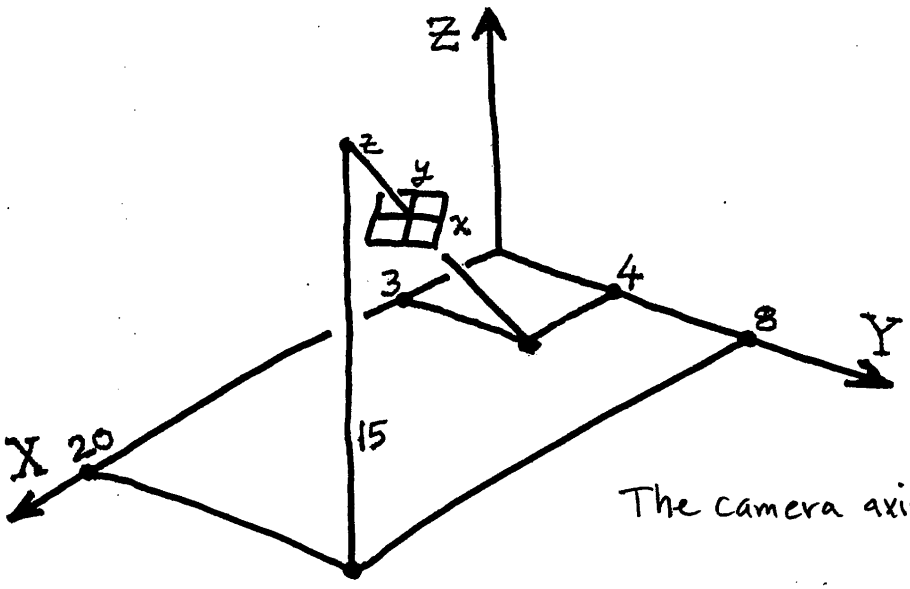
CE 597 Digital Photogrametric Systems EXAM 1

Oct. 19, 2010

1 page notes allowed, 75 minutes

Name _____

1. Using the following sketch, (a) find rotation matrix M, with the sense $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$, and (b) what are ω, ϕ, κ?



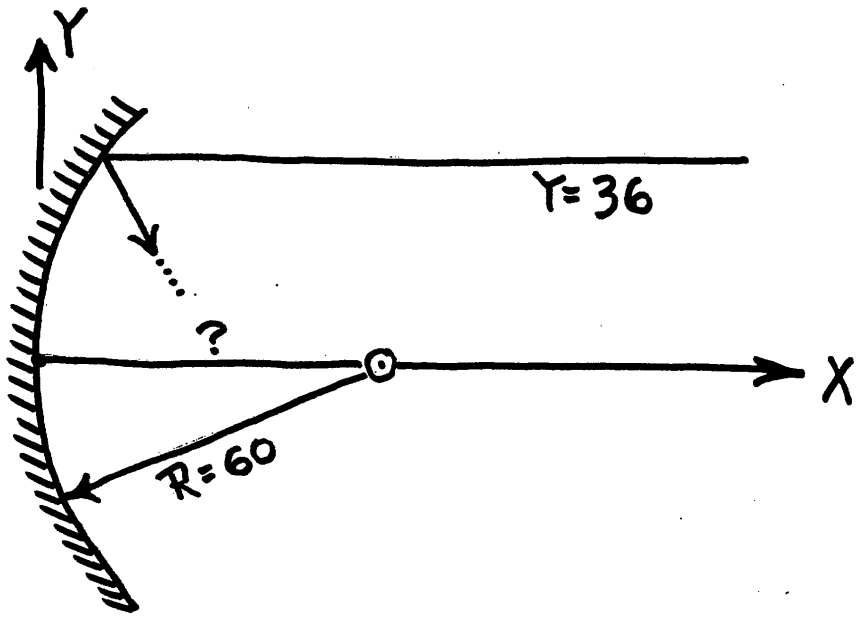
The camera axis x is horizontal

2. An image point is measured as $x = 30.000$ mm, $y = -60.000$ mm, ($x_0 = y_0 = 0$). The combined radial distortion (lens & atmosphere) is $+0.020$ mm. What are the corrected image coordinates?

3. Find the intersection of image point $x = 20$ mm, $y = 30$ mm with the object plane $Z = 50$ m, using the given values for interior and exterior orientation. The sense of M is the same as question 1.

$$M = \begin{bmatrix} .9513 & .2725 & -.1445 \\ -.2549 & .9583 & .1290 \\ .1736 & -.0858 & .9811 \end{bmatrix} \qquad \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} 1000 \\ 2000 \\ 500 \end{bmatrix}_m \qquad \begin{bmatrix} x_0 \\ y_0 \\ f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 152.4 \end{bmatrix}_{mm}$$

4. For the spherical mirror shown in the sketch, where does the reflected ray intersect the x-axis? (all elements of sketch are in the XY plane, and coordinate origin is on the mirror.) Radius of mirror is 60.



5. Match the word or phrase on the left with the word or phrase on the right most closely associated with it.

aberration _____	a. correction to observation
panchromatic _____	b. component of unit vector
panoramic _____	c. all colors
residual _____	d. frame
pinhole _____	e. distortion
direction cosine _____	f. mechanical device to implement euler angles
gimbal _____	g. camera geometry sweeping a linear sensor

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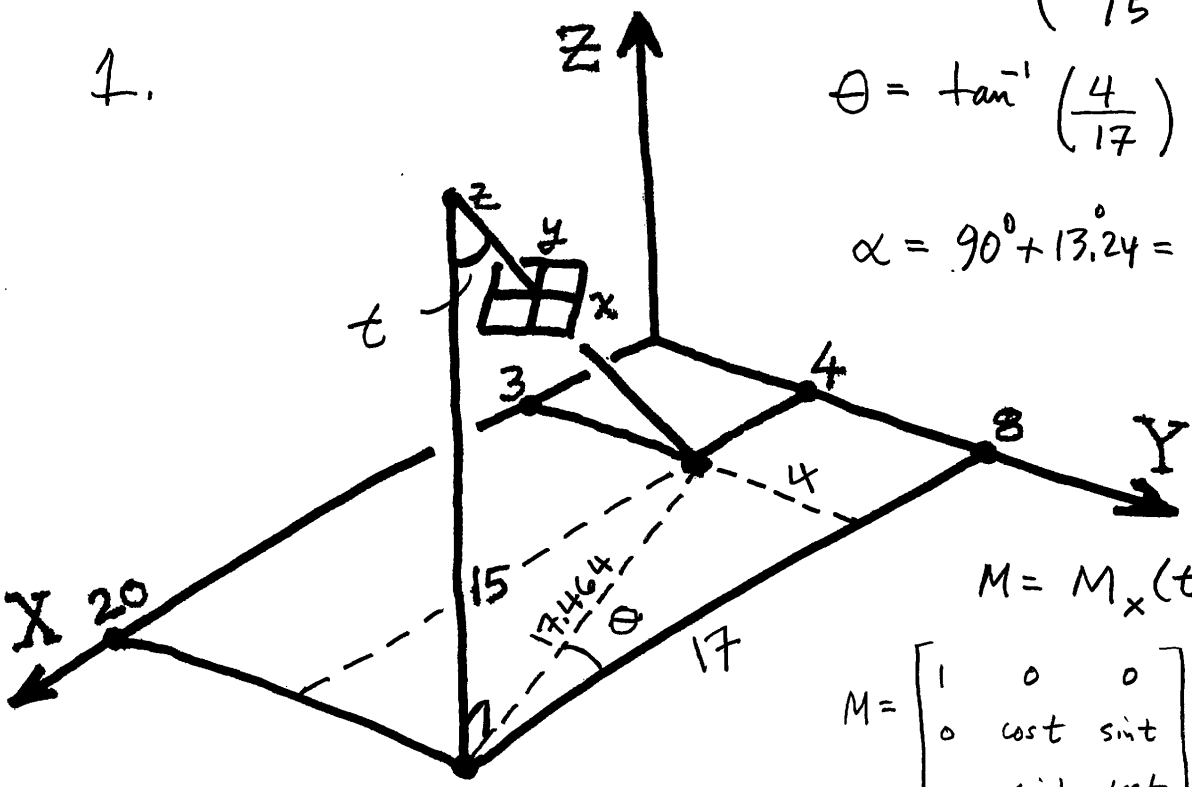
SOLUTION

mean = 72

1.



$$t = \tan^{-1}\left(\frac{17.464}{15}\right) = 49.34^\circ$$

$$\theta = \tan^{-1}\left(\frac{4}{17}\right) = 13.24^\circ$$

$$\alpha = 90^\circ + 13.24^\circ = 103.24^\circ$$

$$M = M_x(t)\, M_z(\alpha)$$

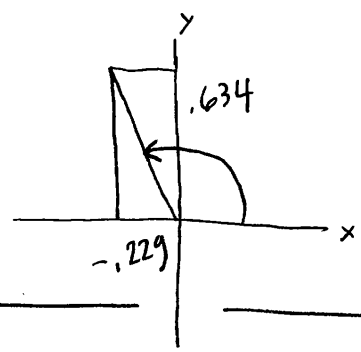
$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos t & \sin t \\ 0 & -\sin t & \cos t \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .6516 & .75860 \\ 0 & -.75860 & .6516 \end{bmatrix} \begin{bmatrix} -.22904 & .97342 & 0 \\ -.97342 & -.22904 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -.22904 & .97342 & 0 \\ -.63424 & -.14923 & .75860 \\ .73844 & .17375 & .65156 \end{bmatrix}$$

$$\phi = \sin^{-1}(.73844) = 47.5987^\circ$$

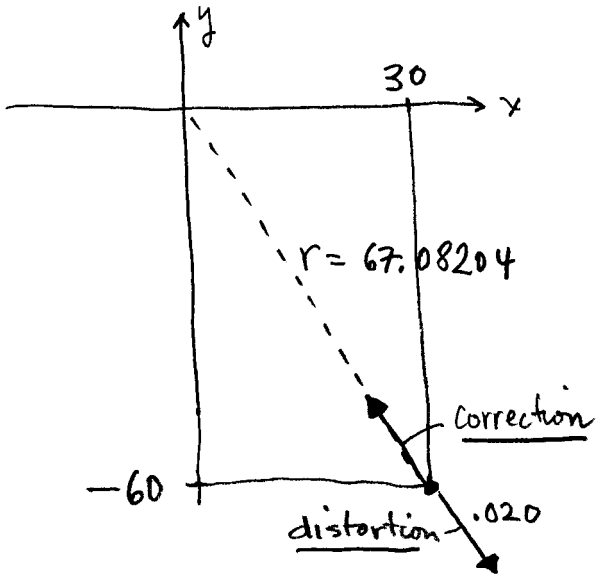
$$\omega = \tan^{-1}\left(\frac{-.17375}{.65156}\right) = -14.9314^\circ$$

$$K = \tan^{-1}\left(\frac{.63424}{-.22904}\right) = 109.8559^\circ$$



note: questions each graded as 10 points each
total score is scaled to 100.

2.



$$dx = x\frac{dr}{r} = 30 \cdot \frac{.020}{67.08} = .009$$

$$dy = y\frac{dr}{r} = -60 \cdot \frac{.020}{67.08} = -.018$$

distortion ↗

$$x_c = x - dx = 30 - .009 = 29.991$$

$$y_c = y - dy = -60 - -.018 = -59.982$$

use sketch to get the signs right !

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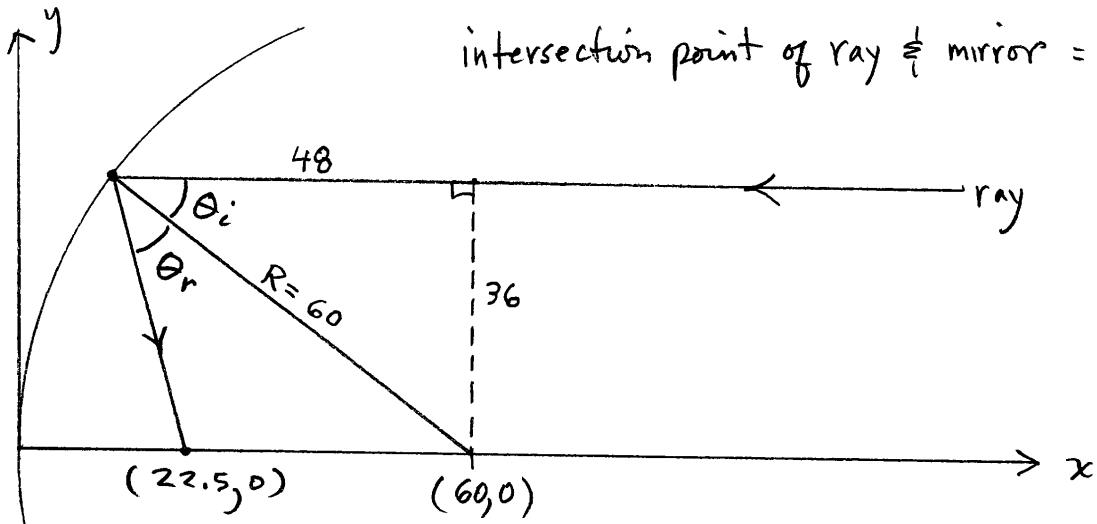
3. $\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} x-x_L \\ Y-Y_L \\ z-z_L \end{pmatrix}$, $\frac{1}{\lambda} M^T \underbrace{\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix}}_{\begin{pmatrix} u \\ v \\ w \end{pmatrix}} = \begin{pmatrix} x-x_L \\ Y-Y_L \\ z-z_L \end{pmatrix}$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{bmatrix} .9513 & -.2549 & .1736 \\ .2725 & .9583 & -.0858 \\ -.1445 & .1290 & .9811 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ -152.4 \end{bmatrix} = \begin{bmatrix} -15.078 \\ 47.275 \\ -148.540 \end{bmatrix}$$

$$\frac{u}{w} = \frac{X - 1000}{50 - 500} \quad , \quad X = 1000 + (-450)\left(\frac{-15.078}{-148.540}\right) = 954.32$$

$$\frac{v}{w} = \frac{Y - 2000}{50 - 500} \quad , \quad Y = 2000 + (-450)\left(\frac{47.275}{-148.540}\right) = 2143.22$$

4. <u>spherical</u> mirror in 3D = <u>circle</u> mirror in 2D 3/3



intersection point of ray & mirror = (12,36)

$\theta_i = \tan^{-1}\left(\frac{36}{48}\right) = 36.86990^\circ$

$\theta_r = 36.86990^\circ$

"bearing" of reflected ray is $-2 \cdot 36.86990^\circ$

$= -73.7398^\circ$

$\Rightarrow$ slope $= m = \tan(-73.7398^\circ) = -3.428572$

equation of line (of reflected ray) $y = mx + b$, $36 = (-3.428572)12 + b$

$\Rightarrow b = 36 + (3.428572)(12) = 77.142857$

Solve for x @ y = 0 : $0 = (-3.428572)x + 77.142857$

$x = -77.142857 / -3.428572 = \underline{\underline{22.5}}$

5.

aberration	-	(e)	distortion
panchromatic	-	(c)	all colors
panoramic	-	(g)	camera geometry sweeping a linear sensor
residual	-	(a)	correction to observation
pinhole	-	(d)	frame
direction cosine	-	(b)	component of unit vector
gimbal	-	(f)	mechanical device to implement euler angles