LS Matching 2D

I_L — Left image window

I_R — Right image window

could carry **brightness** + **contrast**, no: equalize histograms before

$$I_L(x,y) = I_R(x',y')$$ condition equation

$$x' = a_0 + a_1 x + a_2 y \qquad x' = ax + by + c$$
$$y' = b_0 + b_1 x + b_2 y \qquad y' = -bx + ay + d$$

if carry radiometric parameters (brightness + contrast)

$$I_L(x,y) = \underbrace{K_1}_{\substack{\text{scale}\\\text{gain}\\\text{contrast}}} I_R(x',y') + \underbrace{K_2}_{\substack{\text{shift}\\\text{offset}\\\text{brightness}}}$$

usually better to avoid added radiometric parameters K_1, K_2 and just equalize histograms between images (make the intensity range the same for each)

$$F = I_L(x,y) - I_R(x',y') = 0$$ (without k_1, k_2)

$$F = \underset{\uparrow}{I_L}(x,y) - I_R(\underbrace{a_0 + a_1 x + a_2 y}_{x'}, \underbrace{b_0 + b_1 x + b_2 y}_{y'}) = 0$$

observation

a's, b's parameters
x,y constant

assumption: almost aligned within 2-3 pixels max. } this implies that affine transformation is nearly the identity transformation

$$x' = a_0 + a_1 x + a_2 y$$
$$y' = b_0 + b_1 x + b_2 y$$

$$x' \approx 0 + 1 \cdot x + 0 \cdot y$$
$$y' \approx 0 + 0 \cdot x + 1 \cdot y$$

LS: $V + B\Delta = f$

$x' = a_0 + a_1 x + a_2 y$ $\qquad a_0 \approx 0 \qquad a_1 = 1 \qquad a_2 = 0$

$y' = b_0 + b_1 x + b_2 y$ $\qquad b_0 \approx 0 \qquad b_2 = 1 \qquad b_1 = 0$

(identity transformation)

$$\frac{\partial F}{\partial p} = \frac{\partial F}{\partial I_R} \cdot \frac{\partial I_R}{\partial x'} \cdot \frac{\partial x'}{\partial p}$$

← chain rule for partial derivatives

$\frac{\partial F}{\partial I_R}$: (-1)

$\frac{\partial I_R}{\partial x'}$: gradient in x or y, g_x, g_y

$\frac{\partial x'}{\partial p}$: coefficient from 6-par equ.

$\partial F / \partial a_0 = (-1)\, g_x \cdot 1 = -g_x$

$\partial F / \partial a_1 = (-1)\, g_x\, x = -g_x x$

$\partial F / \partial a_2 = (-1)\, g_x\, y = -g_x y$

$\partial F / \partial b_0 = -g_y$

$\partial F / \partial b_1 = -g_y x$

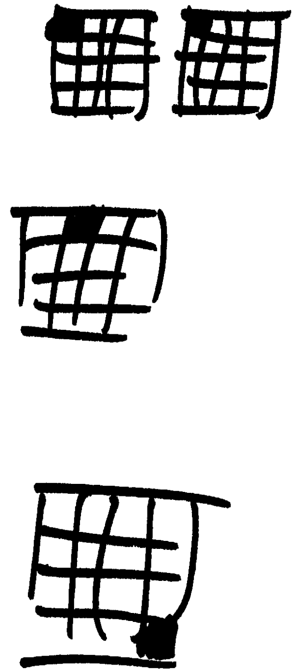
$\partial F / \partial b_2 = -g_y y$

I_1	I_2	I_3	I_4	I_5	I_6	I_7	…
1	2	3	4	5	6	7	…

$g_x(\text{at } 4) = \frac{I_5 - I_4}{1}$, or $\frac{I_4 - I_3}{1}$, or $\frac{I_5 - I_3}{2}$

$V + B\Delta = f$ B matrix: $\left[\frac{\partial F}{\partial a_0} \; \frac{\partial F}{\partial a_1} \; \frac{\partial F}{\partial a_2} \; \frac{\partial F}{\partial b_0} \; \frac{\partial F}{\partial b_1} \; \frac{\partial F}{\partial b_2} \right]$

$$\begin{bmatrix} -g_x & -g_x x_1 & -g_x y_1 & -g_y & -g_y x_1 & -g_y y_1 \\ -g_x & -g_x x_2 & -g_x y_2 & -g_y & -g_y x_2 & -g_y y_2 \\ & \vdots & & & \vdots & \\ -g_x & -g_x x_{16} & -g_x y_{16} & -g_y & -g_y x_{16} & -g_y y_{16} \end{bmatrix}$$



16×6 B-matrix

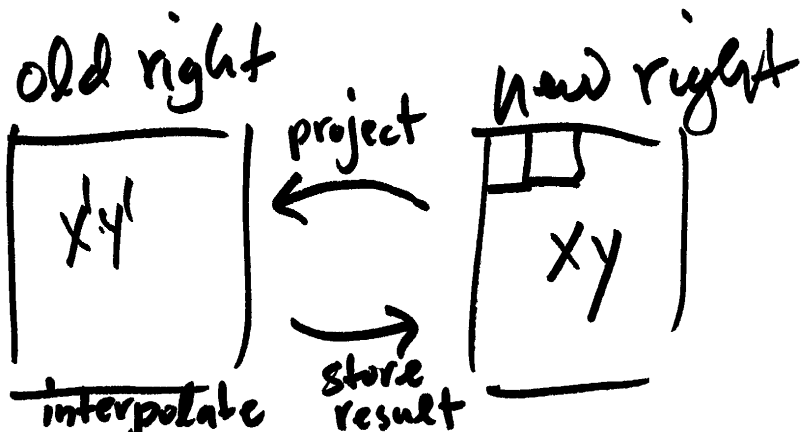
$F = I_L(x,y) - I_R(x',y') = 0$

$f = -F$, $\Delta = (B^T B)^{-1} B^T f$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P_{new} = P_{old} + \Delta$$

take new P resample right image



$$\boxed{\begin{aligned} x' &= a_0 + a_1 x + a_2 y \\ y' &= b_0 + b_1 x + b_2 y \end{aligned}}$$

in addition to rotating image, need accumulate Δ's

$$P = P_0 + \Delta P_1 + \Delta P_2 + \Delta P_3 + \dots$$

practical hint: helps to store intensities as F.P. values