$$\vec{a}_1 \cdot (\vec{b} \times \vec{a}_2) = 0$$ coplanarity equation

$$\underset{1,3}{a_1^T}\ \underset{3,3}{K_b}\ \underset{3,1}{a_2} = \underset{1,1}{0}$$

$$\vec{a}_1 = M_1^T \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ -f \end{pmatrix} = M_1^T \begin{pmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & -f \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

$$= \underbrace{M_1^T C \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}}$$

$$\vec{a}_2 = M_2^T \begin{pmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{pmatrix} = \underbrace{M_2^T C \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}}$$

$$\overbrace{\begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix} C^T M_1}^{a_1^T}\ \overbrace{K_b}^{K_b}\ \overbrace{M_2^T C \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}}^{a_2} = 0$$

$$\begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix} \underbrace{C^T \underbrace{M_1 K_b M_2^T}_{E} C}_{F} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = 0$$

E: essential matrix

F: fundamental matrix

$$[x_1\ y_1\ 1]\ F \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = 0 \quad , \quad [x_1\ y_1\ 1)\ C^T\ E\ C \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = 0$$

$$[x_1 - x_0\ \ y_1 - y_0\ \ -f]\ E \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{bmatrix} = 0 \Longleftarrow$$

5 parameters of R/O are inside E matrix

$$(x_1\ y_1\ -f) \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ -f \end{pmatrix}$$

e_{ij} are unknowns

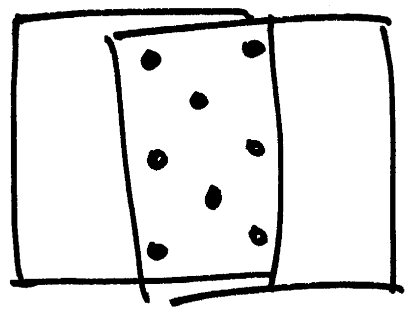
$$x_1 x_2 e_{11} + x_2 y_1 e_{21} - x_2 f e_{31} + y_2 x_1 e_{12} + y_2 y_1 e_{22}$$
$$- y_2 f e_{32} - f x_1 e_{13} - f y_1 e_{23} + f^2 \underbrace{e_{33}}_{=1} = 0$$

if solution E, then λE also a solution

$$\begin{bmatrix} x_1 x_2 & x_2 y_1 & -x_2 f & y_2 x_1 & y_2 y_1 & -y_2 f & -f x_1 & -f y_1 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \\ e_{31} \\ e_{12} \\ e_{22} \\ e_{32} \\ e_{13} \\ e_{23} \end{bmatrix} = -f^2$$

1 linear condition equation in
8 unknowns $\Rightarrow$

8-point algorithm
$\Rightarrow$ unique solution



8^+ points yields
redundant solution

$$E = \underbrace{M_1}_{= I_3} K_b M_2^T = I \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix} M_2^T$$

Make singular value decomposition $\underset{3,3}{E} = \underset{3,3}{U}\,\underset{3,3}{S}\,\underset{3,3}{V^T}$

SVD

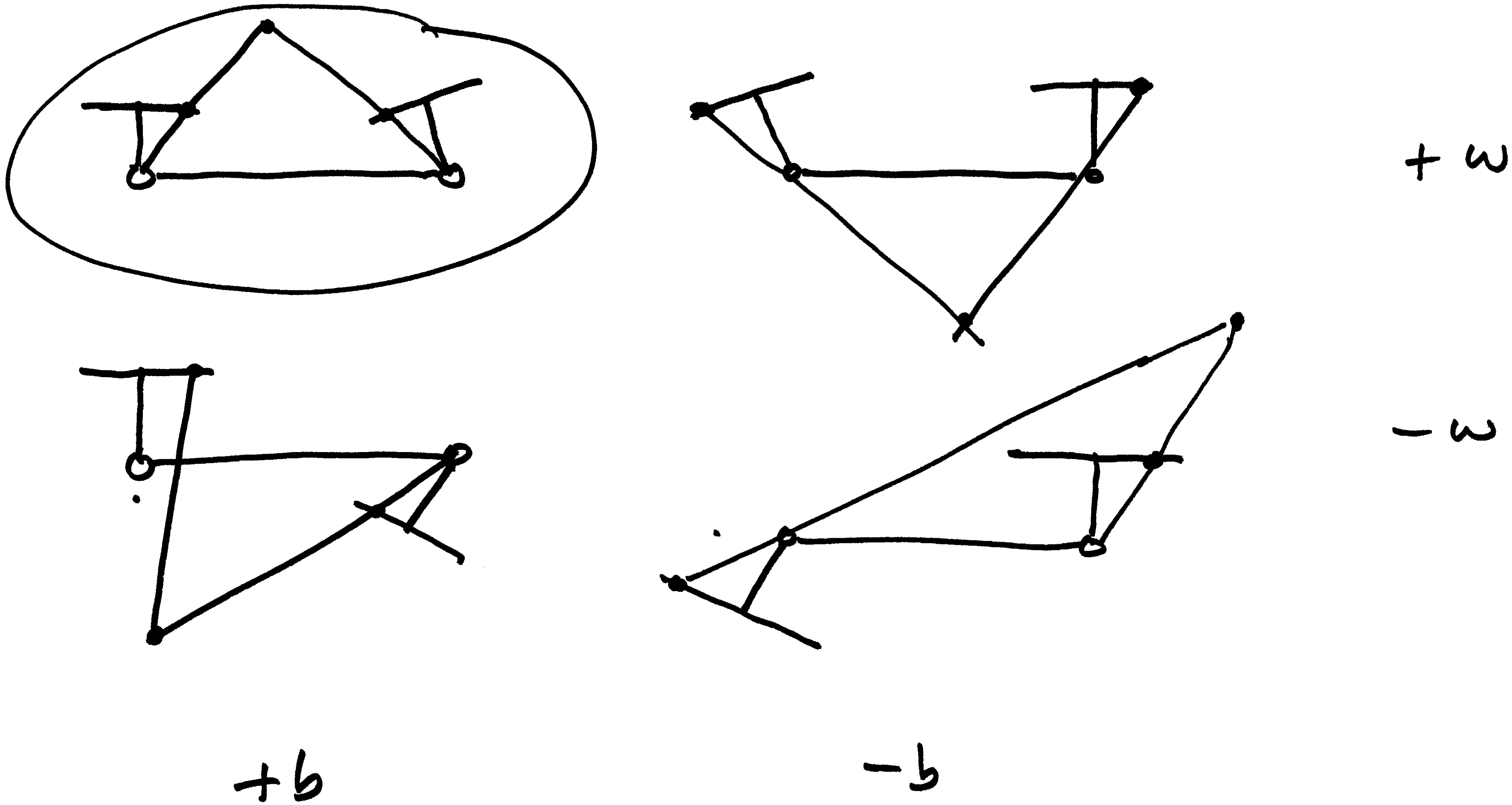
[U,S,V] = svd(E) ;

U*S*V' → E

$M_2 =$ either UWV^T or UW^TV^T, $W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$b =$ either u_3 or $-u_3$

u_3 is last column of U



compute intersection of 1 point
check if intersected point is in
"front" of both cameras.

$$\begin{pmatrix} x \\ y \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

negative 3rd element for both cameras

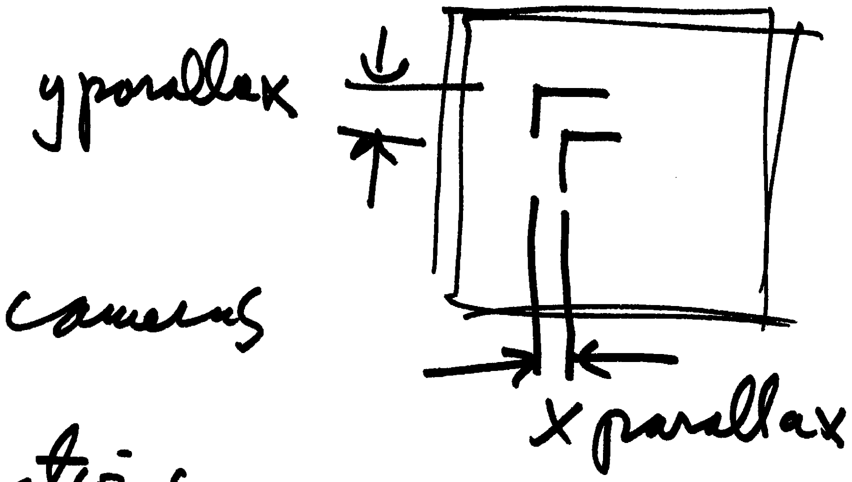


Image Preparation for <u>Stereo Applications</u>

↳ pairwise rectification / normalization / epipolar resampling

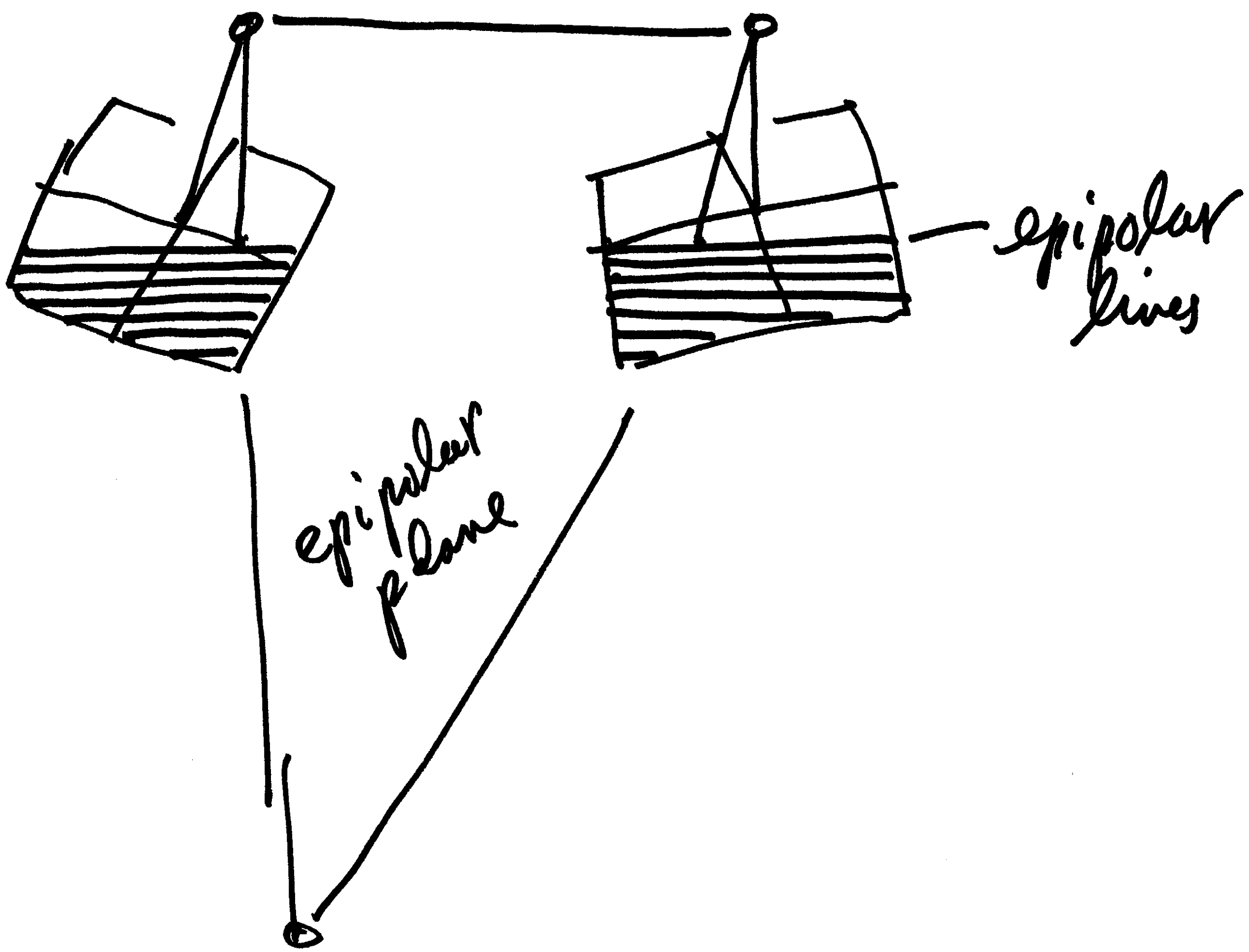
⇓

1. eliminate y-parallax
2. x-parallax ⇒ depth, height

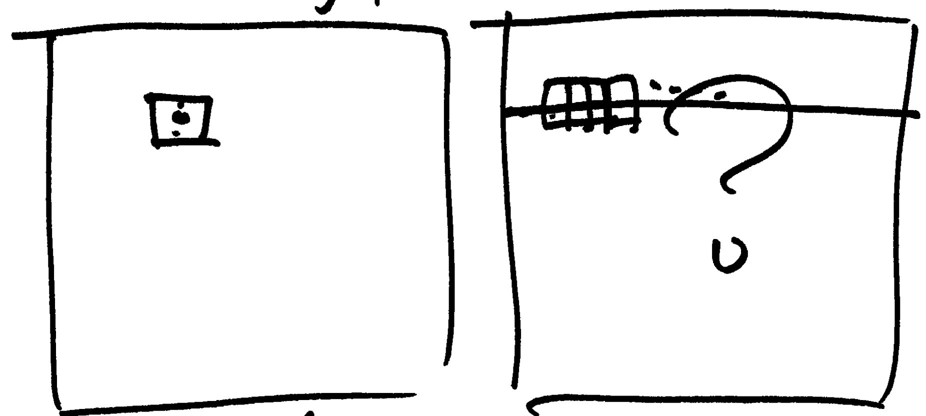
stereo environments

- anaglyph L: red, R: blue
- optical stereoscope (view master)
- flicker 120 hz, active glasses
- polarization bezel, switched passive glasses
- auto stereo (lenticular lenses)
- Nature (U of Az) holographic 3D no eyewear

pairwise rectification
└ 1. epipolar resample
2. normalization



matching problem



simple approach to search is 2D, slow
epipolar resampled, 1D faster

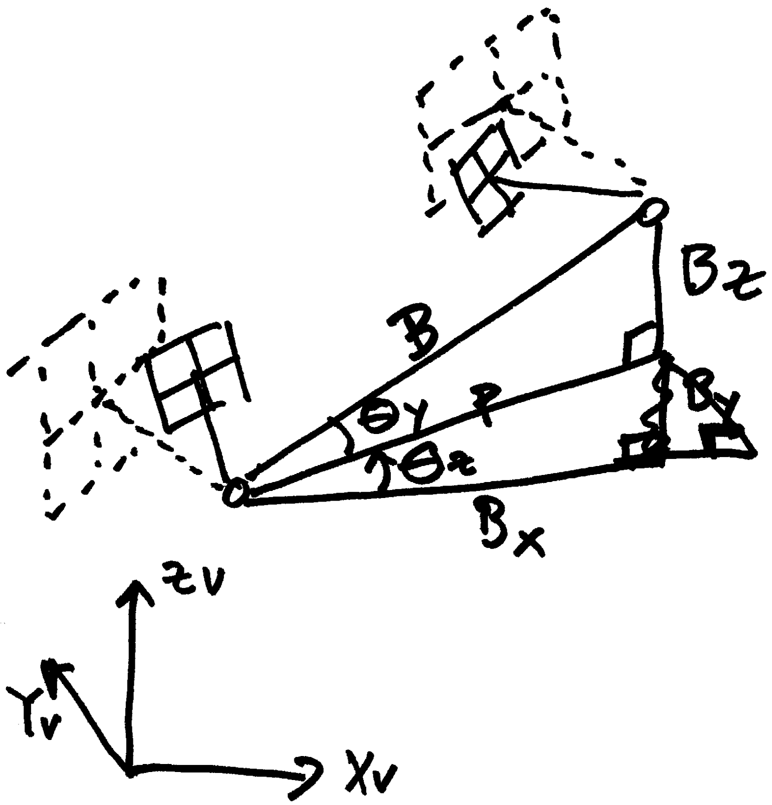
normalization (equiv. to epipolar resampling) ch. 7 text

normalized images:

1. same f
2. Opt. axes parallel
3. Opt. axes $\perp$ base
4. x-axes parallel to base

$$P = \sqrt{B_x^2 + B_y^2} \qquad \theta_z = \tan^{-1} B_y / B_x$$

$$\theta_y = \tan^{-1}\left(\frac{-B_z}{P}\right)$$



$$\theta_x = \frac{\omega_1 + \omega_2}{2}$$

(ω_1, ω_2 tertiary)

$$M_B = M_x(\theta_x) M_y(\theta_y) M_z(\theta_z)$$

$$X_N = M_B X_v$$

normalized axes   reference axes

$$X_p = M X_v, \quad X_v = M^T X_p$$

$$\boxed{X_N = M_B M^T X_p}$$

norm   photo