Rotation Matrix (q_s, q_i, q_j, q_k) (q_0, q_1, q_2, q_3) 4-1

$q_s^2 + q_i^2 + q_j^2 + q_k^2 = 1$ (w, x, y, z)

$$M = \begin{bmatrix} q_s^2+q_i^2-q_j^2-q_k^2 & 2(q_jq_i - q_sq_k) & 2(q_iq_k + q_sq_j) \\ 2(q_iq_j+q_sq_k) & q_s^2-q_i^2+q_j^2-q_k^2 & 2(q_jq_k - q_sq_i) \\ 2(q_iq_k - q_sq_j) & 2(q_jq_k+q_sq_i) & q_s^2-q_i^2-q_j^2+q_k^2 \end{bmatrix}$$

advantage: no singularities

rotation matrix from unit quaternion

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$q_s = \cos\left(\frac{\theta}{2}\right)$ $\alpha, \beta, \gamma, \theta$ axis-angle → quat 4-2

$$\begin{bmatrix} q_i \\ q_j \\ q_k \end{bmatrix} = \sin\frac{\theta}{2}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$\cos\theta = q_s^2 - (q_i^2 + q_j^2 + q_k^2)$ quat → axis-angle

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \frac{1}{\sqrt{q_i^2+q_j^2+q_k^2}}\begin{bmatrix} q_i \\ q_j \\ q_k \end{bmatrix}$$

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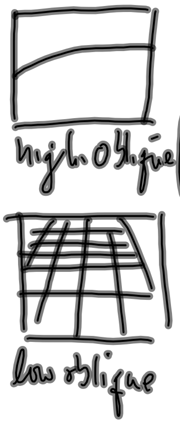
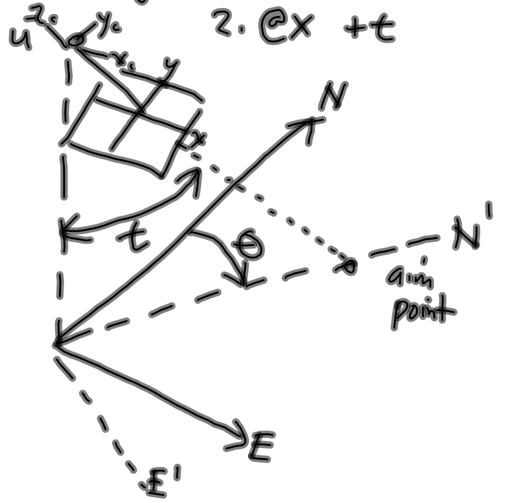
approximate angles for oblique image

4-3

1. @z $-\theta$
2. @x $+t$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} E \\ N \\ U \end{pmatrix}$$



NE, N'E' all in horizontal plane

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4-4

1. $M_3(-\theta)$
2. $M_1(t)$

$M = M_1(t)\, M_3(-\theta)$

$M_x \quad M_z$

$R_1 \quad R_3$

↑ extract any parameters

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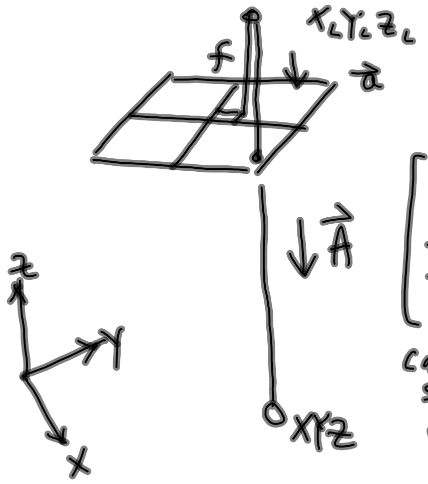

$\vec{a} = \lambda M \vec{A}$; $\frac{1}{\lambda} M^T \vec{a} = \vec{A}$ 4-5

$$\begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix} \checkmark$$

camera space vector; λ ← nuisance param.

$X_L Y_L Z_L$, XYZ } obj. space

collinearity equations

$\phi, \lambda, h \rightarrow$ cartesian
ENH $\rightarrow$ cartesian } before using collinearity equations

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4.6

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix} = \lambda \underbrace{\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}}_{\begin{bmatrix} u \\ v \\ w \end{bmatrix}} = \lambda \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\frac{x - x_0}{-f} = \frac{\lambda u}{\lambda w} = \frac{u}{w}, \quad x - x_0 = -f\frac{u}{w}, \quad x = x_0 - f\frac{u}{w}$$

$$\frac{y - y_0}{-f} = \frac{\lambda v}{\lambda w} = \frac{v}{w}, \quad y - y_0 = -f\frac{v}{w}, \quad y = y_0 - f\frac{v}{w}$$

G2I

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4-7

$x = x_0 - f\, u/w$

$y = y_0 - f\, v/w$

observed

unk: $x_0 y_0 f$: camera calibr. ✓

att, exp sta.: space resection ✓

XYZ : spc. intersection ✓

$x_0 y_0 f$ inner orientation
will also include lens distortion

attitude : $\omega\phi\kappa$ or $\alpha\beta\gamma\theta$ or $q_s q_i q_j q_k$

camera pos : $X_L Y_L Z_L$ (exposure station)

obj. pt. : XYZ

✓ BBA
bundle block adj.
- att & exp. sta unk.
- some obj pts known (GCP), some unknown (pass points)

useful applications of collin. eqn's.
calibration, resection, intersection, BBA

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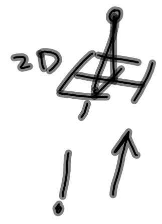
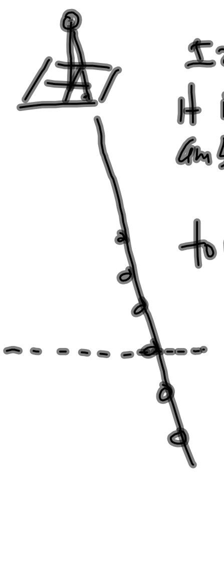
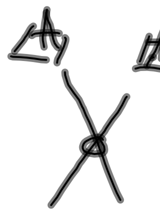
4-8



unique
G2I



I2G
it is ambiguous



to resolve ambiguity :
add more info
① add 1 coord. component of ground point
② add obs. from another photo
(conjugate image pts.)

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4-9

$$\vec{a} = \lambda M \vec{A}$$

$$\frac{1}{\lambda} M^T \vec{a} = \vec{A}$$

$$\frac{1}{\lambda} M^T \underbrace{\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix}}_{\begin{pmatrix} u \\ v \\ w \end{pmatrix}} = \begin{pmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{pmatrix}$$

$$\frac{1}{\lambda} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} X-X_L \\ Y-Y_L \\ Z-Z_L \end{pmatrix}$$

fix Z obj. space

$$\frac{\frac{1}{\lambda}u}{\frac{1}{\lambda}w} = \frac{X-X_L}{Z-Z_L}$$

$$\frac{\frac{1}{\lambda}v}{\frac{1}{\lambda}w} = \frac{Y-Y_L}{Z-Z_L}$$

$$\boxed{\begin{aligned} X &= X_L + (Z-Z_L)\frac{u}{w} \\ Y &= Y_L + (Z-Z_L)\frac{v}{w} \end{aligned}}$$

I2G (fixing Z)

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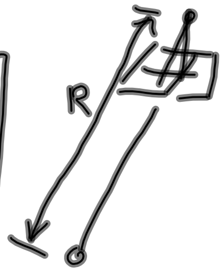
4-10

$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \underbrace{(\lambda R)}_{\text{nuisance constant}} M \underbrace{\begin{bmatrix} (X-X_L)/R \\ (Y-Y_L)/R \\ (Z-Z_L)/R \end{bmatrix}}_{\text{direction cosine } \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}}$$



$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = (\lambda R) M \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix}$$

rearrange equations for directional control

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4-11

$$\frac{x-x_0}{-f} = \frac{m_{11}c_x + m_{12}c_y + m_{13}c_z}{m_{31}c_x + m_{32}c_y + m_{33}c_z}$$

$$\frac{y-y_0}{-f} = \frac{m_{21}c_x + m_{22}c_y + m_{23}c_z}{(\cdot)}$$

got rid of obj. coordinates
exp. sta. coordinates
replaced position description by
angle description of pt.

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4-12

star camera : star catalog
right ascension] c_x
declination] c_y
c_z

direction control

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