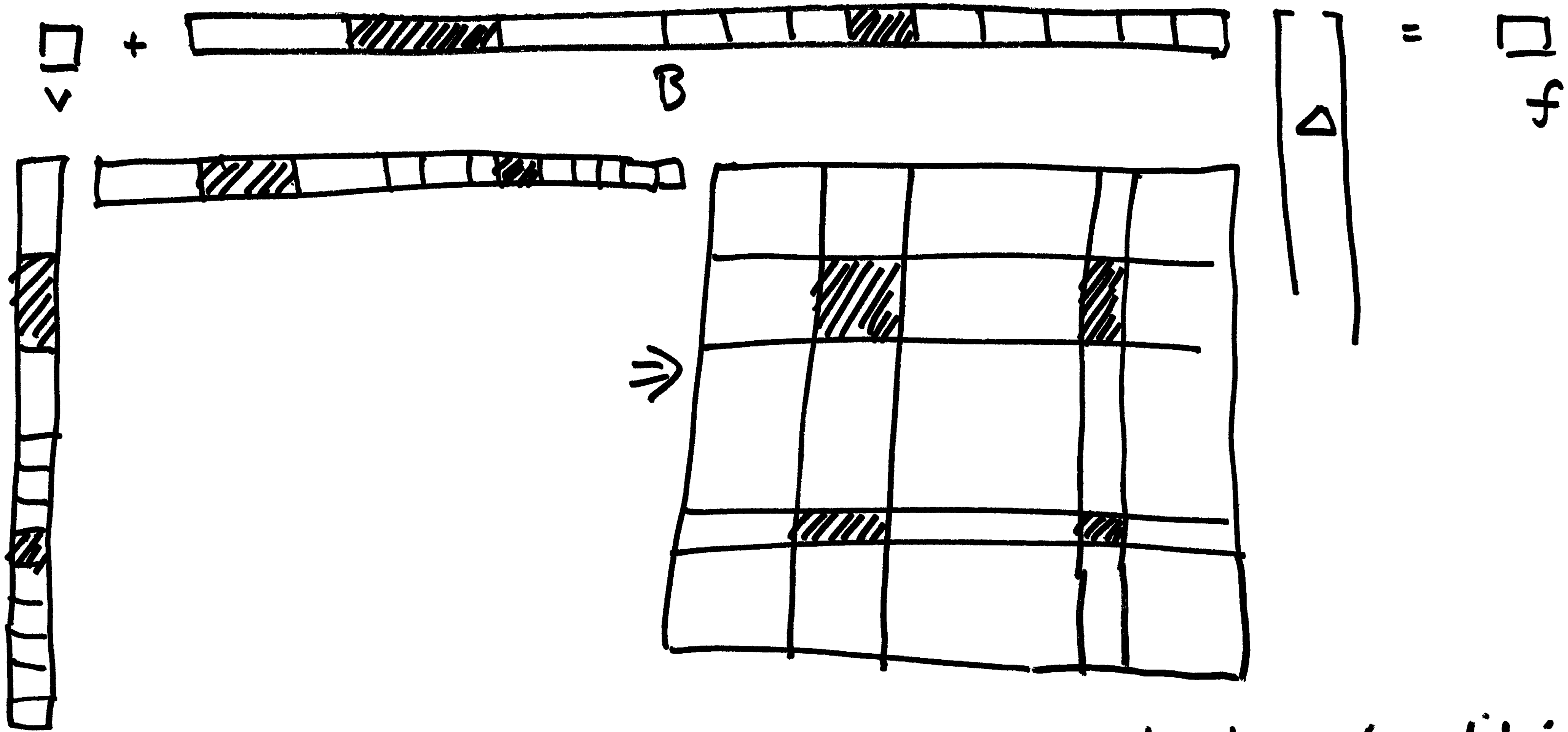
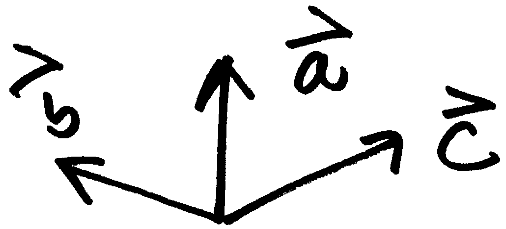
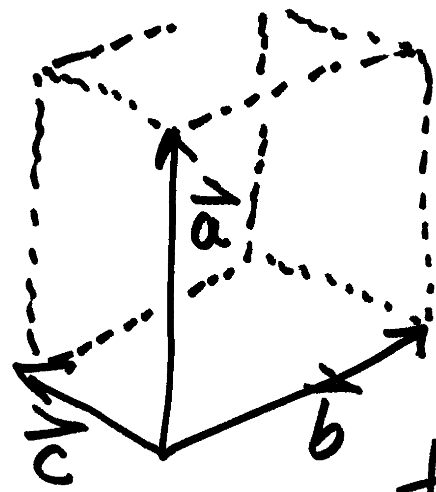
$V + B\Delta = f, \quad B^TWB, \quad B^TB$



structure of condition equation leads to corresponding structure of normal equations, we can consider B^TWB one equation at a time!

Relative Orientation





parallelepiped

<u>volume</u>

$$\vec{a} \cdot (\vec{b} \times \vec{c})$$

triple scalar product

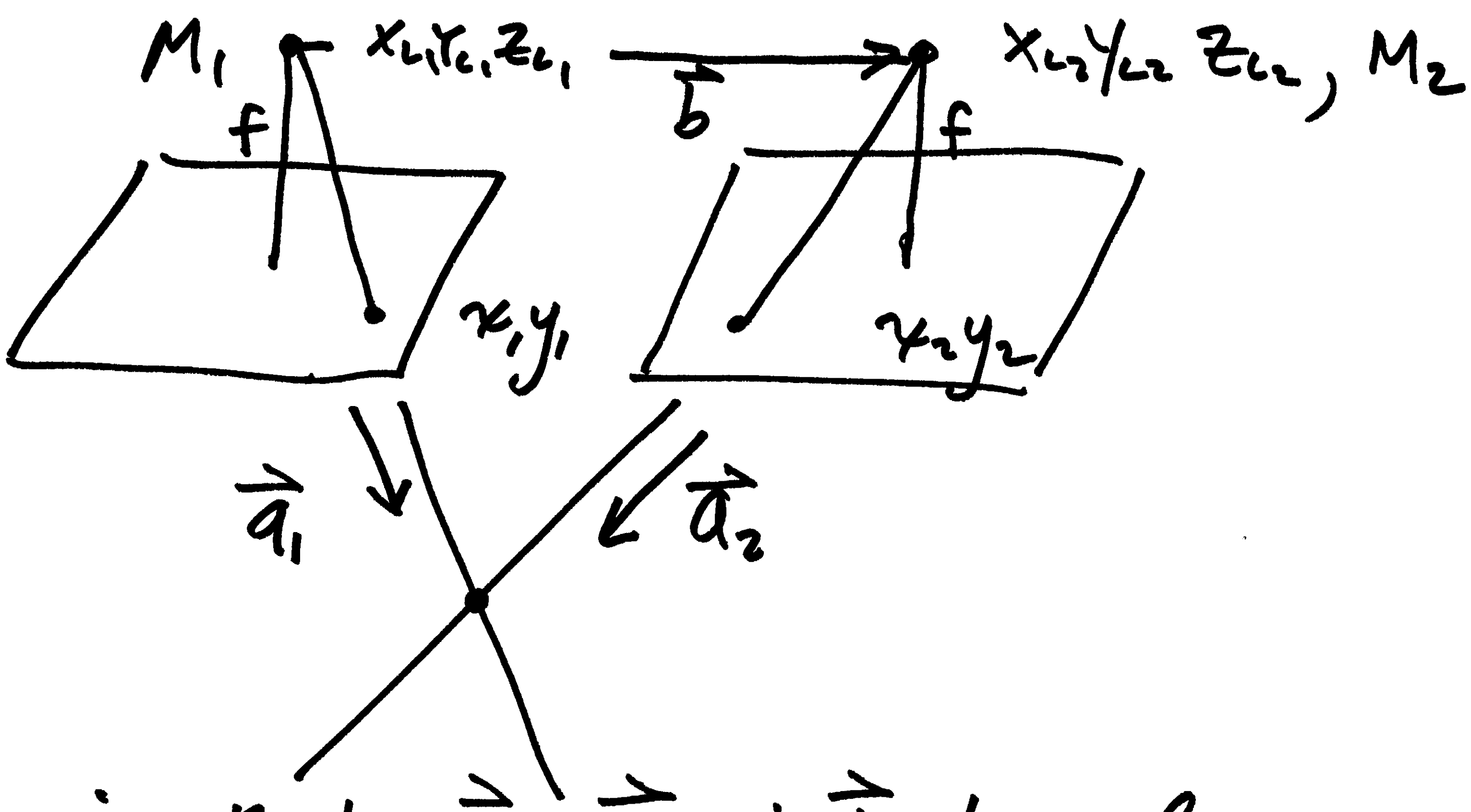
$$\vec{b} \times \vec{c}$$

$$\vec{a} \cdot \begin{vmatrix} i & j & k \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

<u>determinants</u>

If the determinant is zero, then the volume is zero and the 3 vectors are <u>COPLANAR</u>.

When we enforce triple scalar product to be <u>zero</u>, then <u>any</u> order of the 3 vectors is ok.



$$\vec{b} = \begin{bmatrix} X_{L_2} - X_{L_1} \\ Y_{L_2} - Y_{L_1} \\ Z_{L_2} - Z_{L_1} \end{bmatrix}$$

base vector

$$\vec{b} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

require that $\vec{a}_1$, $\vec{a}_2$, & $\vec{b}$ be coplanar
for 5 pts., at least

enforce coplanarity for $\vec{b}, \vec{a}_1, \vec{a}_2$

$$\vec{a}_1 = M_1^T \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ -f \end{pmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix}, \quad \vec{a}_2 = M_2^T \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix}, \quad \vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

<u>coplanarity condition equation</u>

$$F = \begin{vmatrix} b_x & b_y & b_z \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \end{vmatrix} = 0, \quad D = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$$

$$\frac{\partial D}{\partial p} = \begin{vmatrix} \partial R_1/\partial p \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ \partial R_2/\partial p \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ \partial R_3/\partial p \end{vmatrix} \longleftarrow$$

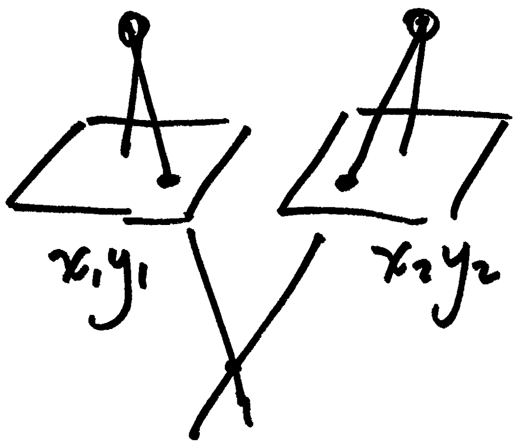
how to take partial derivatives of a determinant so we can linearize the coplanarity equations.

need to choose 5 parameters (independent)

ω_1 ω_2 b_x
φ_1 φ_2 b_y
κ_1 κ_2 b_z

independent: $\kappa_1 \varphi_1 \kappa_2 \varphi_2 \omega_2$

b_x = constant (100)
$b_y = b_z = 0$



dependent: $\omega_2 \varphi_2 \kappa_2 \, b_y \, b_z$

fix b_x constant, arbitrary
fix $\omega_1, \varphi_1, \kappa_1$

4 observations into coplanarity cond. eqn.

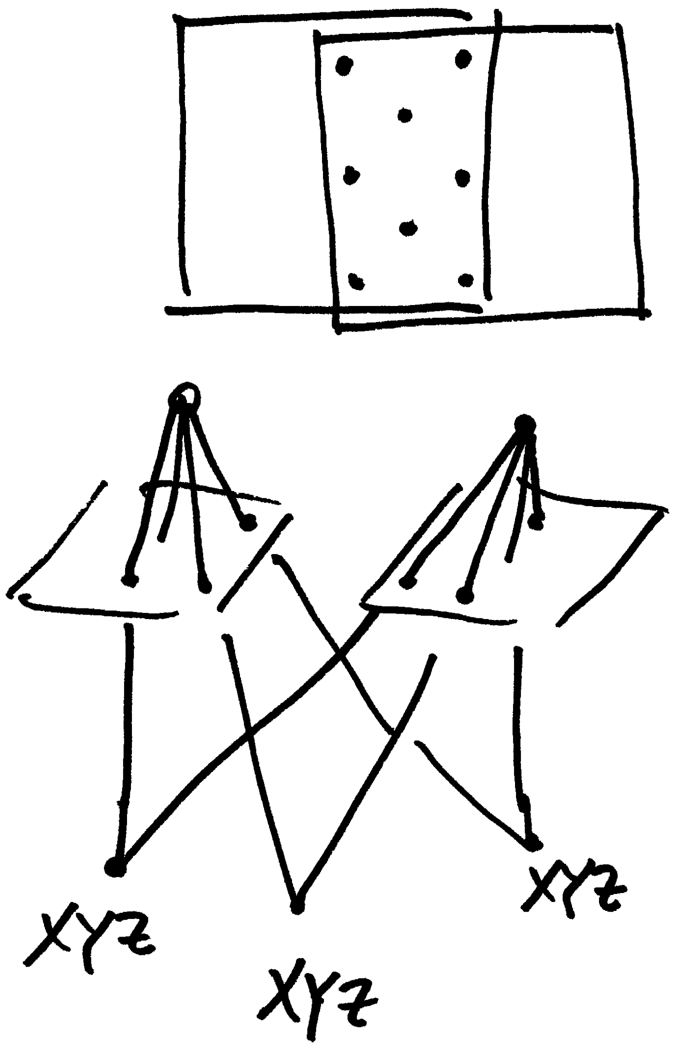
⇒ cannot use Indirect Observation

⇒ use General LS, mixed model

$Av + Bo = f$, General LS (mixed model)

Follow R/O with space intersection

$\Rightarrow$ model coordinates



object space (arbitrary)

$$\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{7\text{-par.}} \begin{pmatrix} \underline{X} \\ \underline{Y} \\ \underline{Z} \end{pmatrix}}_{\text{This part would be Absolute Orientation}}$$

model coordinates

ground coordinates
reference coords.

7 par. transf.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \lambda M \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} t_X \\ t_Y \\ t_Z \end{pmatrix}$$

ground coords.

model coords

λ : scale

Ω, Φ, K : rotations

t_X, t_Y, t_Z : shifts

Absolute Orientation

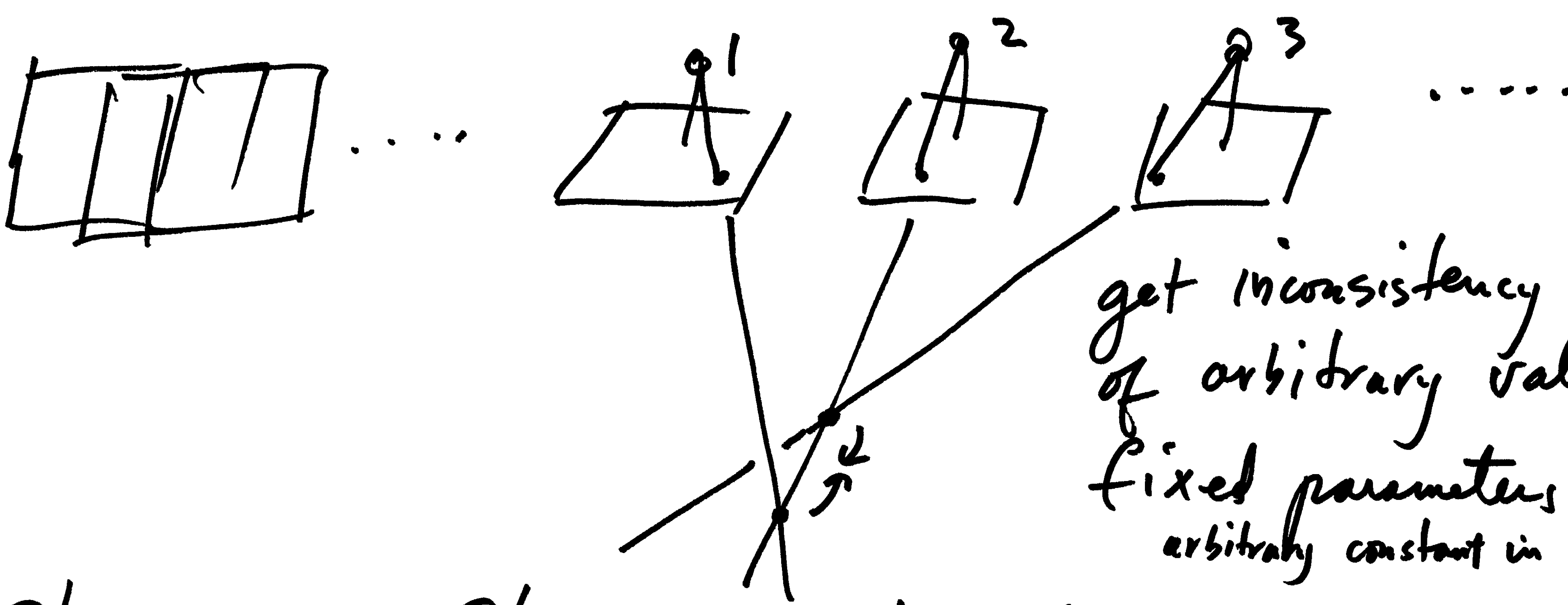
$$\left.\begin{array}{l} \text{Relative Orientation} \\ + \\ \text{Absolute Orientation} \end{array}\right\} = \text{Bundle Block Adj.}$$

2 photo bundle block

$$\left.\begin{array}{l} \omega_1, \phi_1, \kappa_1, X_{L_1}, Y_{L_1}, Z_{L_1}, \\ \omega_2\ \phi_2\ \kappa_2\ X_{L_2} Y_{L_2} Z_{L_2} \end{array}\right\} 12$$

2 photo R/O + A/O

5 par + 7 par = 12



get inconsistency because of arbitrary values of fixed parameters, i.e. base is an arbitrary constant in R/O

R/O 1 & 2, R/O 2 & 3 together include <u>Scale Restraint Equations</u>

↳ see chapter 4 in text for details

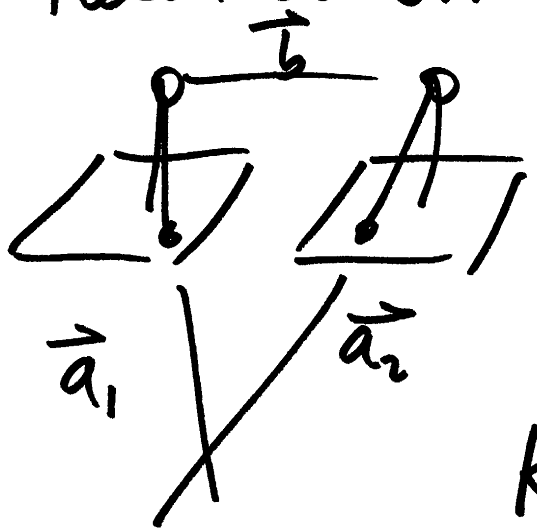
Why R/O + A/O instead of BBA ?

acquire imagery before GCP's

separate photo measurement errors from GCP errors

* with BBA you can do something equivalent with
(1) minimally constrained adjustment
(2) then fully constrained adjustment (all GCP's)

Relative Orientation from Computer Vision



$$\vec{b} \cdot (\vec{a}_1 \times \vec{a}_2) = 0$$

$$K_b : \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = K_b \vec{a} = \underline{\vec{b} \times \vec{a}}$$

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

as per prior note, we can change the order of vectors in the T.S.P. as long as we enforce it to be zero, i.e. no sign issues arise.

$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & -f \end{bmatrix}}_{C} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

image space vector
homogeneous coordinates

$$\vec{a}_1 \cdot (\vec{b} \times \vec{a}_2) \qquad \vec{a} \cdot \vec{b} = a^T b$$

$$\boxed{a_1^T K_b a_2 = 0}$$