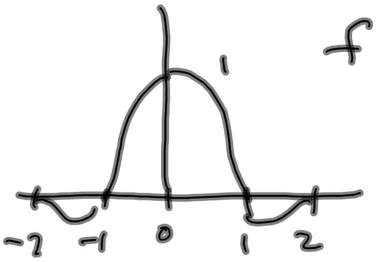
12-1



$$f = |x|^3 - 2|x|^2 + 1 \qquad 0 \le |x| \le 1$$

$$-|x|^3 + 5|x|^2 - 8|x| + 4 \qquad 1 < |x| \le 2$$

$$0 \quad \text{otherwise}$$

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12-2

(r_0, c_0) - point to interpolate

$c_1 = \text{fix}(c_0) - 1$	$r_1 = \text{fix}(r_0) - 1$
$c_2 = c_1 + 1$	$r_2 = r_1 + 1$
$c_3 = c_1 + 2$	$r_3 = r_1 + 2$
$c_4 = c_1 + 3$	$r_4 = r_1 + 2$

if $\text{fix}(c_0) = c_0$ then no interpolation necessary

rows and columns for the image "window" to use for the interpolation

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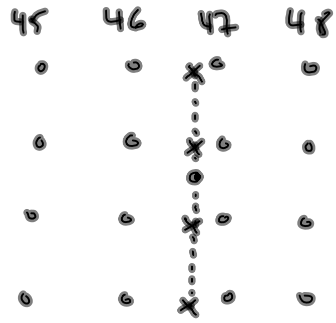
12-3

	45	46	47	48
49	242	213	152	148
50	220	194	152	156
51	192	147	160	154
52	226	202	153	141



$$\left[\begin{array}{l} r_0 = 50.3 \\ c_0 = \underline{46.8} \end{array}\right.$$

$$\underline{f_c \cdot \begin{bmatrix} r_{49} \cdot f_r \\ r_{50} \cdot f_r \\ r_{51} \cdot f_r \\ r_{52} \cdot f_r \end{bmatrix}} = I_{Bc}$$

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12-4

along the rows

$$f_r = \begin{bmatrix} f(45 - 46.8) \\ f(46 - 46.8) \\ f(47 - 46.8) \\ f(48 - 46.8) \end{bmatrix} = \begin{bmatrix} f(c_1 - c_0) \\ f(c_2 - c_0) \\ f(c_3 - c_0) \\ f(c_4 - c_0) \end{bmatrix}$$

4 col values $\quad c_0$

$$f_c = \begin{bmatrix} f(r_1 - r_0) \\ f(r_2 - r_0) \\ f(r_3 - r_0) \\ f(r_4 - r_0) \end{bmatrix}$$

along the columns

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12-5

$$[f_{c_1}\ f_{c_2}\ f_{c_3}\ f_{c_4}]\begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}\begin{bmatrix} f_{r_1} \\ f_{r_2} \\ f_{r_3} \\ f_{r_4} \end{bmatrix}$$

$$\boxed{I_{Bc} = f_c^T I f_r}$$

do this individually for R,G,B

I : image sub matrix
or
window

can operate first on rows or columns — same result!

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12-6

$I_{Bc}(50.3, 46.8) =$

$$[-.1470\ \ .8470\ \ .3630\ \ -.0630]\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}\begin{bmatrix} -.0320 \\ .2720 \\ .9280 \\ -.1280 \end{bmatrix}$$

$= 157.2$

uint8 0-255

$R = \text{round}(I_{Bc})$

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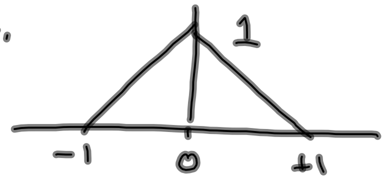
Bilinear Interp. 12-7



$$f_r = \begin{bmatrix} f(c_1 - c_0) \\ f(c_2 - c_0) \end{bmatrix}$$

$$f = \begin{cases} -|x| + 1 & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_c = \begin{bmatrix} f(r_1 - r_0) \\ f(r_2 - r_0) \end{bmatrix}$$

r_0, c_0 :
$r_1 = \text{fix}(r_0)$
$r_2 = r_1 + 1$
$c_1 = \text{fix}(c_0)$
$c_2 = c_1 + 1$

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$$I_{BL} = \underbrace{f_c^T}\,\underbrace{\underline{I} f_r}$$

12-8

$$\begin{bmatrix} .7 & .3 \end{bmatrix} \begin{bmatrix} 194 & 152 \\ 147 & 160 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} = \underline{159.5}$$

Camera calibration : USGS

$\underline{x_0, y_0}, f, k_1, k_2, k_3, P_1, P_2$

resolving power / resolution

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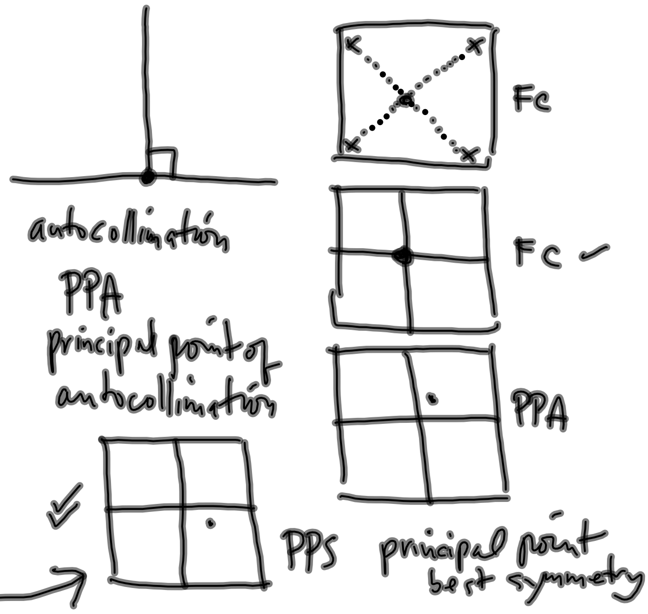
12-9



we use this one

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12-10

lens distortion model

Duane Brown
Clive Fraser
1997

$$x - x_0 = -f\,\frac{u}{w}$$
$$y - y_0 = -f\,\frac{v}{w}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M \cdot \begin{pmatrix} X - X_c \\ Y - Y_c \\ Z - Z_c \end{pmatrix}$$

$$F_x = x - x_0 + f\,\frac{u}{w} = 0$$
$$F_y = y - y_0 + f\,\frac{v}{w} = 0$$

$$F_x = x - x_0 + \Delta x + f\,\frac{u}{w} = 0$$
$$F_y = y - y_0 + \underbrace{\Delta y}_{\text{lens distortion}} + f\,\frac{v}{w} = 0$$

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12-11

$$\Delta x = \Delta x_r + \Delta x_d + \Delta x_u + \Delta x_f$$
$$\Delta y = \Delta y_r + \Delta y_d + \Delta y_u + \Delta y_f$$

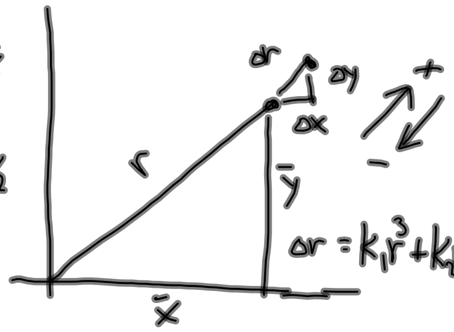
radial ↑ decentering ↑ unflatness ↑ in-plane ↑

radial:

$x - x_0 = \bar{x}$

$y - y_0 = \bar{y}$

$r = \left[\bar{x}^2 + \bar{y}^2\right]^{1/2}$



$\Delta r = k_1 r^3 + k_2 r^5 + k_3 r^7 \ldots$

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12-12

$\frac{\bar{x}}{r} = \frac{\Delta x}{\Delta r}$, $\Delta x = \frac{\Delta r}{r} \cdot \bar{x}$

$\frac{\bar{y}}{r} = \frac{\Delta y}{\Delta r}$, $\Delta y = \frac{\Delta r}{r} \cdot \bar{y}$

$$\Delta x_r = \frac{k_1 r^3 + k_2 r^5 + k_3 r^7}{r} \cdot \bar{x}$$

$$\Delta y_r = \frac{k_1 r^3 + k_2 r^5 + k_3 r^7}{r} \cdot \bar{y}$$

$$\boxed{\begin{aligned} \Delta x_r &= \bar{x} \cdot \left[k_1 r^2 + k_2 r^4 + k_3 r^6\right] \\ \Delta y_r &= \bar{y} \cdot \left[k_1 r^2 + k_2 r^4 + k_3 r^6\right] \end{aligned}}$$

radial distortion

unknowns k_1, k_2, k_3

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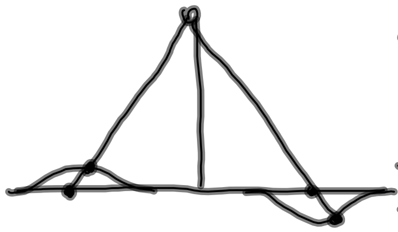
$$\boxed{\begin{aligned}\Delta x_d &= P_1(r^2 + 2\bar{x}^2) + 2P_2\bar{x}\bar{y}\\ \Delta y_d &= 2P_1\bar{x}\bar{y} + P_2(r^2 + 2\bar{y}^2)\end{aligned}}$$

decentering distortion

(not same as textbook)

12-13

focal plane <u>un</u>flatness (out of plane)



$\Delta x = \bar{x}\cdot\frac{\Delta r}{r}$

$\Delta y = \bar{y}\cdot\frac{\Delta r}{r}$

< 2μm

$$\Delta r = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6x^3 + a_7y^3 + a_8x^2y + a_9xy^3$$

$$\boxed{\begin{aligned}\Delta x_u &= \frac{\bar{x}}{r}\cdot\Delta r\\ \Delta y_u &= \frac{\bar{y}}{r}\cdot\Delta r\end{aligned}}$$

unflatness distortion

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