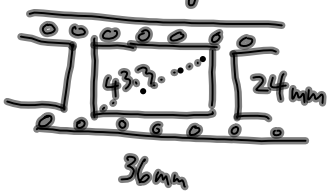
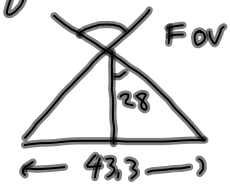
7-1

focal length vs. 35mm equivalent



3:2



most digital sensors 4:3

$2 \cdot \tan^{-1}\left(\frac{\frac{1}{2}43.3}{28}\right) \approx 75^\circ$

Wide angle 28mm → FOV 47°

normal angle 50mm → FOV 47°

telephoto 200mm → 12°

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4.291

5.760 mm

f: 6 mm

(width) $f_{35} = 36 \cdot \frac{f}{w}$

(diagonal) $f_{35} = 34.6 \frac{f}{w}$

$f_{35} = 36mm$

f	FOV	
88	123°	SWA
152.4	94°	WA
210	75°	...

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FOV ~ 17°

Intersection

recast equations so may become linear

$$X = X_L + (Z - Z_L)\left(\frac{u}{w}\right) - C_1$$
$$Y = Y_L + (Z - Z_L)\left(\frac{v}{w}\right) - C_2$$

$$X = X_L + (Z - Z_L)C_1$$
$$Y = Y_L + (Z - Z_L)C_2$$
$$X = X_L + Z \cdot C_1 - Z_L \cdot C_1$$
$$Y = Y_L + Z \cdot C_2 - Z_L \cdot C_2$$

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$$X - C_1 Z = X_L - C_1 Z_L$$
$$Y - C_2 Z = Y_L - C_2 Z_L$$

$$\begin{bmatrix} 1 & 0 & -C_1 \\ 0 & 1 & -C_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_L - C_1 Z_L \\ Y_L - C_2 Z_L \end{bmatrix}$$

really linear : matrix elements constants

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$$\begin{bmatrix} 1 & 0 & -c_{11} \\ 0 & 1 & -c_{21} \\ 1 & 0 & -c_{12} \\ 0 & 1 & -c_{22} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \approx \begin{bmatrix} X_{L_1} - c_{11} Z_{L_1} \\ Y_{L_1} - c_{21} Z_{L_1} \\ X_{L_2} - c_{12} Z_{L_2} \\ Y_{L_2} - c_{22} Z_{L_2} \end{bmatrix}$$

7.5

↑ constant

↑ known

Linear LS $Bx = f$

$$x = (B^T B)^{-1} B^T f$$

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Benefit: NO iterations
NO initial approximations
NO partial derivatives

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Not rigorous solution
C's contain observations
stochastic model wrong

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Coordinate transformations

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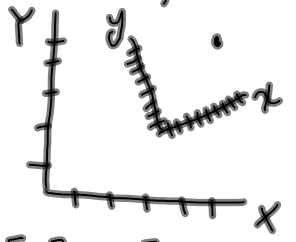
2D conformal coord. transf.



rotation Θ
scale λ
2 shifts t_x, t_y

$$\begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} X+T_X \\ Y+T_Y \end{bmatrix}$$

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$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \overbrace{\lambda\cos\Theta}^{a} & \overbrace{\lambda\sin\Theta}^{b} \\ -\lambda\sin\Theta & \lambda\cos\Theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \begin{matrix} \leftarrow c \\ \leftarrow d \end{matrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$$ form application

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} X & Y & 1 & 0 \\ Y & -X & 0 & 1 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$ form to use for estimation

XY fixed, xy observed

each point: 2 equations

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7.9

2 points ⇒ 4 equations

$$\begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & Y_1 & 1 & 0 \\ Y_1 & -X_1 & 0 & 1 \\ X_2 & Y_2 & 1 & 0 \\ Y_2 & -X_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

⇒ unique solution for a, b, c, d

3 pts ⇒ 6 eqn's, LS estimation

n pts ⇒ 2n eqn's ...

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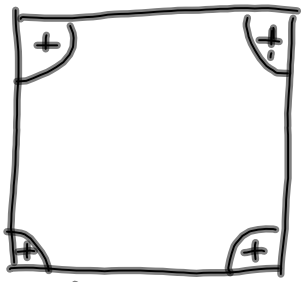
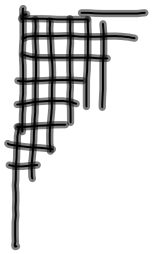


fiducial marks
XY



observations
xy

typical application of 2D coord. transf.
4 parameter transf.
film : deformation

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2D projective transformation
8 parameter transf.

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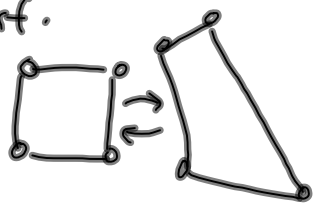
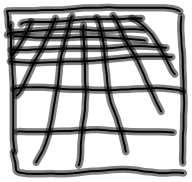


applications

1. rectification *
(flat terrain)
resampling : constant scale



2. texture mapping
3. approximations for EO of indiv. images

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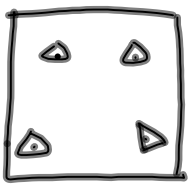

8 par. ⇒ E.O.

7 -12

$$x = \frac{a_0 + a_1 X + a_2 Y}{1 + c_1 X + c_2 Y} \qquad y = \frac{b_0 + b_1 X + b_2 Y}{1 + c_1 X + c_2 Y}$$

$a_0 a_1 a_2 b_0 b_1 b_2 c_1 c_2$ — 8 par

applying

$$x + x c_1 X + x c_2 Y = a_0 + a_1 X + a_2 Y$$

$$y + y c_1 X + y c_2 Y = b_0 + b_1 X + b_2 Y$$

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$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 1 & x & y & 0 & 0 & 0 & -xX & -xY \\ 0 & 0 & 0 & 1 & x & y & -yX & -yY \end{pmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix}$$

7.13

each point generates 2 Eq's

4 pts ⇒ 8 eqn's

8 eq / 8 unknowns

x, y obs., XY constant

obs

"

RV

treat as constants

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