bundle block adjustment (LS)

- simultaneous resection / intersection
- any number of images 1, 2 — 1000's
- any number of object points < pass, tie / control.
- muliple sensors
- base on collinearity equation

$$\overset{x'}{x - x_0} = -f\frac{u}{w}, \quad \overset{y'}{y - y_0} = -f\frac{v}{w}$$

$$F_x : \; x' + f\frac{u}{w} = 0$$

$$F_y : \; y' + f\frac{v}{w} = 0$$

$$\boxed{\begin{matrix} X_L & \omega \\ Y_L & \varphi \\ Z_L & K \end{matrix}}$$

Exterior Orientation Group

$$\boxed{\begin{matrix} X \\ Y \\ Z \end{matrix}}$$

Ground / Object pnt. group

$$\boxed{\begin{matrix} x_0 & K_1 & P_1 \\ y_0 & K_2 & P_2 \\ f & K_3 & \end{matrix}}$$

internal camera param. group

Interior Orient

$$\boxed{x, y}$$

observation group

extended collin. equations

$$x' = x - x_0 \,, \quad y' = y - y_0 \,, \quad \begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix} , \quad r = \sqrt{(x')^2 + (y')^2}$$

$$F_x : \; x' - dx_r - dx_d - dx_{atm} + f\frac{u}{w} = 0$$

$$F_y : \; y' - dy_r - dy_d - dy_{atm} + f\frac{v}{w} = 0$$

assume nadir imagery

$$dx_r = x' \cdot \frac{dr}{r} = x' \cdot \frac{k_1 r^3 + k_2 r^5 + k_3 r^7}{r}$$

$$dy_r = y' \cdot \frac{dr}{r} = y' \cdot \frac{\vdots}{\vdots}$$

$$dx_d = P_1 (r^2 + 2(x')^2) + 2P_2 x'y'$$

$$dy_d = P_2 (r^2 + 2(y')^2) + 2P_1 x'y'$$

if nadir img.

$$dx_{atm} = x' \cdot \frac{dr}{r} = x' \cdot \frac{K(r + r^3/f^2)}{r}$$

K: atm. refr. formula

$$dy_{atm} = y' \cdot \frac{dr}{r} = y' \cdot \frac{''}{''}$$

linearized cond. eqn. (x) point i, image j

$$V_{x_i} + \Bigg[\underbrace{\frac{\partial F_{x_i}}{\partial X_{Lj}} \quad \frac{\partial F_{x_i}}{\partial Y_{Lj}} \quad \frac{\partial F_{x_i}}{\partial Z_{Lj}} \quad \frac{\partial F_{x_i}}{\partial \omega_j} \quad \frac{\partial F_{x_i}}{\partial \varphi_j} \quad \frac{\partial F_{x_i}}{\partial \kappa_j}}_{E/O} \quad \cdots$$

$$\underbrace{\frac{\partial F_{x_i}}{\partial X_i} \quad \frac{\partial F_{x_i}}{\partial Y_i} \quad \frac{\partial F_{x_i}}{\partial Z_i}}_{\text{ground/object point}} \quad \underbrace{\frac{\partial F_{x_i}}{\partial x_0} \quad \frac{\partial F_{x_i}}{\partial y_0} \quad \frac{\partial F_{x_i}}{\partial f_{oc}} \quad \frac{\partial F_{x_i}}{\partial k_1} \quad \cdots}_{\text{camera calibration}} \Bigg]$$

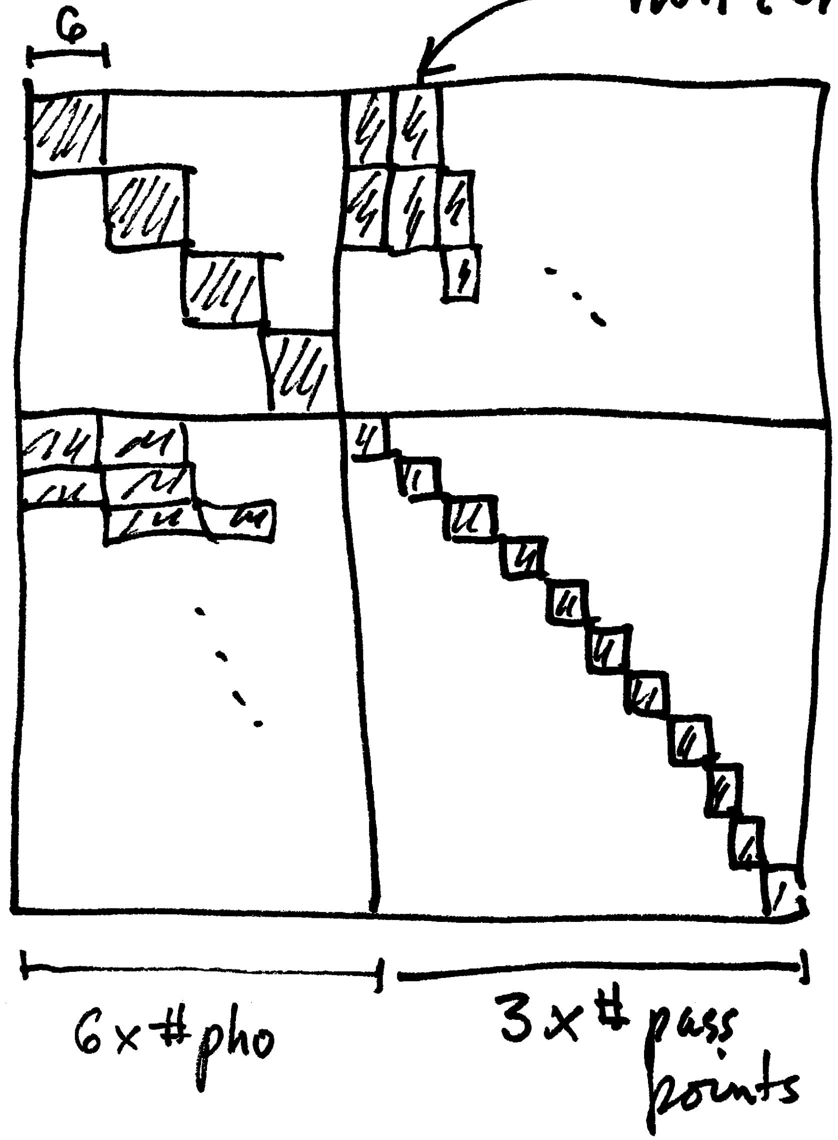
$$\begin{bmatrix} \Delta X_{Lj} \\ \Delta Y_{Lj} \\ \Delta Z_{Lj} \\ \Delta \omega_j \\ \Delta \varphi_j \\ \vdots \end{bmatrix} = f$$

$$f = -F_{x_i}$$

$$\boxed{V + B\Delta = f}$$

$$N = B^T W B$$

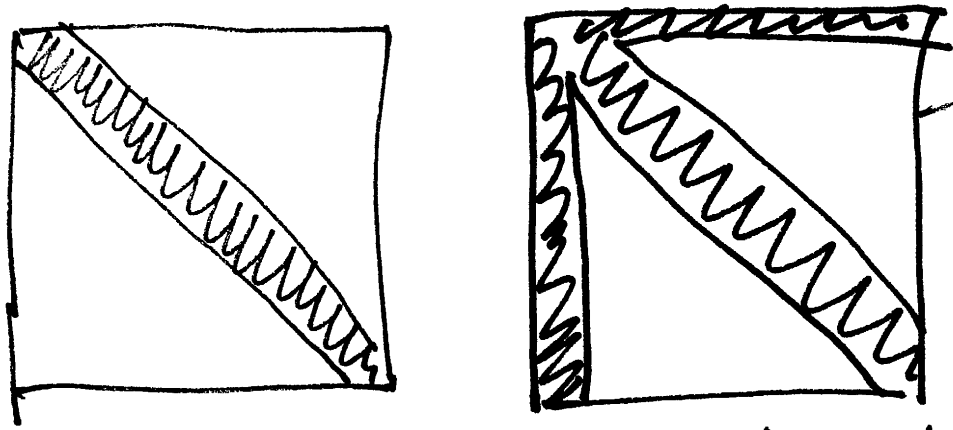
N normal equation matrix



nonzero if point i falls on photo j

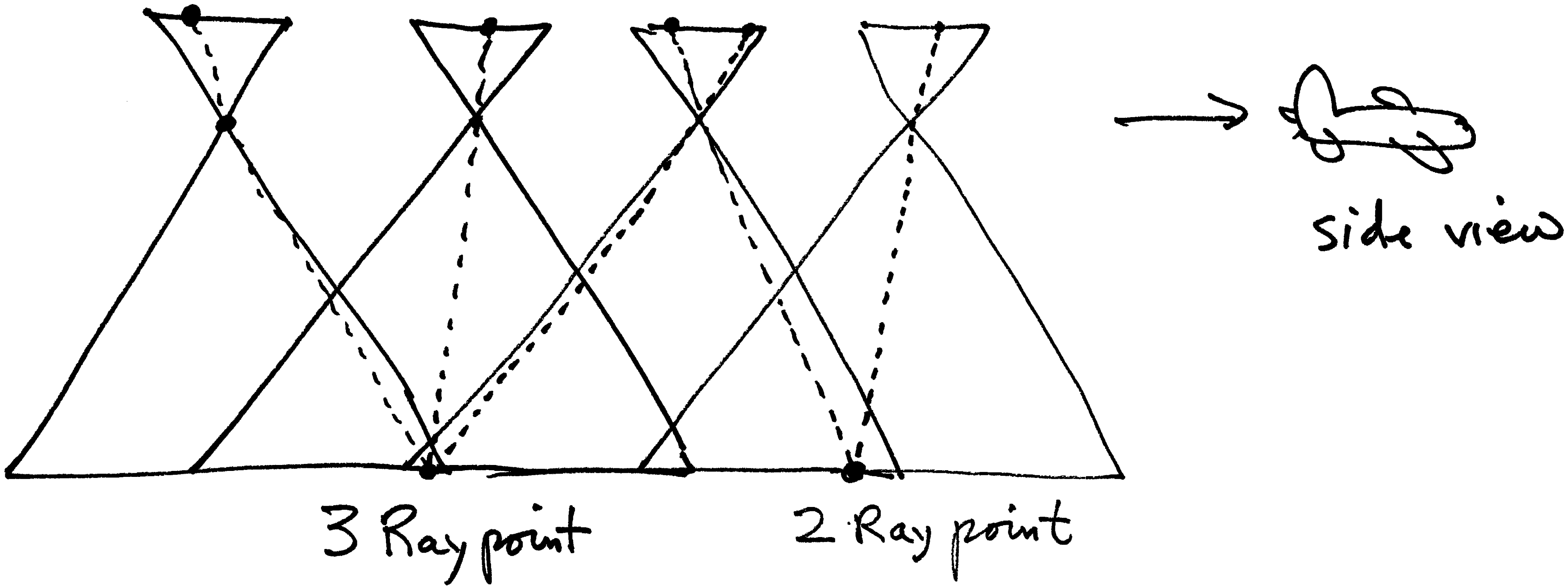
Matlab: spy(N)

large blocks
- block gauss elim.
- eliminate ground points
- fold into camera/image

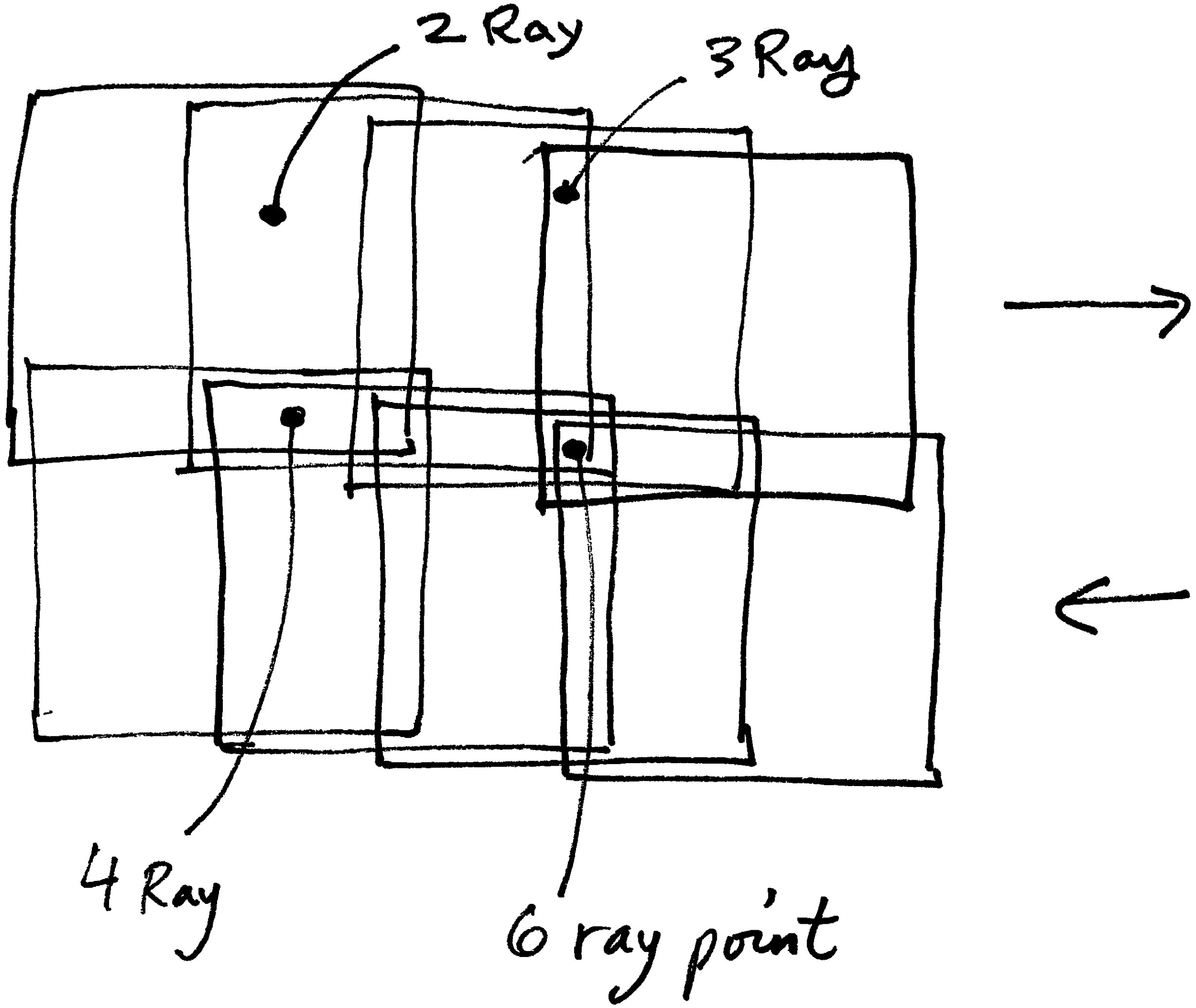


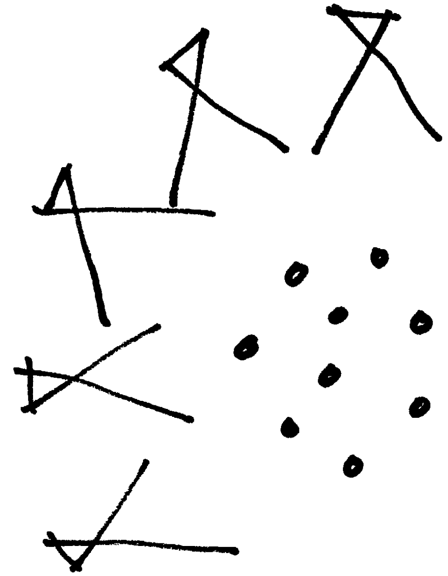
Structure of N after elimination

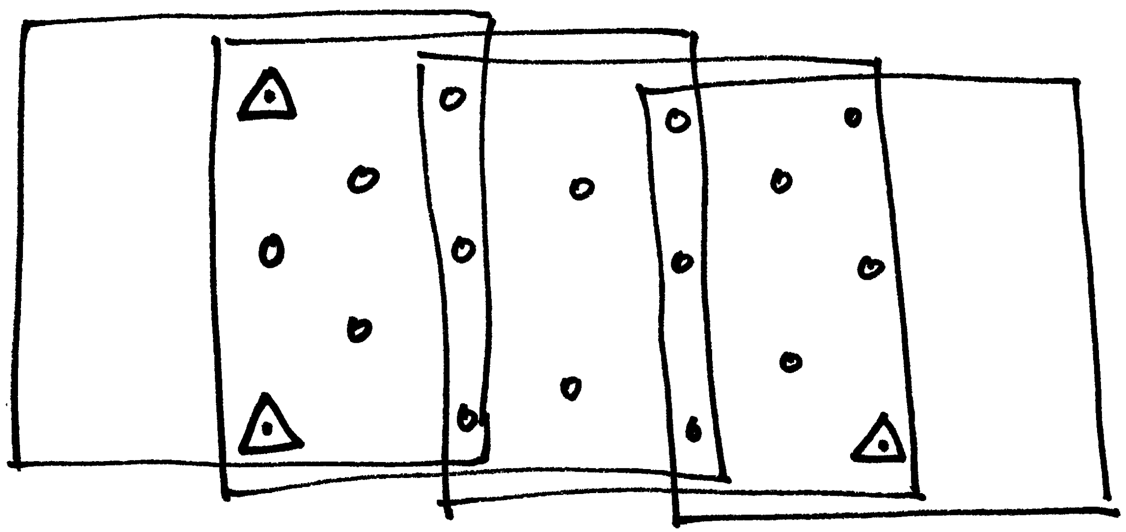
use sparse matrix techniques


side view
3 Ray point
2 Ray point

60% Forward Overlap







o = pass point, tie point

△ = control point

Equs

2 Ray : $12 \times 4 = 48$

3 Ray : $6 \times 6 = 36$

84 equs

redundancy = 15

Unknowns

cam/pho $6 \times 4 = 24$

ground $3 \times 15 = 45$

69 unknowns