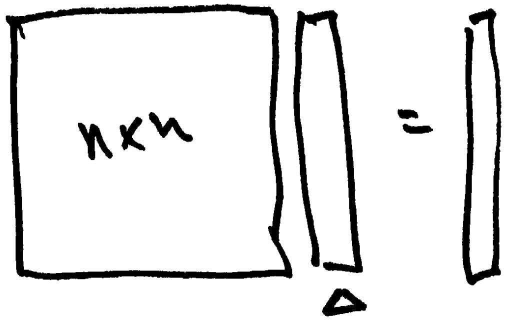
nD $$\boxed{\Delta = -J^{-1}F^0}$$

J = jacobian

Show 2D version of prior 1D Newton Iteration, extend to nD

1D $$\boxed{\Delta x = \frac{-F(x_0)}{F'(x_0)}}$$

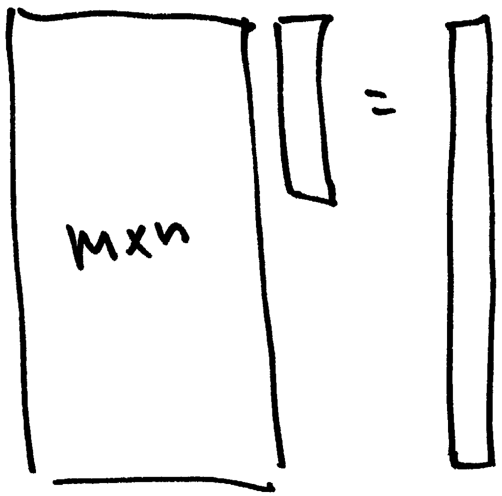
conventional Newton Iteration



solve a uniquely determined system

NL LS Newton Iteration

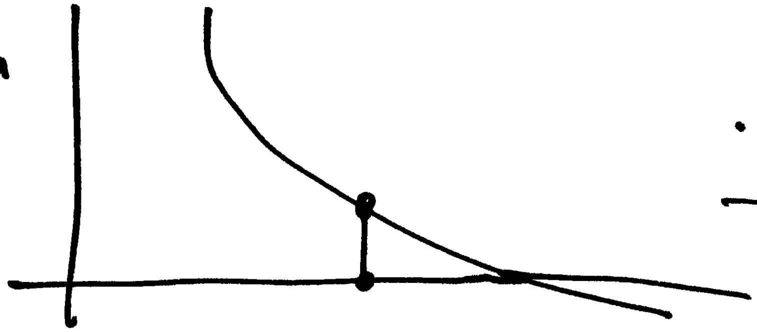
$m > n$

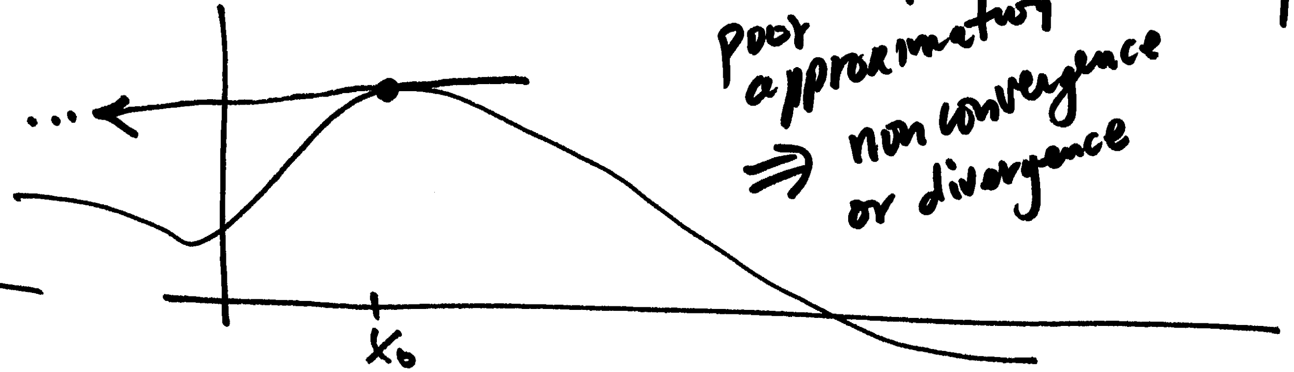


solve an overdetermined system

equations to be solved at each step of Newton Iteration.

good approximation
$\Rightarrow$ favorable convergence





poor approximation
$\Rightarrow$ non convergence or divergence

NL LS for indirect observations (LS method)

$$F(\hat{l}, x) = \hat{l} - G(x) = 0$$

$$F(\hat{l}, x) = 0 \approx F(l^0, x^0) + \underbrace{\frac{\partial F}{\partial l}}_{= I_n} \Delta l + \underbrace{\frac{\partial F}{\partial x}} \Delta x$$

$$\frac{\partial F}{\partial x} = B = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_u} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_c}{\partial x_1} & \cdots & \frac{\partial F_c}{\partial x_u} \end{bmatrix}$$

$$0 = \underbrace{F(l^0, x^0)}_{l^0 - G(x^0)} + \Delta l + B\,\Delta x \qquad \underbrace{-F(l, x^0)}_{f} = V + B\,\Delta$$

$$\boxed{V + B\Delta = f}$$

$$x^0 + \Delta x = \hat{x}$$

$$\underbrace{l^0 + \Delta l}_{} = \hat{l}\,, \quad \underbrace{l + v}_{} = \hat{l} \qquad \Delta l = V + l - l^0$$

$$0 = l^0 - G(x^0) + V + l - l^0 + B\Delta$$

$$0 = \underbrace{l - G(x^0)}_{F(l, x^0)} + v + B\Delta$$

here we make an important distinction between v and Δl. They both correct l, but in significantly different ways.

$V + B\Delta = f$, $\quad \Delta = (B^T W B)^{-1} B^T W f$ (have W)

$X^0 + \Delta = X^1$

continue until Δ small

$V = f - B\Delta$

$\hat{l} = l + v$

statistical test

E.P.

NL LS flow chart for Ind. Obs.

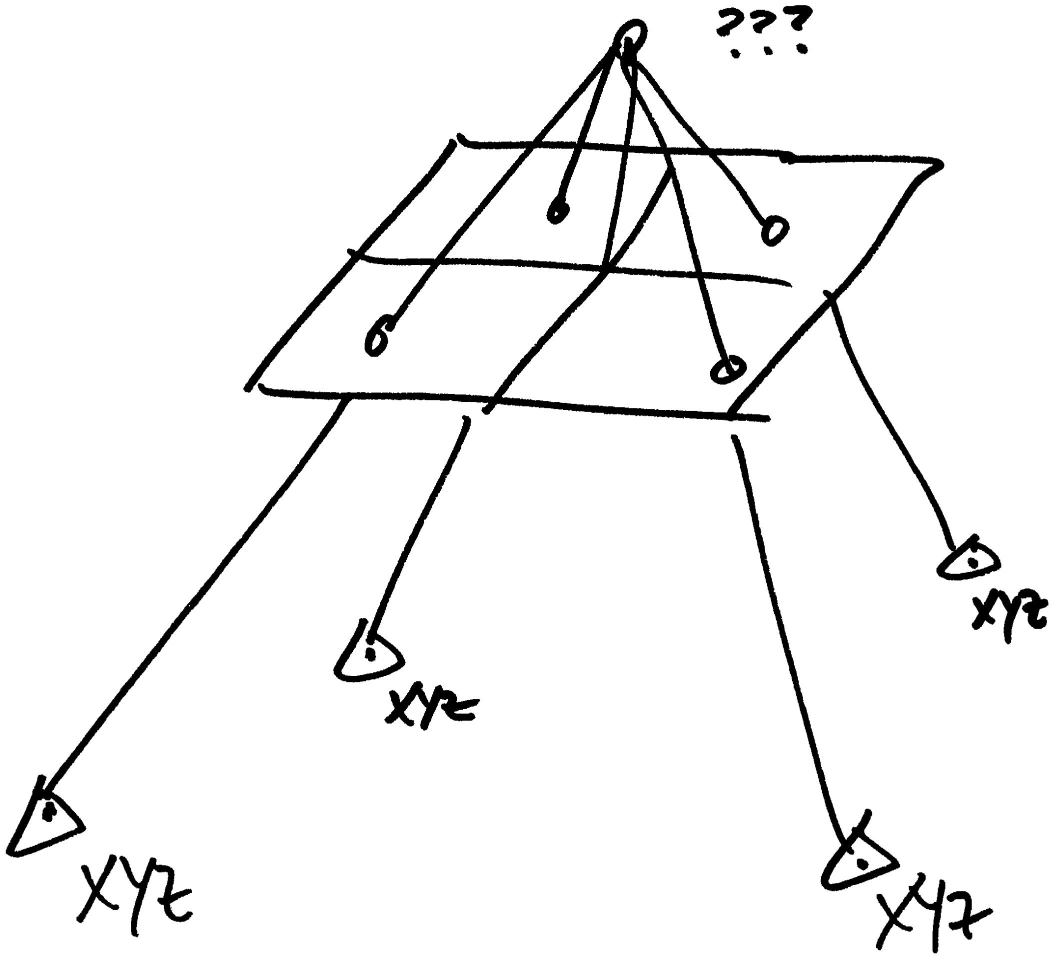
1. $F(\hat{l}, x) = \hat{l} - G(x) = 0$
2. initial approx. for X : X°
3. obtain weights $w_i = \sigma_0^2 / \sigma_i^2$, W
4. eval $B = \partial F / \partial x \big|_{x^{\circ}}$, eval $f = -F(l, x^{\circ})$
5. solve "linear" LS $\Delta = (B^T W B)^{-1} B^T W f$
6. update param's $X_o\,(\text{new}) = X_o\,(\text{old}) + \Delta$

no

7. check convergence, Δ small?

yes

8. $V = f - B\Delta$, $\hat{l} = l + v$
9. QA, stat. test, E.P.



resection problem

Solving for Exterior Orientation

Elements $X_L Y_L Z_L, \omega \varphi \kappa$

$$F_x = x - x_0 + f\frac{u}{w} = 0$$

$$F_y = y - y_0 + f\frac{v}{w} = 0$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} x - x_L \\ y - y_L \\ z - z_L \end{pmatrix}$$

quotient rule

$$\frac{d}{dx}\frac{u}{v} = \frac{v\,du - u\,dv}{v^2}$$

$$\frac{\partial F_x}{\partial p} = f\left(\frac{w\frac{du}{\partial p} - u\frac{dw}{\partial p}}{w^2}\right)$$

$$= \frac{f}{w}\left(\frac{du}{dp} - \frac{u}{w}\frac{dw}{dp}\right)$$

$$\frac{\partial F_y}{\partial p} = f\left(\frac{w\frac{dv}{dp} - v\frac{dw}{\partial p}}{w^2}\right)$$

$$\frac{f}{w}\left(\frac{dv}{\partial p} - \frac{v}{w}\frac{\partial w}{\partial p}\right)$$

for the W^2 in the denominator: bring one outside the parenthesis, combine the other with factors in the numerator.

$$U = m_{11}(X - X_c) + m_{12}(Y - Y_c) + m_{13}(Z - Z_c)$$

$$V = m_{21}(X - X_c) + m_{22}(Y - Y_c) + m_{23}(Z - Z_c)$$

$$W = m_{31}(X - X_c) + \cdots$$

$$M = M_K M_\varphi M_\omega, \quad \frac{\partial M}{\partial \omega} = M_K M_\varphi \frac{\partial M_\omega}{\partial \omega}$$

$$M_\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix}$$

$$\frac{\partial M_\omega}{\partial \omega} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\omega & \cos\omega \\ 0 & -\cos\omega & -\sin\omega \end{bmatrix}$$

first derive $\frac{\partial M_x}{\partial x}$ where x is any of the 3 orientation angles.

$$M = M_K M_\varphi M_\omega \quad , \quad \frac{\partial M}{\partial \varphi} = M_K \frac{\partial M_\varphi}{\partial \varphi} M_\omega$$

$$M_\varphi = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix}$$

$$\frac{\partial M_\varphi}{\partial \varphi} = \begin{bmatrix} -\sin\varphi & 0 & -\cos\varphi \\ 0 & 0 & 0 \\ \cos\varphi & 0 & -\sin\varphi \end{bmatrix}$$

now for ϕ

$$\frac{\partial M}{\partial K} = \frac{\partial M_K}{\partial K} M_\varphi M_\omega$$

$$M_K = \begin{bmatrix} \cos K & \sin K & 0 \\ -\sin K & \cos K & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial M_K}{\partial K} = \begin{bmatrix} -\sin K & \cos K & 0 \\ -\cos K & -\sin K & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial F_x}{\partial p} = \frac{f}{w}\left(\frac{du}{dp} - \frac{u}{w}\frac{dw}{dp}\right)$$

$$\frac{\partial}{\partial \omega}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{\partial M}{\partial \omega}\begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

$$\frac{\partial}{\partial K}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{\partial M}{\partial K}\begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

$$\frac{\partial}{\partial \varphi}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{\partial M}{\partial \varphi}\begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

now K,

now get partials of the elements of $\begin{bmatrix} u \\ v \\ w \end{bmatrix}$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

$$\frac{\partial}{\partial X_L} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -m_{11} \\ -m_{21} \\ -m_{31} \end{pmatrix}$$

$$\frac{\partial}{\partial Y_L} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -m_{12} \\ -m_{22} \\ -m_{32} \end{pmatrix}$$

$$\frac{\partial}{\partial Z_L} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -m_{13} \\ -m_{23} \\ -m_{33} \end{pmatrix}$$

partials of $\begin{pmatrix} u \\ v \\ w \end{pmatrix}$ with respect to X_L, Y_L, Z_L

for resection

$$
B \quad
\begin{array}{l}
\\ \\ \text{point 2}\begin{cases} \\ \end{cases} \\ \text{point 3}\begin{cases} \\ \end{cases} \\ \\ \text{point } n\begin{cases} \\ \end{cases}
\end{array}
\begin{bmatrix}
\partial F_{x_1}/\partial \omega & \partial F_{x_1}/\partial \varphi & \partial F_{x_1}/\partial \kappa & \partial F_{x_1}/\partial X_L & \partial F_{x_1}/\partial Y_L & \partial F_{x_1}/\partial Z_L \\
\partial F_{y_1}/\partial \omega & \partial F_{y_1}/\partial \varphi & \partial F_{y_1}/\partial \kappa & \partial F_{y_1}/\partial X_L & \partial F_{y_1}/\partial Y_L & \partial F_{y_1}/\partial Z_L \\
- & & & & & \\
- & & & & & \\
- & & & & & \\
- & & & & & \\
\vdots & & & & & \\
- & & & & & \\
- & & & & &
\end{bmatrix}
$$

right hand side vector $\quad V + B\sigma = f$

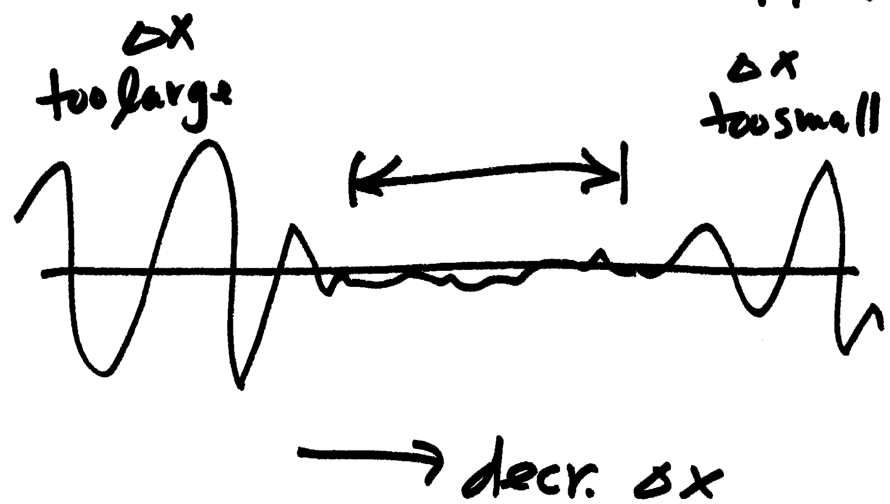
$$f = \begin{bmatrix} -F_{x_1} \\ -F_{y_1} \\ -F_{x_2} \\ -F_{y_2} \\ \vdots \\ \vdots \end{bmatrix} = -\begin{bmatrix} x_1 - x_0 + f\,u_1/w_1 \\ y_1 - y_0 + f\,v_1/w_1 \\ x_2 - x_0 + f\,u_2/w_2 \\ y_2 - y_0 + f\,v_2/w_2 \\ \vdots \\ \vdots \end{bmatrix}$$

$\underbrace{W, B, f}$

Δ

for B & f
evaluate at current value of parameters

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



<u>use small Δx</u>

$$\frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{\partial F}{\partial p} \approx \frac{F(p+\Delta p, q, r) - F(p,q,r)}{\Delta p}$$

$$\frac{\partial F}{\partial q} \approx \frac{F(p, q+\Delta q, r) - F(p,q,r)}{\Delta q}$$

$$\vdots$$

Numerical approximation of derivatives and partial derivatives. It is a VERY POWERFUL technique.