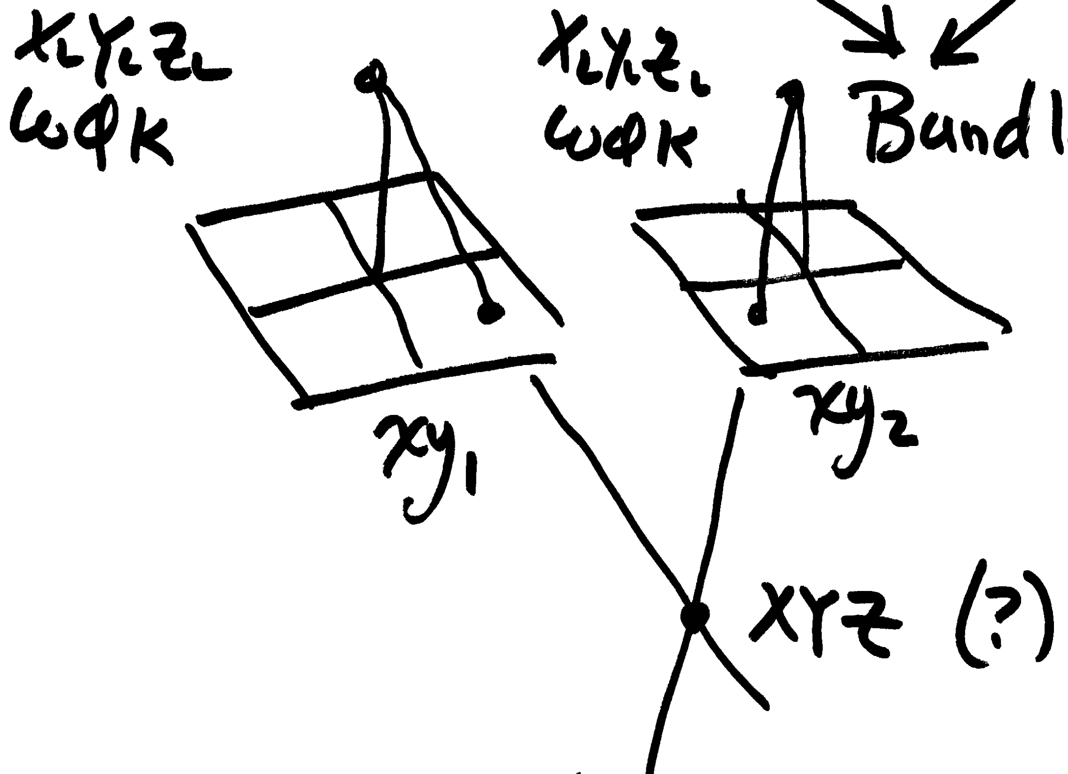
Space Intersection (Resection)

Bundle Block Adjustment



$$M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$F_x : x - x_0 + f \frac{u}{w} = 0$$

$$F_y : y - y_0 + f \frac{v}{w} = 0$$

<u>Know</u>

$$\left[\underbrace{X_L Y_L Z_L\, \omega\, \varphi\, \kappa}_{\text{Exterior Orientation}}\ \underbrace{x_0\, y_0\, f}_{\text{Interior Orientation}}\right]_{1\,\&\,2}$$

observations $x_1 y_1$ + $x_2 y_2$

unknowns : XYZ

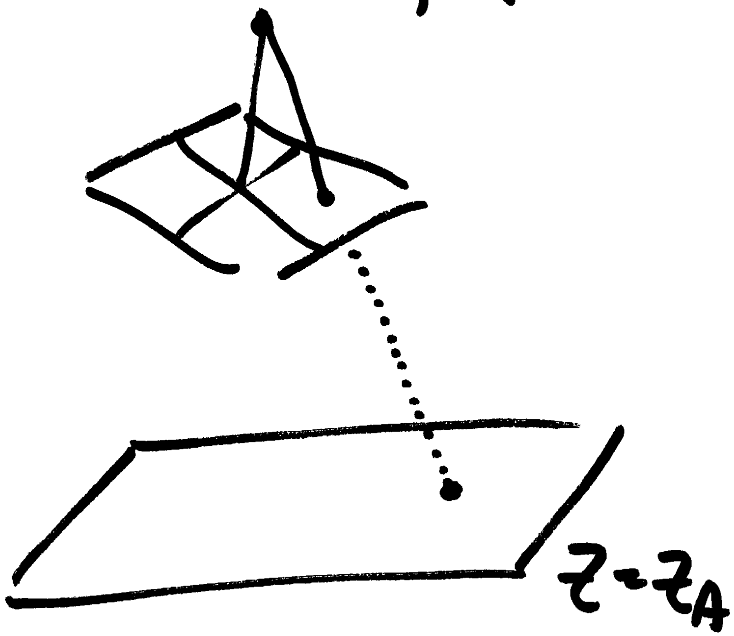
$$
\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \end{bmatrix}
+
\begin{bmatrix}
\partial F_{x_1}/\partial x & \dfrac{\partial F_{x_1}}{\partial y} & \dfrac{\partial F_{x_1}}{\partial z} \\
\partial F_{y_1}/\partial x & \partial F_{y_1}/\partial y & \partial F_{y_1}/\partial z \\
\dfrac{\partial F_{x_2}}{\partial x} & \dfrac{\partial F_{x_2}}{\partial y} & \partial F_{x_2}/\partial z \\
\partial F_{y_2}/\partial x & \dfrac{\partial F_{y_2}}{\partial y} & \dfrac{\partial F_{y_2}}{\partial z}
\end{bmatrix}
\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}
=
\begin{bmatrix} -F_{x_1} \\ -F_{y_1} \\ -F_{x_2} \\ -F_{y_2} \end{bmatrix}
$$

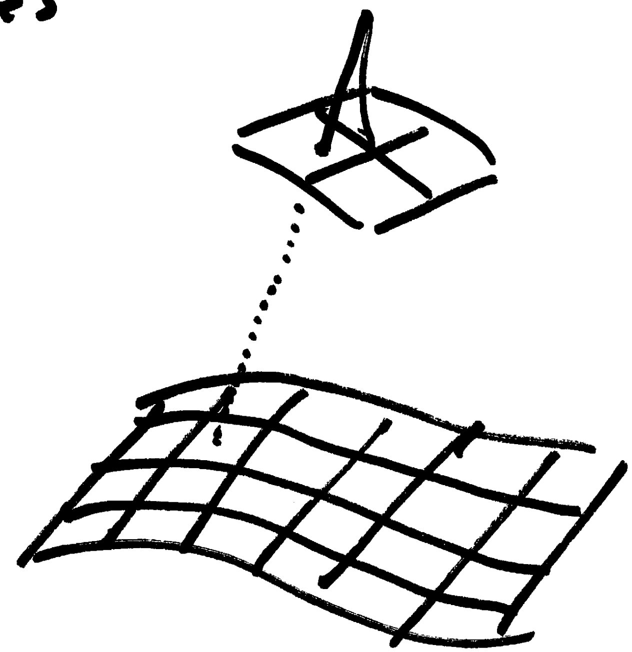
$$V + B \cdot \Delta = f$$

need initial approximations for X, Y, Z

$n = 4$, $n_0 = 3$, $r = 1$, $c = n = 4$ ind. obs.

Intersection, special cases





$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

$$x' = x - x_0$$
$$y' = y - y_0$$

$$\underbrace{\frac{1}{\lambda} M^T \begin{pmatrix} x' \\ y' \\ -f \end{pmatrix}}_{\begin{pmatrix} u \\ v \\ w \end{pmatrix}} = \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}, \qquad \frac{1}{\lambda} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z_A - Z_L \end{pmatrix}$$

fix

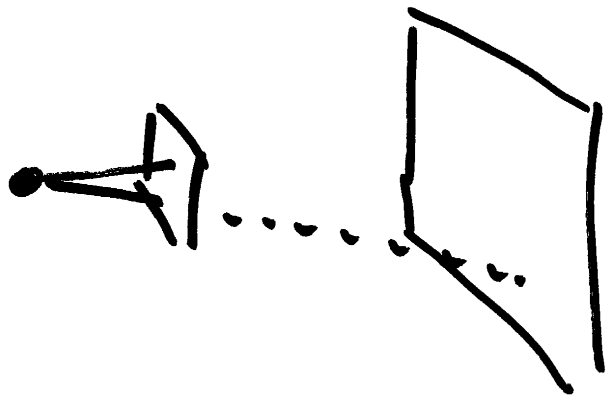
$$\frac{u}{w} = \frac{X - X_L}{Z_A - Z_L}, \quad X - X_L = (Z_A - Z_L)\frac{u}{w}$$

$$\frac{v}{w} = \frac{Y - Y_L}{Z_A - Z_L}, \quad Y - Y_L = (Z_A - Z_L)\frac{v}{w}$$

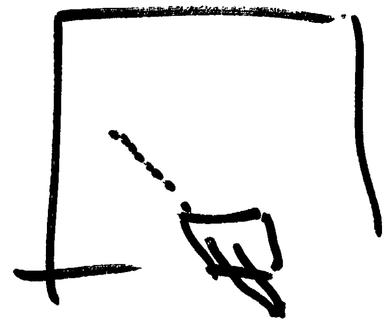
$$X = X_L + (Z_A - Z_L)\, u/w$$

$$Y = Y_L + (Z_A - Z_L)\, v/w$$

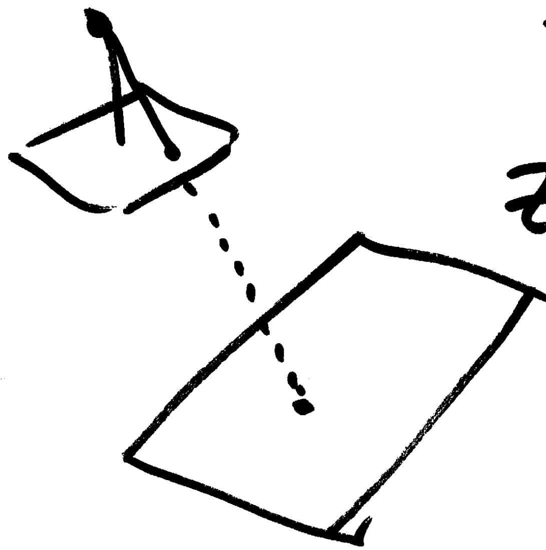
Closed form
no iterations



$X = X_A$



$Y = Y_A$



$Z = a_0 + a_1 X + a_2 Y$

intersection with arbitrary plane

$$X = X_L + (a_0 + a_1 X + a_2 Y - Z_L)\,\frac{u}{w}$$

$$Y = Y_L + (a_0 + a_1 X + a_2 Y - Z_L)\,\frac{v}{w}$$

$$X - a_1 \frac{u}{w} X - a_2 \frac{u}{w} Y = X_L + a_0 \frac{u}{w} - \frac{u}{w} Z_L$$

$$Y - a_1 \frac{v}{w} X - a_2 \frac{v}{w} Y = Y_L + a_0 \frac{v}{w} - \frac{v}{w} Z_L$$

$$\begin{bmatrix} 1 - a_1 \frac{u}{w} & -a_2 \frac{u}{w} \\ -a_1 \frac{v}{w} & 1 - a_2 \frac{v}{w} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

solve 2 eqn + 2 unk for XY

$$X = X_L + (Z - Z_L)\frac{u}{w} \rightarrow C_1$$

$$Y = Y + (Z - Z_L)\frac{v}{w} \rightarrow C_2$$

$$X = X_L + C_1 Z - C_1 Z_L$$

unk: X, Y, Z

$$Y = Y_L + C_2 Z - C_2 Z_L$$

$$X - C_1 Z = X_L - C_1 Z_L$$

$$Y - C_2 Z = Y_L - C_2 Z_L$$

$$\begin{bmatrix} 1 & 0 & -C_1 \\ 0 & 1 & -C_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_L - C_1 Z_L \\ Y_L - C_2 Z_L \end{bmatrix}$$

2 "linear" eqns
3 unknowns

if 2 (or more)

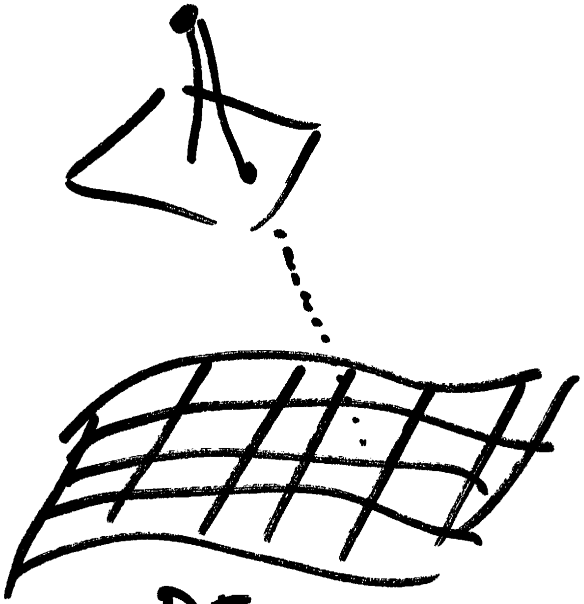
↳ 4 equations in 3 unknowns

linear LS problem

⇒ gives good approximations for XYZ

so nonlinear intersection problem

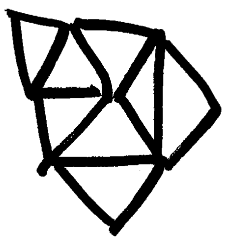
converges



DEM

grid 

TIN



Estimate Z_0

→ Intersect ray with Z_0

$\Rightarrow X, Y$

Evaluate DEM at XY

another for Z_0

iterate to convergence

$X_0 Y_0 Z_0$ don't change