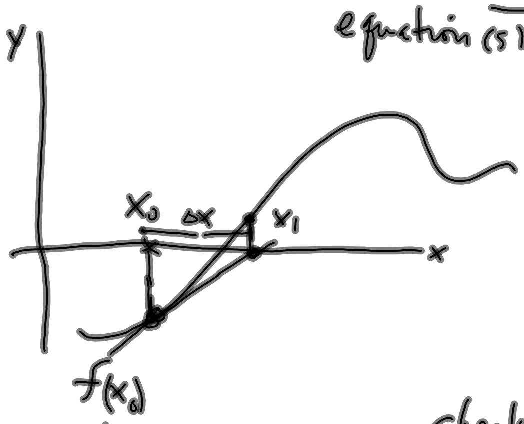
Newton Iteration: solve for root(s) of non-linear equation(s) 5-0



$f'(x_0) = \frac{-f(x_0)}{\Delta x}$

$$\boxed{\Delta x = \frac{-f(x_0)}{f'(x_0)}}$$

$x_1 = x_0 + \Delta x$

⋮

check for convergence: Δx small

$f'(x_0)$ $f(x)=0$

1D newton iteration, 1 eqn. 1 unknown

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$F_1(x_1, x_2) = 0$

$F_2(x_1, x_2) = 0$

$$\boxed{\Delta x = \frac{-f(x_0)}{f'(x_0)}}$$

5-1

$F_1 \approx F_1(x_1^0, x_2^0) + \frac{\partial F_1}{\partial x_1}\Delta x_1 + \frac{\partial F_1}{\partial x_2}\Delta x_2 = 0$

$F_2 \approx F_2(x_1^0, x_2^0) + \frac{\partial F_2}{\partial x_1}\Delta x_1 + \frac{\partial F_2}{\partial x_2}\Delta x_2 = 0$

$$\begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} -F_1(x_1^0, x_2^0) \\ -F_2(x_1^0, x_2^0) \end{bmatrix}$$

$J \cdot \Delta = -F$

$$\boxed{\Delta = -J^{-1} \cdot F}$$

Newton iteration 2 eqns 2 unknowns — can be extended to n eqns n unknowns

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5-2

solve 1×1, 2×2, $n\times n$

Newton Iteration Loop

solve over det. LS prob.

Newton Iter. Loop

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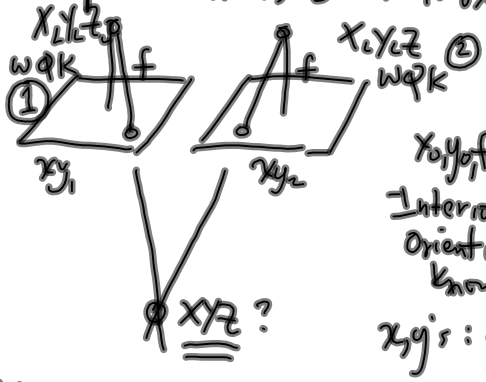
Space Intersection Problem

5-3



Exterior Orientation fixed Known

$x_0 y_0 f$ Interior Orientation Known

xy's : observed

Known $\sigma = 0$
observed $\sigma =$ finite
unknown $\sigma = \infty$

$$x = x_0 - f\frac{u}{w}$$
$$y = y_0 - f\frac{v}{w}$$

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$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

5-4

Rewrite Eq. so = zero

$$\left.\begin{aligned} F_{x_1} &= x_1 - x_0 + f\frac{u_1}{w_1} = 0 \\ F_{y_1} &= y_1 - y_0 + f\, {}^{v_1}/_{w_1} = 0 \\ F_{x_2} &= x_2 - x_0 + f\, {}^{u_2}/_{w_2} = 0 \\ F_{y_2} &= y_2 - y_0 + f\, {}^{v_2}/_{w_2} = 0 \end{aligned}\right\}$$

Linearized by Taylor Series Approx.

Trunc @ 1st deriv. terms

$\boxed{\text{4 condition equation, 3 unknowns}}$

overdetermined, so need to solve by LS

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$$F_{x_1} \approx F_{x_1}^0 + \boxed{\frac{\partial F_{x_1}}{\partial x_1}}^{1} \Delta x_1 + \boxed{\frac{\partial F_{x_1}}{\partial y_1}}^{0} \Delta y_1 + \frac{\partial F_{x_1}}{\partial X}\Delta X + \frac{\partial F_{x_1}}{\partial Y}\Delta Y + \frac{\partial F_{x_1}}{\partial Z}\Delta Z$$

$$F_{y_1} \approx F_{y_1}^0 + \underset{0}{\boxed{\frac{\partial F_{y_1}}{\partial x_1}}} \Delta x_1 + \underset{1}{\boxed{\frac{\partial F_{y_1}}{\partial y_1}}} \Delta y_1 + \frac{\partial F_{y_1}}{\partial x}\Delta X + \frac{\partial F_{y_1}}{\partial y}\Delta Y_1 + \overset{=0}{\frac{\partial F_{y_1}}{\partial z}}\Delta Z \quad = 0$$

(4 Eqns.)

$\Delta x_1 \rightarrow v_{x_1}$

$\Delta y_1 \rightarrow v_{y_1}$

$$\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \end{bmatrix} + \begin{bmatrix} \partial F_{x_1}/\partial x & \partial F_{x_1}/\partial y & \partial F_{x_1}/\partial z \\ \partial F_{y_1}/\partial x & \partial F_{y_1}/\partial y & \partial F_{y_1}/\partial z \\ \partial F_{x_2}/\partial x & \partial F_{x_2}/\partial y & \partial F_{x_2}/\partial z \\ \partial F_{y_2}/\partial x & \partial F_{y_2}/\partial y & \partial F_{y_2}/\partial z \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} -F_{x_1}^0 \\ -F_{y_1}^0 \\ -F_{x_2}^0 \\ -F_{y_2}^0 \end{bmatrix}$$

$$V + B \Delta = f$$

$$\left\{\begin{aligned} \Delta x_1 &\longrightarrow v_{x_1} \\ \Delta y_1 &\longrightarrow v_{y_1} \\ \Delta x_2 &\longrightarrow v_{x_2} \\ \Delta y_2 &\longrightarrow v_{y_2} \end{aligned}\right.$$

2 equations from left photo

2 equations from right photo

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$$\Delta = (B^T W B)^{-1} B^T W f \quad W: \text{weight matrix}$$

5-6

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{update} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{current} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

monitor magnitude of $\Delta X, \Delta Y, \Delta Z$

solve 1 iteration of the NL LS problem & update the parameters (unknowns) for the next iteration.

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NL-LS

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(a) need good approx of unknowns before we start

(b) need matrix B partial derivatives Jacobian — we will compute by numerical approximation ✓ + misclosure vector f on the right

(c) monitor convergence ✓

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$y = f(x)$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

choose Δx small, evaluate ratio

$$\frac{\partial f}{\partial x_i} = \lim_{\Delta x_i \to 0} \frac{f(x_1, x_2, \ldots, x_i + \Delta x_i, \ldots, x_n) - f(x_1 \ldots x_n)}{\Delta x_i}$$

use definition of derivative to make numerical approximations. Usually insignificant accuracy and performance issues — very quick & simple to implement

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5-9

numerical approx. for partial derivs, Intersect problem:

$F_x = x - x_0 + f\,u/w$, $F_y = y - y_0 + f\,v/w$

$p = [x; y; x_0; y_0; f; \omega; \varphi; \kappa; X_L; Y_L; Z_L; X; Y; Z]$

dp = ones(14,1) * 1e-06;

Construct function Collinearity Eqns.

```
% col.m
function result = col(p);
x = p(1);      om = p(6);     ZL = p(11);
y = p(2);      ph = p(7);     X = p(12);
x0 = p(3);     kp = p(8);     Y = p(13);
y0 = p(4);     XL = p(9);     Z = p(14);
foc = p(5);    YL = p(10);
```

need a function to evaluate the condition equation(s)

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m1 = [1 0 0 ; 0 cos(om) sin(om) ; 0 -sin(om) cos(om)];

m2 = [cos(ph) 0 -sin(ph) ; 0 1 0 ; sin(ph) 0 cos(ph)];

m3 = [cos(kp) sin(kp) 0 ; -sin(kp) cos(kp) 0 ; 0 0 1];

M = m3 * m2 * m1 ;

uvw = M * [X-XL ; Y-YL ; Z-ZL]

5-10

put together intermediate variables to evaluate the collinearity condition equations.

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