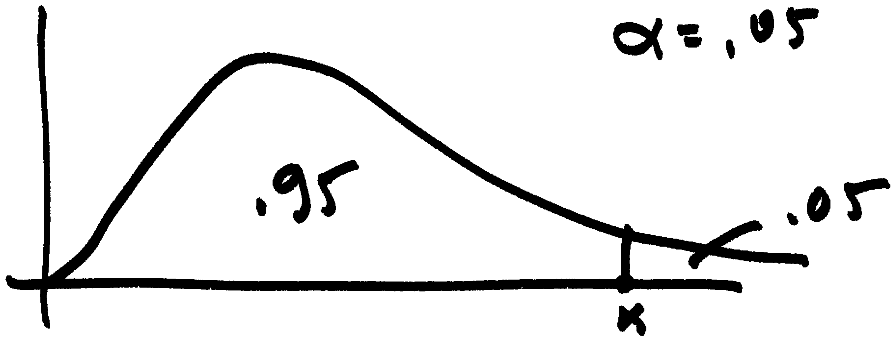
test statistic $\frac{v^T W v}{\sigma_0^2} \sim \chi^2_r$

HW#3 $r = 20$

$\alpha = .05$



$x = \text{chi2inv}(.95, 20)$

one sided test $H_1\ \sigma^2 > \sigma_0^2$

$\alpha = .05$

$x_1 = \text{chi2inv}(.025, 20)$

$x_2 = \text{chi2inv}(.975, 20)$

χ_r^{*} = test statistic = $\dfrac{v^TWv}{\sigma_0^2} = 14.45$

$1.5 = \sigma_0$

$v^TWv = 32.51$

$CV_1 = 31.4$

$CV_1 = 9.6$

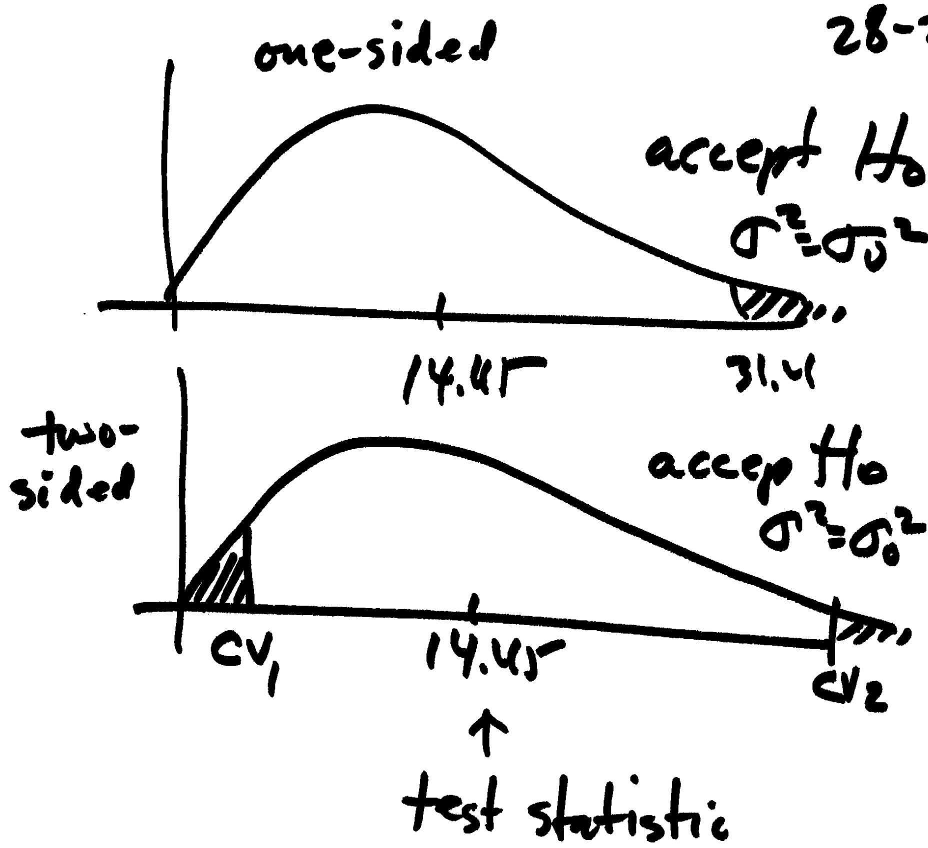
$CV_2 = 34.2$

<u>proceed to error propagation</u>

$N = B^TWB$

$Q_{\Delta\Delta} = N^{-1}$

$$\boxed{\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}}$$



$$\text{par}: \begin{bmatrix} \omega \\ \varphi \\ \kappa \\ X_L \\ Y_L \\ Z_L \end{bmatrix}, \quad \Sigma_{\Delta 0} = \begin{bmatrix} \sigma_\omega^2 & \sigma_{\omega\varphi} & \sigma_{\omega\kappa} & \cdot & \cdot & \cdot \\ \cdot & \sigma_\varphi^2 & \sigma_{\varphi\kappa} & & & \\ & & \sigma_\kappa^2 & & & \\ & & & \sigma_{X_L}^2 & & \\ & & & & \sigma_{Y_L}^2 & \\ & & & & & \sigma_{Z_L}^2 \end{bmatrix}$$

Square
Symmetric
6x6

2x2 submatrix for only $\Sigma_{\substack{X\\Y}}$

$$\sigma_\omega = \sqrt{\sigma_{11}^2}$$

$$\sigma_{X_L} = \sqrt{\sigma_{44}^2} = 1.89 \text{ m}$$

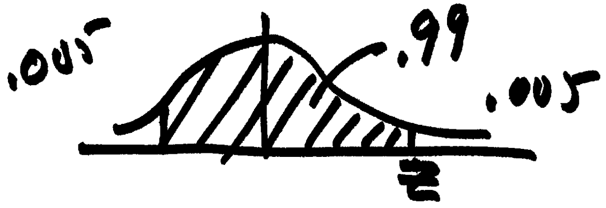
$$\sigma_{Y_L} = \sqrt{\sigma_{55}^2} = 0.9$$

$$\sigma_{Z_L} = \sqrt{\sigma_{66}^2} = 1.46$$

numerical results from HW3

confidence interval @ 99% for μ_{x_2}

$$\hat{\bar{X}} \pm \bar{z} \cdot \bar{\sigma}_{\bar{x}}$$



z: 1. select $P = 0.99$

2. obtain z

$z =$ normin (.995, 0, 1) [mean zero & std. deviation one = standard normal]

2.576

3. construct interval

$$\hat{X} = 506500.589 \pm (2.576)(1.89)$$

$$= 506500.589 \pm 4.87$$

$$506495.72 \rightarrow 506505.46$$

99% conf. interval for μ_x

confidence region for X + Y simultaneously

28-4

extract 2×2 submatrix from full $\Sigma_{\Delta\Delta}$

$$\Sigma_{XY} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix}$$

$$\begin{bmatrix} 3.572 & -.643 \\ -.643 & 0.777 \end{bmatrix}$$

cov. matrix S, $[V, D] = \text{eig}(S)$

D: diag. matrix, w eigenvalues on diag

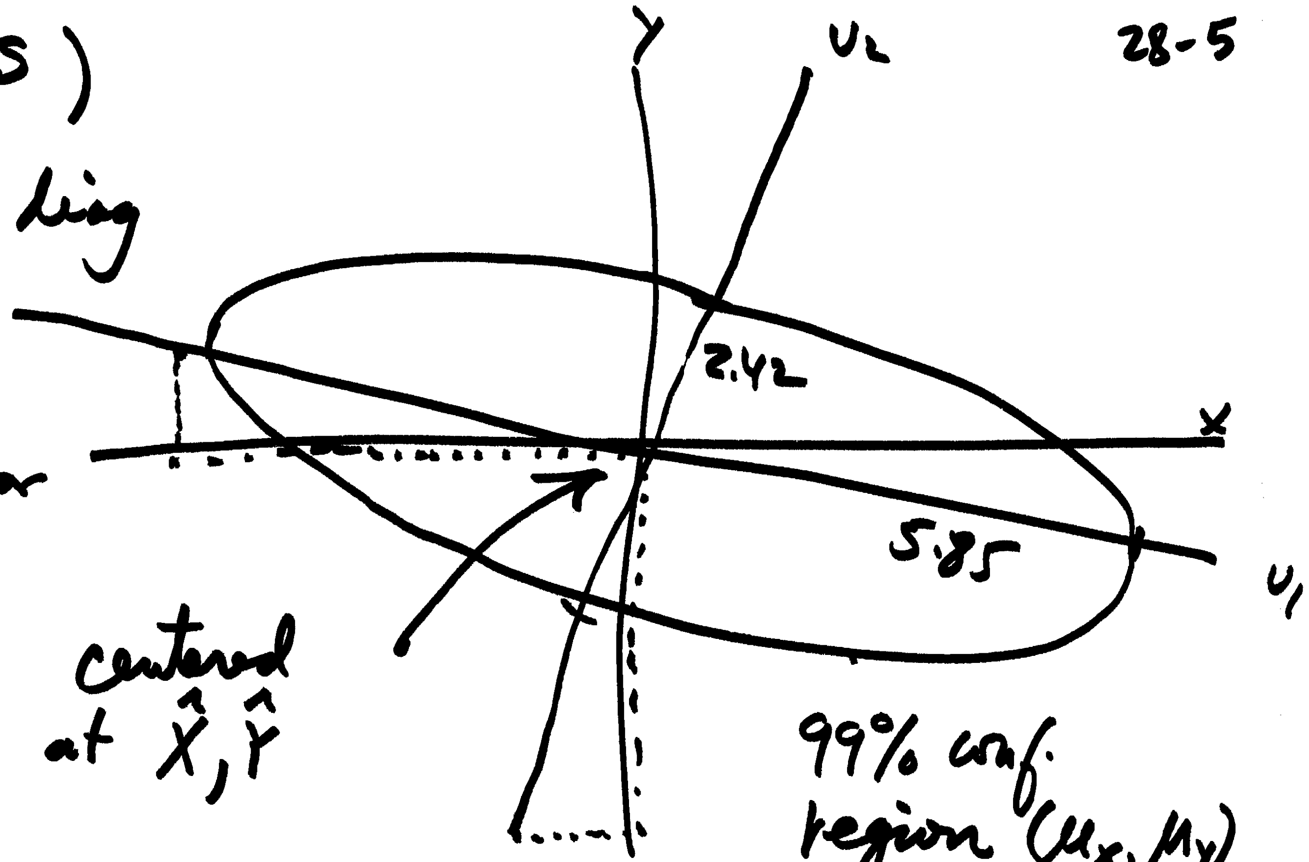
V: columns are eigenvectors

$(Ax = \lambda x)$ → λ = eigenvalue of A
x = corresponding eigenvector

$$V = \begin{array}{c} \begin{array}{cc} \quad v_2 & \quad v_1 \end{array} \\ \begin{bmatrix} -.214 & -.977 \\ -.977 & +.214 \end{bmatrix} \end{array}$$

$$D = \begin{bmatrix} .637 & 0 \\ 0 & 3.713 \end{bmatrix}$$

λ_2 λ_1



centered at $\hat{X}, \hat{Y}$

99% conf. region (μ_x, μ_y)

lengths of axes :

$$\text{length}_1 = \sqrt{\chi^2_{p,2} \cdot \lambda_1}$$

$$\text{length}_2 = \sqrt{\chi^2_{p,2} \cdot \lambda_2}$$

like critical value

$$c = \text{chi2inv}(.99, 2) = 9.210$$

$$\text{length}_1 = 5.85$$

$$\text{length}_2 = 2.42$$

circular confidence region

$$\Sigma \begin{bmatrix} 3.572 & -.643 \\ -.643 & .777 \end{bmatrix}$$

CE90 90% conf. circle

CE99 99% conf. circle

etc.

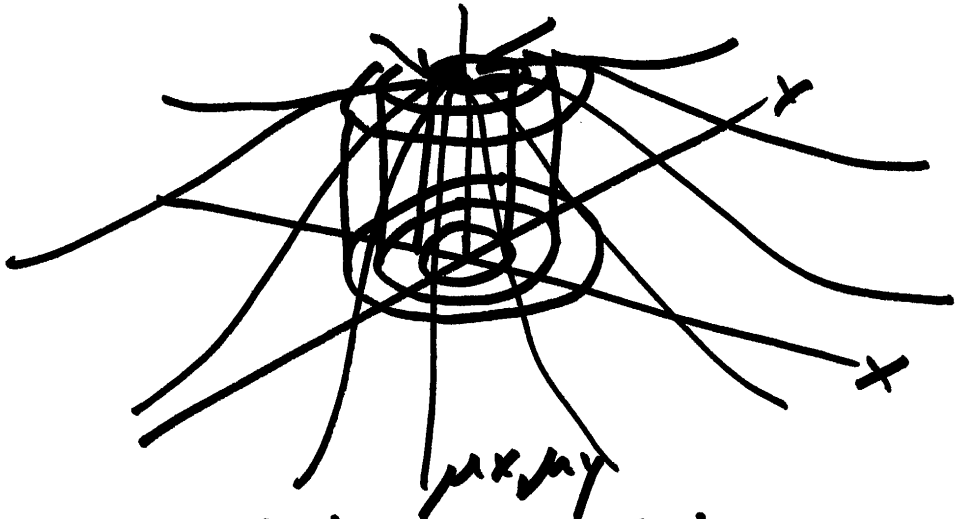
⋮

$$f(X) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} \cdot e^{-0.5[(X-\mu_X)^T \Sigma^{-1}(X-\mu_X)]}$$

$\downarrow$

$\begin{pmatrix}x_1\\x_2\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix}$, $\text{radius}_{.99} = 5.07$

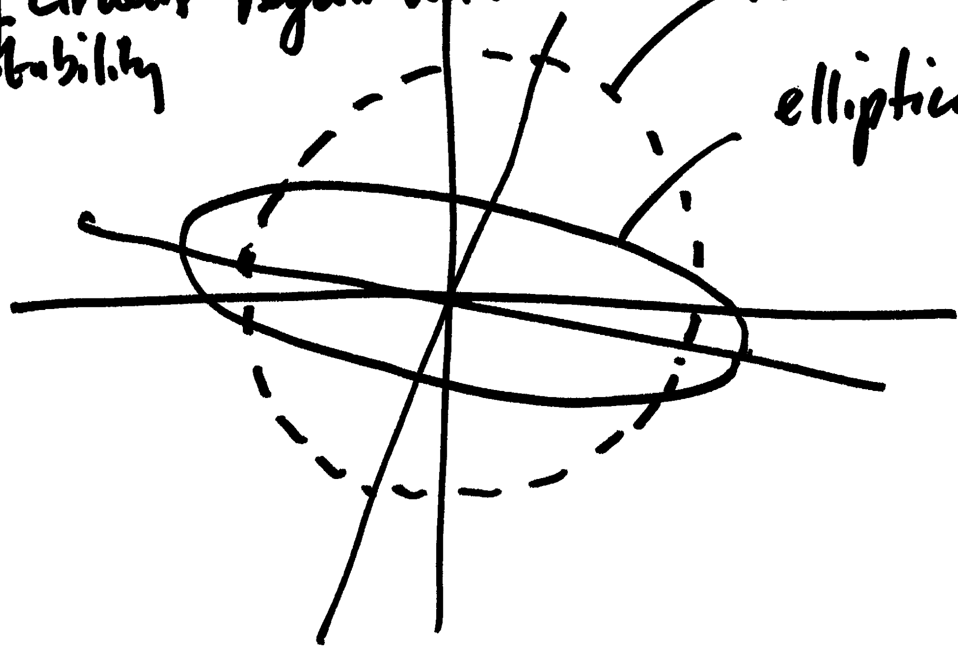
* [note: forgot $(2\pi)^{n/2}$ in class]



numerical technique to obtain radius of circular region with given probability value

circular 99% confidence region (see posted cep2.m for algorithm & code)

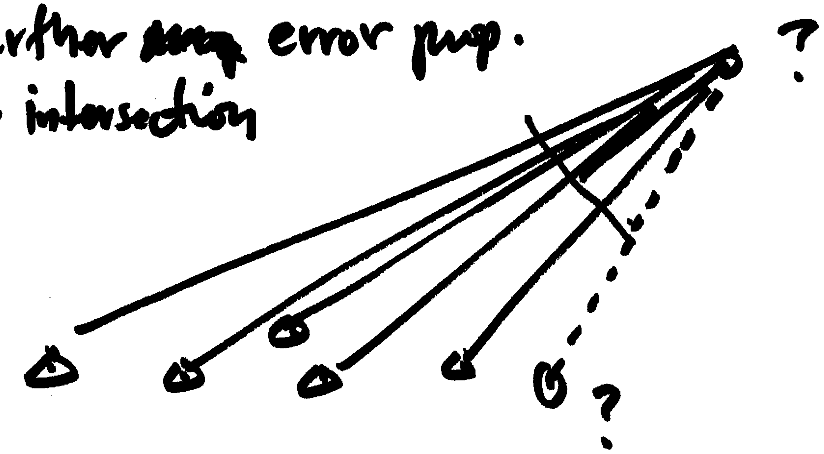
elliptical 99% confidence region



$$* \quad \boxed{f(X) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} e^{-0.5[(X-\mu_X)^T \Sigma^{-1}(X-\mu_X)]}}$$

multivariate normal distribution
actually it is the density function.

further error prop.
for intersection



$$\begin{pmatrix} X \\ Y \end{pmatrix} = F(\omega, \varphi, \kappa, X_c, Y_c\, Z_c\, Z\; x\; y)$$

$$\Sigma\ ? \longleftarrow \Sigma$$

$$\boxed{\Sigma_{XY} = J_{xp}\,\Sigma_{pp}\,J_{xp}^{T}}$$

$$J_{xp} = \begin{bmatrix} \partial X/\partial\omega & \partial X/\partial\varphi & \partial X/\partial\kappa & \partial X/\partial X_c & \cdots\cdots & \partial X/\partial y \\ \partial Y/\partial\omega & \partial Y/\partial\varphi & \partial Y/\partial\kappa & \partial Y/\partial X_c & \cdots\cdots & \partial Y/\partial y \end{bmatrix}$$