3-1

$$\begin{bmatrix} \cos\phi\cos\kappa & - & - \\ -\cos\phi\sin\kappa & - & - \\ \sin\phi & -\cos\phi\sin\omega & \cos\phi\cos\omega \end{bmatrix} = M$$

$\phi = \arcsin(m_{31})$ $(-\pi/2 \to +\pi/2)$

$\boxed{\omega = \arctan(-m_{32}/m_{33})}$

what if $m_{31} = \pm 1$, $\Rightarrow \phi = \pm 90^\circ$, then

$\cos\phi = 0$ argument to arctan would fail

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3-2

$m_{32}, m_{33}, m_{11}, m_{21} = 0$ cannot solve for ω, k

there are an infinite number of solutions for ω, k

$\Rightarrow$ "gimbal lock"

if you encounter this situation:

1. change rotation order ω, ϕ, k x-y-z
2. rotate reference coord. system
3. don't use euler angles, use quaternions

euler parameters $\longrightarrow$

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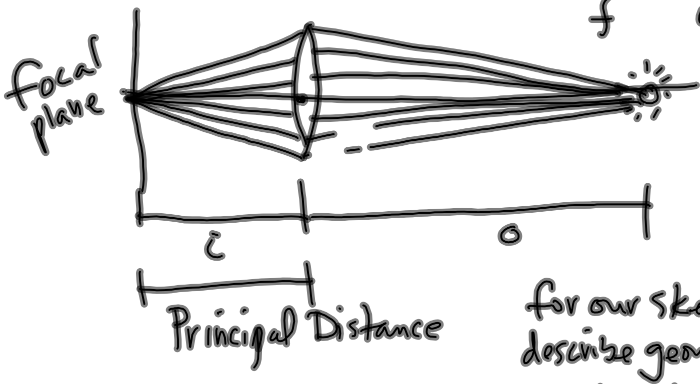
advantage of quaternions
no critical orientation

thin lens equation

3-3

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$$



if $o = \infty$
$i = f$

for our sketches to describe geometry: use only chief ray

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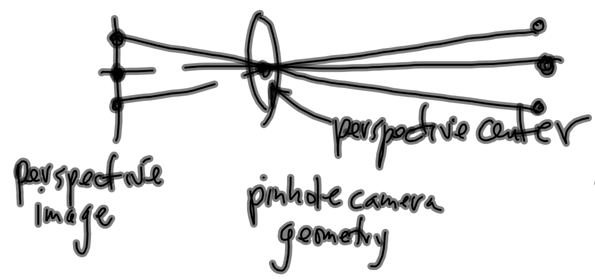
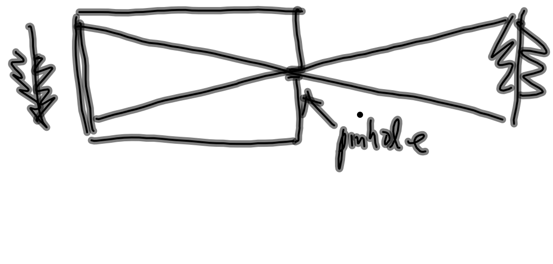
3-4



obj. point
persp. center
image
→ collinear



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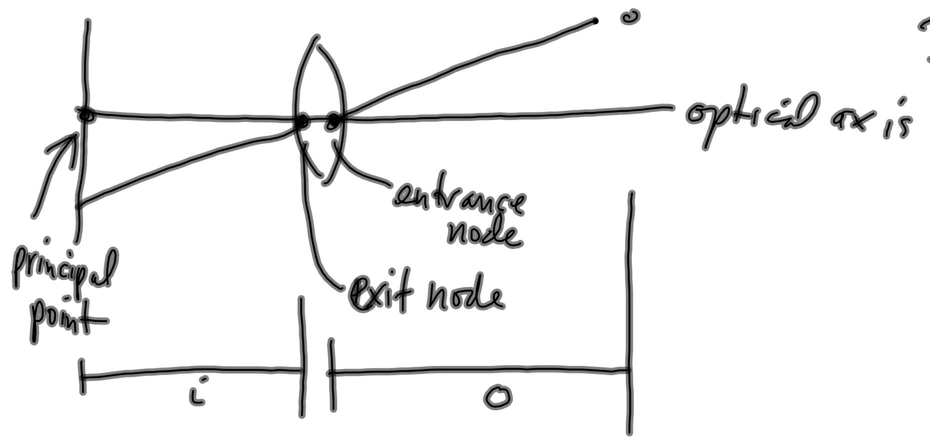
3-5



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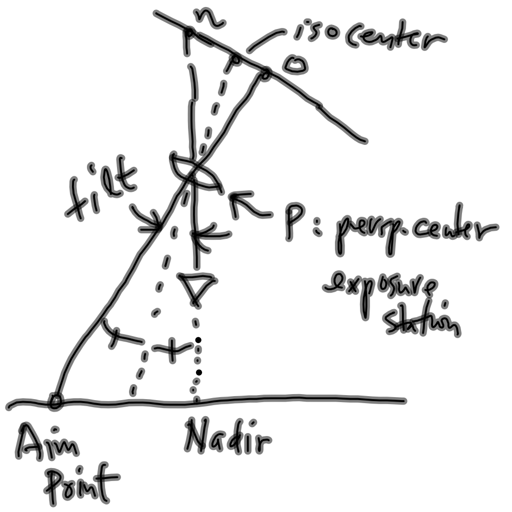
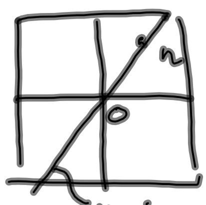
3-6





plane n, o, p
principle plane

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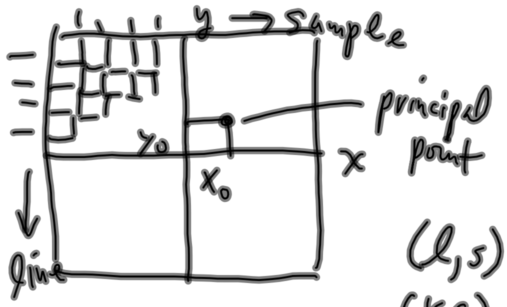
3-7

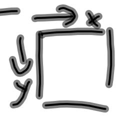


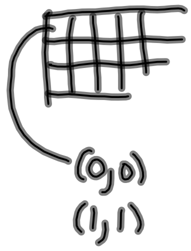
↓ → column
row

raw image meas.
⇓
refined

(l,s) : right handed
(r,c) : " "
(x,y) : " "

(s,l) : left handed
(c,r) : " "
(x,y) photoshop : left handed





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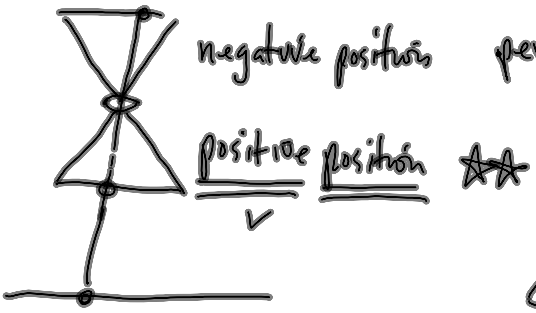
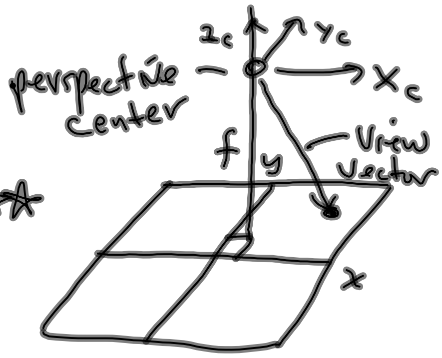
3-8





$$\begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix}$$

3D camera coordinate

sensor coordinate

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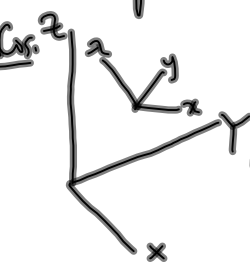
Rotation Matrix

3-9

1. Euler angles, sequential rot.
2. Direction cosines (coordinates of unit vectors)
3. Axis-angle
4. Algebraic parameters
5. Euler parameters (4) = Unit quaternion

DirCos.



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

(to) (from)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(I) columns of rotation matrix are coordinates in the "to" system of unit basis vectors in "from" system

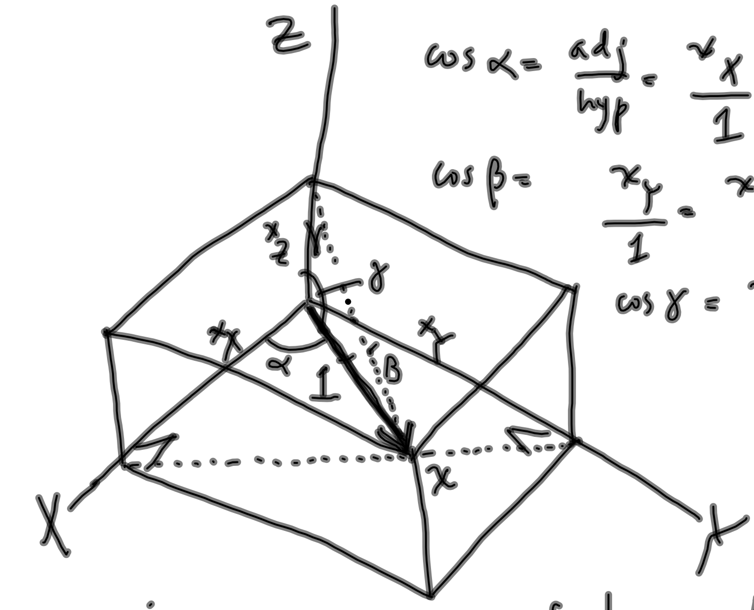
3-10

(II) rows of rotation matrix are coordinates in the "from" system of unit basis vectors in the "to" system

$$M = \begin{bmatrix} x_X & x_Y & x_Z \\ y_X & y_Y & y_Z \\ z_X & z_Y & z_Z \end{bmatrix} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

(I)

3-11



$$\cos\alpha = \frac{adj}{hyp} = \frac{x_X}{1} = x_X$$

$$\cos\beta = \frac{x_Y}{1} = x_Y$$

$$\cos\gamma = x_Z$$

showing equivalence between direction cosines and components of basis unit vectors.

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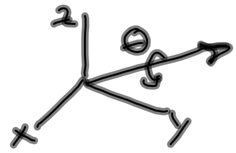
3-12

axis-angle

$\cos\theta = C\theta$
$\sin\theta = S\theta$



$$\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} \quad \alpha^2+\beta^2+\gamma^2 = 1$$

$$= \; *$$

$$M = \begin{bmatrix} \alpha^2(1-C\theta)+C\theta & \alpha\beta(1-C\theta)-\gamma S\theta & \alpha\gamma(1-C\theta)+\beta S\theta \\ \alpha\beta(1-C\theta)+\gamma S\theta & \beta^2(1-C\theta)+C\theta & \beta\gamma(1-C\theta)-\alpha S\theta \\ \alpha\gamma(1-C\theta)-\beta S\theta & \beta\gamma(1-C\theta)+\alpha S\theta & \gamma^2(1-C\theta)+C\theta \end{bmatrix}$$

* appendix A p.375 mistake — singularity occurs
appendix E p.449 ✓ — $M = I_3$

α, β, γ here (vector components) are <u>different</u> from αβγ on previous page (angles).

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quaternion q_s, q_i, q_j, q_k, $q_s^2 + q_i^2 + q_j^2 + q_k^2 = 1$ 3-13

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