HW3 : 1. conversion of object coordinates to cartesian or pseudo cartesian system

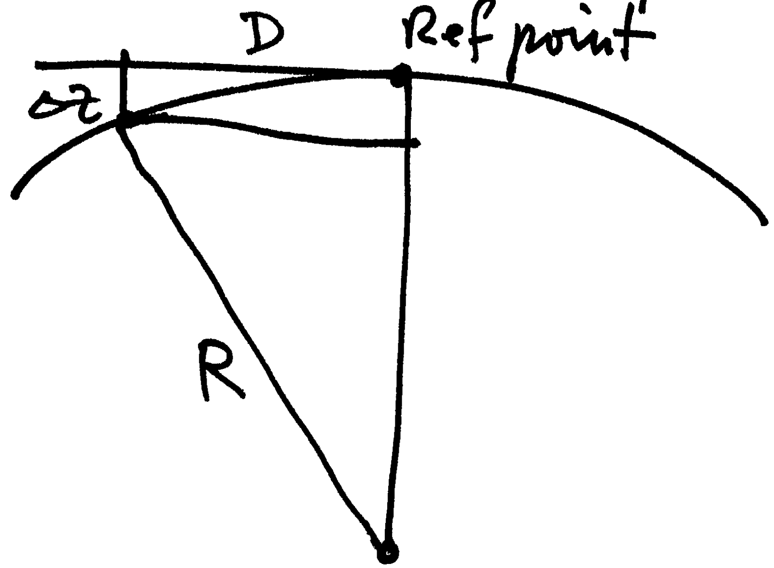
(a)

$$R = 6371000\,m$$

$$D = \sqrt{dx^2 + dy^2}$$

$$\Delta z = D^2/2R$$

$$z_{new} = z_{orig} - \Delta z$$



tangent plane approx

(b) topocentric

$$\begin{bmatrix} e \\ n \\ u \end{bmatrix} = M_x(90^\circ - \varphi)\, M_z(\lambda + 90^\circ) \left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{ECF} - \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}_{ECF} \right]$$

$\uparrow$ data $\quad \uparrow$ Ref

need φ, λ, h & $X_0 Y_0 Z_0$ of Ref. Point

2. <u>measurements</u> + quality

if residuals larger than expected ⟹ investigate

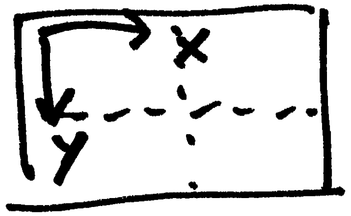
3. <u>intersection</u> $\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M^T \begin{pmatrix} x' \\ y' \\ -f \end{pmatrix}_R$

$$X = X_L + (Z - Z_L)\, u/w$$
$$Y = Y_L + (Z - Z_L)\, v/w$$

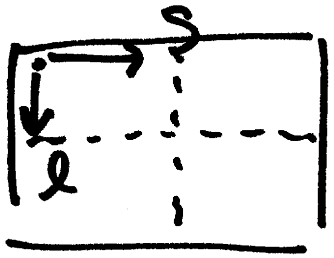
$$x' = x - x_0$$
$$y' = y - y_0$$

<u>Measurement</u>

Photoshop

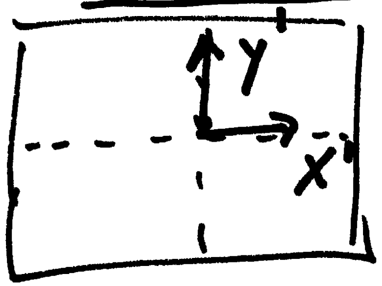


$$x' = (x - x_0)$$
$$y' = -(y - y_0)$$



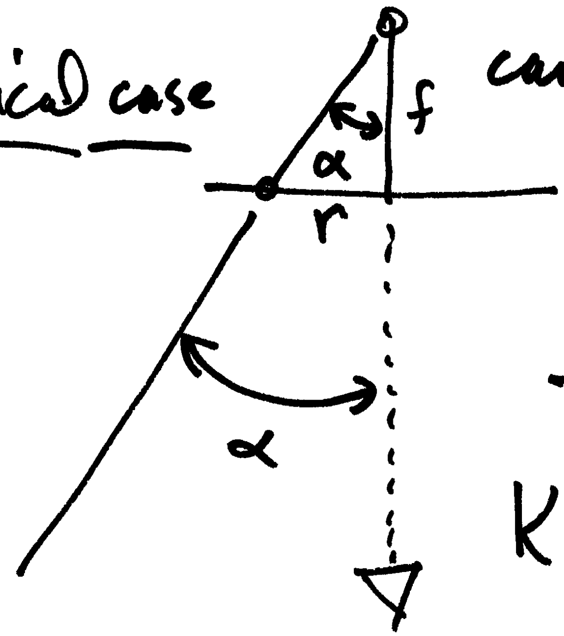
$$x' = (s - s_0)$$
$$y' = -(l - l_0)$$

<u>P.P. centered Cartesian System</u>



4. Atmospheric Refraction

Vertical case



can determine α in **image space**

$$\alpha = \tan^{-1}(r/f)$$

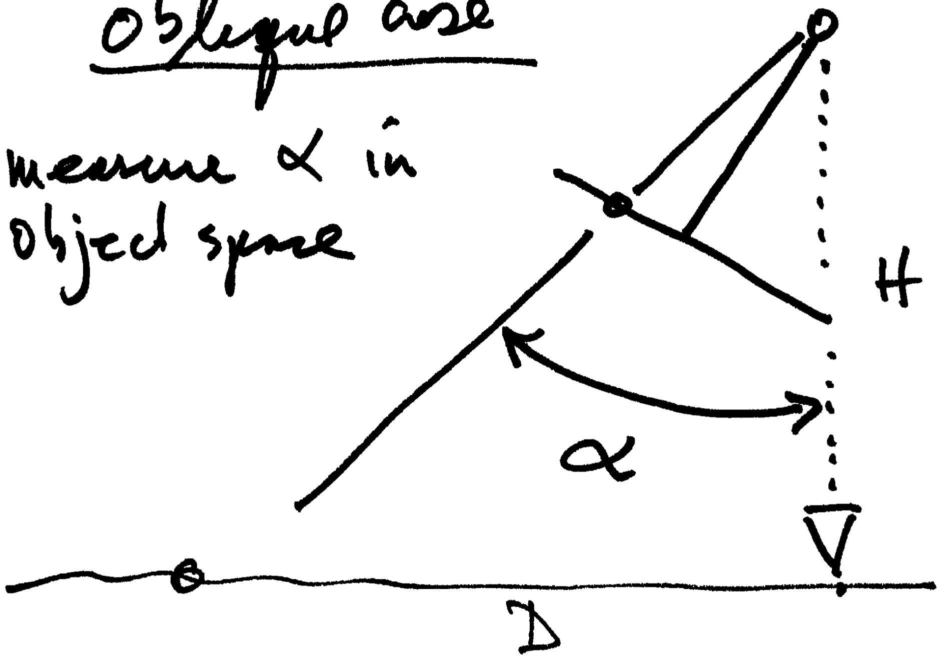
$$\rightarrow \Delta\alpha = K \cdot \tan\alpha$$

$$K = \frac{2410 \cdot H}{H^2 - 6H + 250} - \cdots \quad \times 10^{-6}$$

$$\Delta\alpha \rightarrow \Delta r$$

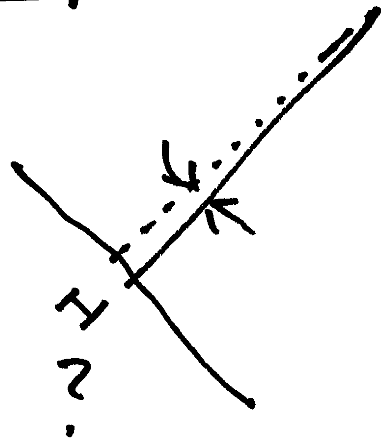
Oblique case

measure α in object space



$$\alpha = \tan^{-1}(D/H), \quad \Delta\alpha = K \cdot \tan\alpha$$

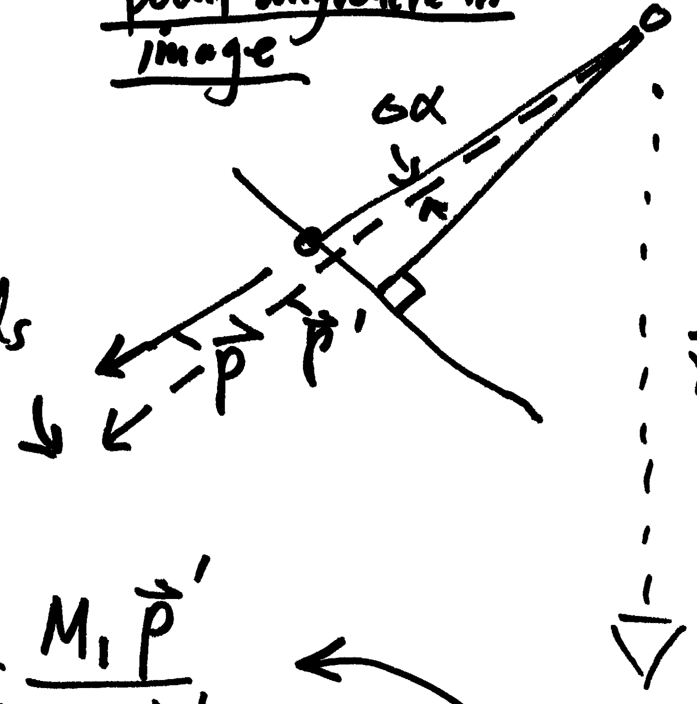
<u>on optical axis</u>



$r = f \tan \Delta\alpha$

$\simeq 0.14$ pixels

<u>point anywhere in image</u>



$\vec{p} = M^T \begin{pmatrix} x' \\ y' \\ -f \end{pmatrix}$

$\vec{n} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

rotation axis

$\vec{p} \times \vec{n} = \vec{a}$

$\Longrightarrow$ axis/angle rotation

$M(a_1, a_2, a_3, \Delta\alpha)$

$\vec{p}\,' = M \vec{p}$ rotate object space vector

project back into image space

M_i = i^{th} row of rotation matrix

$$x' = -f \frac{M_1 \vec{p}\,'}{M_3 \vec{p}\,'}$$

$$y' = -f \frac{M_2 \vec{p}\,'}{M_3 \vec{p}\,'}$$

refined coordinates corrected for Atmos. Refr.

make sure the sign (+/-) of $\Delta\alpha$ is consistent with the way you have implemented axis/angle formula — so that $\vec{p}\,'$ is rotated <u>TOWARD</u> <u>NADIR</u> compared to $\vec{p}$

Post Adjustment Statistics + Error Propagation

$$W = I_n = \begin{bmatrix} 1 & 0 & 0 & & \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \\ & & & \ddots & 0 \\ & & & 0 & 1 \end{bmatrix}$$

$$w_{ii} = \frac{\sigma_0^2}{\sigma_i^2} \qquad \sigma_0^2 = \sigma_i^2$$

$\sigma_i = 2$ pixels , $\sigma_i^2 = 4$ pixels^2 , $\sigma_0^2 = 4$

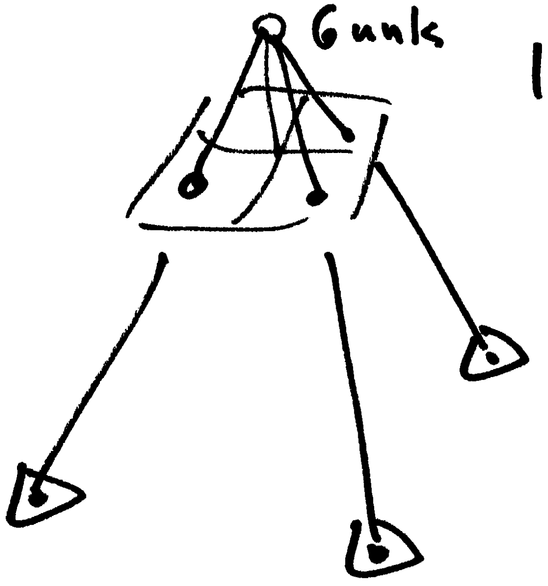
post adjustment estimate of ref var $\frac{v^TWv}{r} = \hat{\sigma}_0^2$, test statistic (global test) $\boxed{\frac{v^TWv}{\sigma_0^2} \sim \chi^2_r}$

$$rms = \sqrt{\frac{v^Tv}{n}}$$

$$s = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}}$$ sample mean

$$\boxed{\hat{\sigma}_0 = \sqrt{\frac{v^TWv}{r}}}$$

r: redundancy
degrees of freedom



13 points, 2 obs. per point

n_0, 3 points, 6 obs

$n = 26$

$n_0 = 6$

$r = 20$

$$\frac{v^T W v}{\sigma_0^2} \sim \chi^2_{20}$$

$H_0: \sigma^2 = \sigma_0^2$

$H_1: \sigma^2 \neq \sigma_0^2$

$H_1: \sigma^2 > \sigma_0^2$

hypothesis test

pick α

level of significance

$\alpha = .05$

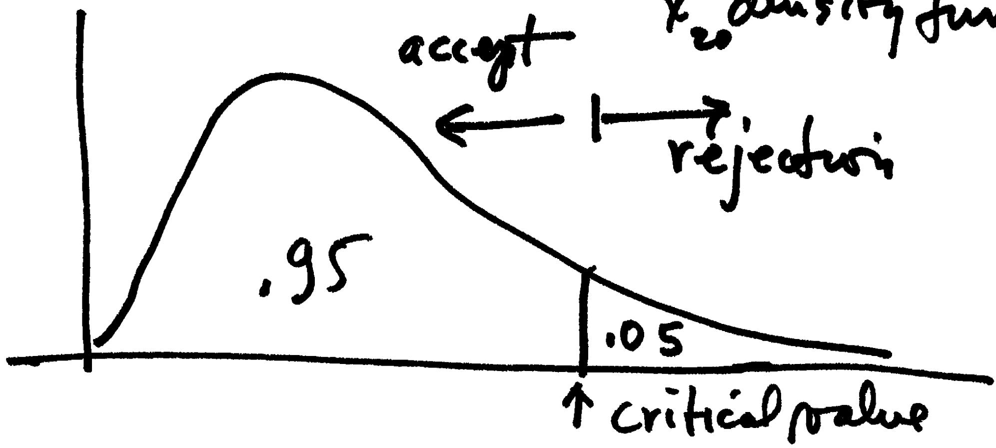
α: prob. Type 1 error

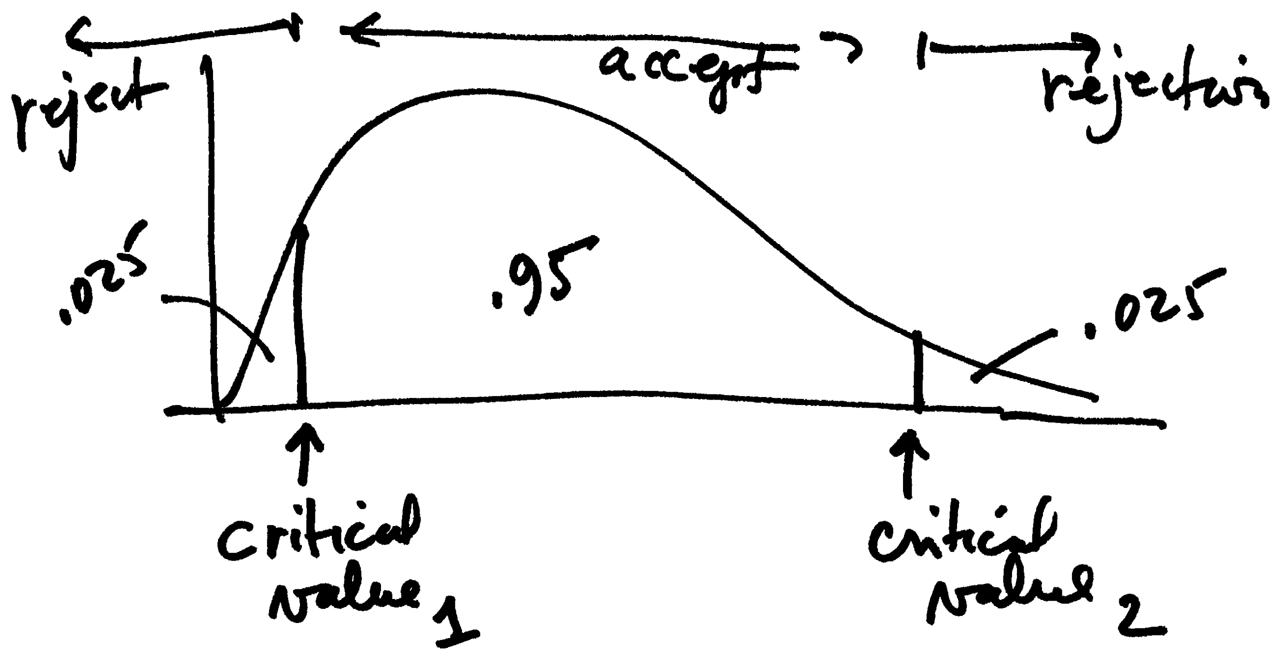
$\alpha = .05 \Rightarrow$ 1 out of 20 reject results even if good

reject when data good

$H_1: \sigma^2 > \sigma_0^2$ one sided test

χ^2_{20} density funct.





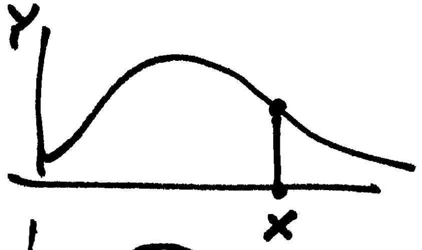
χ^2 matlab

chi2pdf (density)

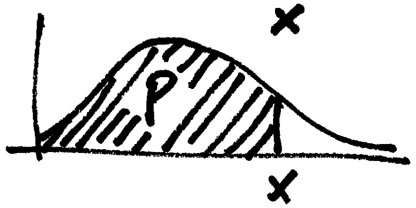
chi2cdf (cumulative distr. function)

chi2inv (inverse of ↑)

$Y = \text{chi2pdf}(x, v)$ ← d.o.f.



$P = \text{chi2cdf}(x, v)$



$$\boxed{x = \text{chi2inv}(P, v)}$$

use for critical values.