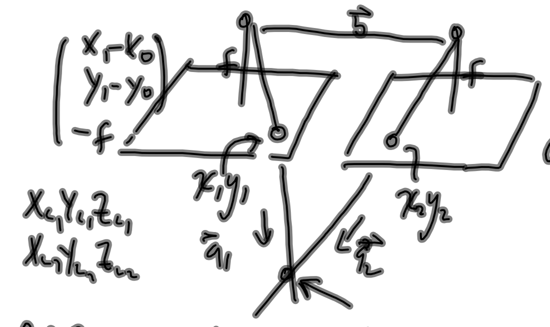


$x_1\, y_1$

$x_2\, y_2$

conjugate points

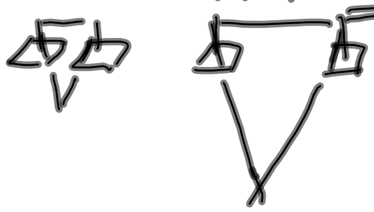
14-1

0,0,0

b_x, b_y

b_z

Relative Orientation : 5 parameter



$$\vec{b} = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

$\vec{b} \cdot (\vec{a}_1 \times \vec{a}_2) = 0$

any order

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14-2

$$\vec{a}_1 = M_1^T \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ -f \end{pmatrix}, \quad \vec{a}_2 = M_2^T \begin{pmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{pmatrix}$$

$$F_{cp} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix} = 0 \qquad b_y, b_z, \omega_2, \varphi_2, k_2$$

determinant $D = \begin{vmatrix} R_1 \\ R_1 \\ R_1 \end{vmatrix}$ $\quad R_1 = 1^{st}$ row

$$\frac{\partial D}{\partial p} = \begin{vmatrix} \partial R_1/\partial p \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ \partial R_2/\partial p \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ \partial R_3/\partial p \end{vmatrix}$$

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14-3

$p = [x_0; y_0; f; bx; by; bz; \omega_2, \varphi_2, k_2, x_1; y_1; x_2; y_2]$

function $F_{cp} = \text{coplan}(p)$

unpack p

$M_1 = I = \text{eye}(3)$

$M_2 = M_z(k_2) \cdot M_y(\varphi_2) \cdot M_x(\omega_2)$

$\boxed{bx = 100}$

$a_1 = M_1^T \cdot [x_1 - x_0; y_1 - y_0; -f]$

$a_2 = M_2^T \cdot [x_2 - x_0; y_2 - y_0; -f]$

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14-4

$F_{cp} = \det([bx\ by\ bz;$
$a_1(1)\ a_1(2)\ a_1(3);$
$a_2(1)\ a_2(2)\ a_2(3)]);$

end of function

$$\frac{\partial F_{cp}}{\partial \omega_2} \approx \frac{F_{cp}(\omega + \Delta\omega, \ldots\ldots) - F_{cp}(\omega, \ldots\ldots)}{\Delta\omega}$$

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parameter selection

14-5

dependent R/O : unknowns $b_y, b_z, \omega_2, \varphi_2, \kappa_2$

independent R/O : unknowns $\kappa_1 \kappa_2 \varphi_1 \varphi_2 \omega_1$

b_x : fixed any value

$b_y = b_z = 0$ (independent)

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R/O : have 4 obs. per equation

does not work with indirect obs. ($V + B\Delta = f$)

14-6

$\Rightarrow$ General LS (mixed model)

$$\left.\begin{array}{l} F_1(x_{1_1} y_{1_1} x_{2_1} y_{2_1} \cdots P_1 P_2 P_3 P_4 P_5) = 0 \\ F_2(x_{1_2} y_{1_2} x_{2_2} y_{2_2} \cdots P_1 \cdots P_5) = 0 \\ \vdots \\ F_5(x_{1_5} y_{1_5} x_{2_5} y_{2_5} \cdots P_1 \cdots P_5) = 0 \\ F_6 \\ \vdots \end{array}\right\} \text{Taylor Series}$$

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14-7

$$\begin{bmatrix} \partial F/\partial l \\ A \end{bmatrix} \begin{bmatrix} v \end{bmatrix} + \begin{bmatrix} \partial F/\partial x \\ B \end{bmatrix} \begin{bmatrix} \Delta \end{bmatrix} = \underbrace{\begin{bmatrix} -F^\circ \end{bmatrix}}_{f}$$

W : weight matrix for observations

↳ initial approximation for unknowns

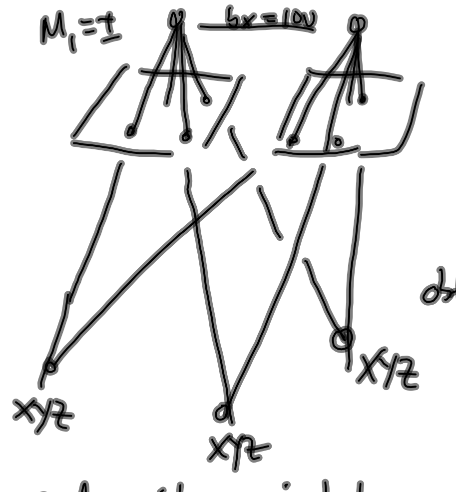
LS solution: $k = W_e(f - B\Delta)$

$Q = W^{-1}$ $\qquad$ $V = QA^T k$

$Q_e = AQA^T$ $\qquad$ residuals

$\Delta = (B^T W_e B)^{-1} B^T W_e f$ $\qquad$ iterate to conv.

14-8



obtain points in object space:

intersection problem = model coordinates

↓

A/O

↓

ground coordinates

scale, position, orientation : arbitrary

↳ fixing these = Absolute Orientation

Absolute Orientation

14-9

$$\begin{array}{cc} A/O & R/O \\ \underbrace{7 \qquad\quad 5}_{\text{BBA } 12} \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

model coordinates: result of R/O + intersection

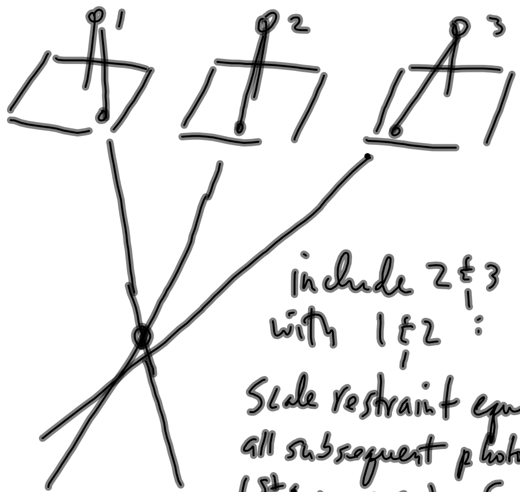
ground coordinates: well defined reference coord. system

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda M \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$$

λ
$\theta_1 \theta_2 \theta_3$
$T_x T_y T_z$

7 parameter transf.
3D conformal coord. transf.
Rigid Body transformation

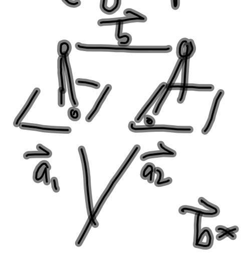
14-10



include 2 & 3 along with 1 & 2:
Scale restraint equation
all subsequent photos, after 1st two: solve for 6 par's rather than 5

Relative Orientation as C.V.
(eight point algorithm)

14-11



$\vec{b}\cdot(\vec{a}_1\times\vec{a}_2)=0$

any order

cross product:

$$\vec{b}\times\vec{a}=\begin{bmatrix}0 & -b_z & b_y\\ b_z & 0 & -b_x\\ -b_y & b_x & 0\end{bmatrix}\begin{bmatrix}a_x\\ a_y\\ a_z\end{bmatrix}$$

$K_b a=\vec{b}\times\vec{a}$ $\qquad K_b$

$$\vec{a}_1\cdot(\underbrace{\vec{b}\times\vec{a}_2}) = \boxed{a_1^T K_b a_2 = 0}$$

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express image coordinates in homogeneous coordinate form

14-12

$$\begin{pmatrix}x\\ y\end{pmatrix}\rightarrow\begin{pmatrix}x\\ y\\ 1\end{pmatrix} \qquad \begin{pmatrix}x-x_0\\ y-y_0\\ -f\end{pmatrix}=\begin{pmatrix}1 & 0 & -x_0\\ 0 & 1 & -y_0\\ 0 & 0 & -f\end{pmatrix}\begin{pmatrix}x\\ y\\ 1\end{pmatrix}$$

C

$$\vec{a}_1 = M_1^T\begin{pmatrix}x_1-x_0\\ y_1-y_0\\ -f\end{pmatrix} = M_1^T C\begin{pmatrix}x_1\\ y_1\\ 1\end{pmatrix}$$

$$\vec{a}_2 = M_2^T\begin{pmatrix}x_2-x_0\\ y_2-y_0\\ -f\end{pmatrix} = M_2^T C\begin{pmatrix}x_2\\ y_2\\ 1\end{pmatrix}$$

$a_1^T K_b a_2 = 0$

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$q_1 = M_1^T C \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$ $q_1^T = [x_1\ y_1\ 1]\ C^T M_1$ 14-13

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