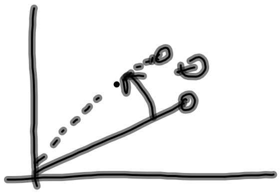
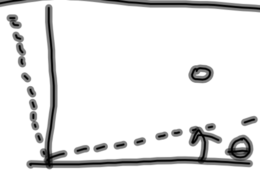
HW2: precision: provide enough digits

15-1



$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

notes + textbook use this one

Two choices to define "positive" rotation. Pick one and be consistent. My notes + textbook use the 2nd convention.

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R/D: computer vision

15-2

$$\vec{a}_1 \cdot (\vec{b} \times \vec{a}_2) = 0$$

$$\underbrace{\vec{a}_1^T K_b \vec{a}_2} = 0$$

$$\vec{a}_1 = M_1^T \begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix}$$

$$= M_1^T \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & -f \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

$$\vec{a}_2 = M_2^T \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

$$[x_1\ y_1\ 1]\ \underbrace{C^T \underbrace{M_1 K_b M_2^T}_{E} C}_{F} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = 0$$

Essential Matrix

Fundamental Matrix

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$$[x_1\ y_1\ 1]\, C^T E\, C \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = 0$$

$$\boxed{[x_1 - x_0\ \ y_1 - y_0\ \ -f]\ E \begin{pmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -f \end{pmatrix} = 0}$$

5 parameters of R/0 inside E

$$[x_1\ y_1\ 1]\ F \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = 0$$

x, y: obs

$$(\bar{x}_1\ \bar{y}_1\ -f) \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{bmatrix} \bar{x}_2 \\ \bar{y}_2 \\ -f \end{bmatrix} = 0$$

e_{ij} unknowns

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$$\bar{x}_1 \bar{x}_2 e_{11} + \bar{x}_2 \bar{y}_1 e_{21} - \bar{x}_2 f e_{31} + \bar{y}_2 \bar{x}_1 e_{12} +$$
$$\bar{y}_2 \bar{y}_1 e_{22} - \bar{y}_2 f e_{32} - f \bar{x}_1 e_{13} - f \bar{y}_1 e_{23} +$$
$$f^2 \underbrace{e_{33}}_{\rightarrow 1} = 0$$

If E were solution, then λE also solution

resolve scale ambiguity $e_{33} = 1$

<u>8 unknowns</u>

$$[\bar{x}_1\bar{x}_2\ \ \bar{x}_2\bar{y}_1\ \ -\bar{x}_2 f\ \ \bar{y}_2\bar{x}_1\ \ \bar{y}_2\bar{y}_1\ \ -\bar{y}_2 f\ \ -f\bar{x}_1\ \ -f\bar{y}_1] \begin{bmatrix} e_{11} \\ e_{21} \\ e_{31} \\ e_{12} \\ e_{22} \\ e_{32} \\ e_{13} \\ e_{23} \end{bmatrix} = -f^2$$

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8 points ⇒ 8 equations ⇒ 8 unknowns

⇒ unique solution

or with redundancy solve 8 e_{ij}'s

$$E = M_1 K_3 M_2^T = I \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix} M_2^T$$

$M_1 = I$

b_x, b_y, b_z M_2

make SVD singular value decomp.

$E = USV^T$

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$[u, s, v] = \text{svd}(E)$ matlab syntax

u, s, v

M_2 either uWv^T OR uW^Tv^T ✓

where $W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b either u_3 OR $-u_3$ ✓

u_3: 3rd column of u

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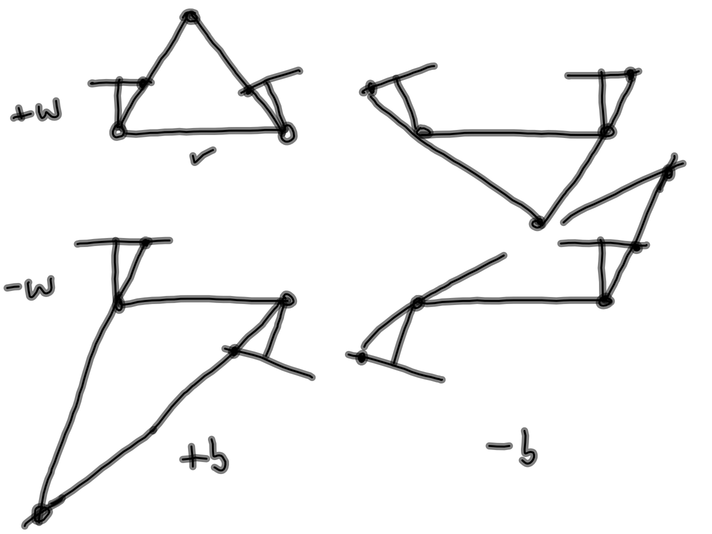

resolve ambiguity to get correct R/O parameters

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<u>intersection eqn's</u> $\begin{pmatrix} u \\ v \\ w \end{pmatrix} = M^T \begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix}$

$$X = X_L + (Z - Z_L)\, u/w \quad \cdots\cdots$$
$$Y = Y_L + (Z - Z_L)\, v/w$$
$$\vdots$$
$$\begin{bmatrix} 1 & 0 & -u/w \\ 0 & 1 & -v/w \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_L - Z_L \cdot u/w \\ Y_L - Z_L \cdot v/w \end{bmatrix}$$

use these intersection equations to get an object point to resolve ambiguity on previous page.

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$$\begin{bmatrix}\square\\ \square\\ \square\end{bmatrix} = M \begin{bmatrix} X-X_c\\ Y-Y_c\\ Z-Z_c\end{bmatrix}$$

if 3rd element negative for both cameras, that is the solution

$b, M \rightarrow \omega, \phi, \kappa$

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15-10

stereo environments

anaglyph : L: red, R: blue

optical : stereoscopes
viewmaster

flicker : 120 Hz, active glasses
synchronized w/ monitor

polarization : bezel, switches between
2 polarization states

passive

auto stereo : lenticular lenses

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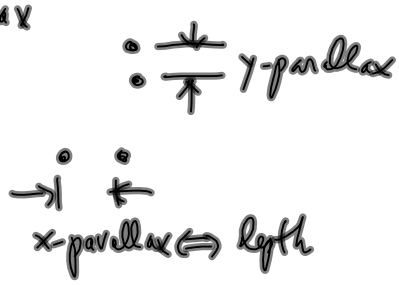
for any stereo environment, must present L+R images oriented so no parallax



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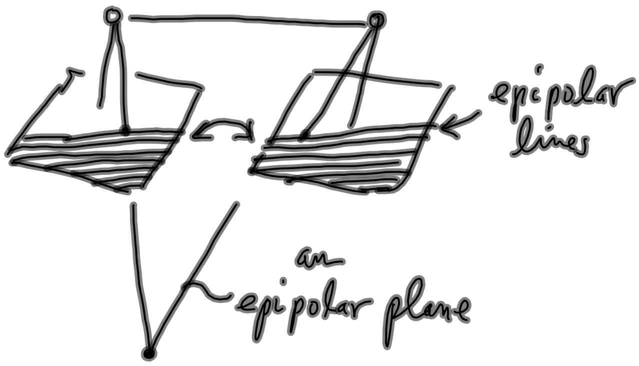

conjugate epipolar lines

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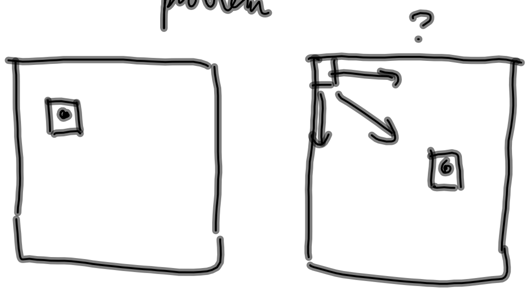
2 major applications for epipolar resampling

15-13

(a) stereo environments

(b) matching / correspondence problem



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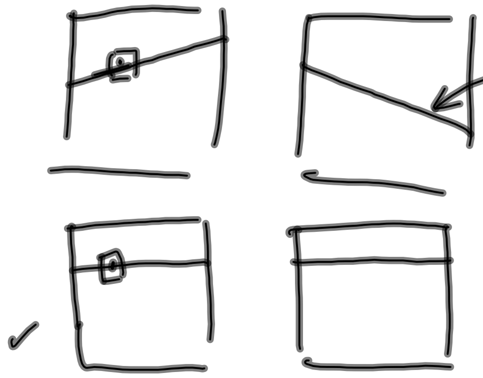

conjugate pt must be along the conj. epi-p. line

resampled

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