22-1

$$\Sigma_{xx} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{12} & \sigma_2^2 & & \\ & & \ddots & \\ \sigma_{1n} & \cdots & & \sigma_n^2 \end{bmatrix}$$ covariance matrix $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$\vec{y} = A\vec{x}$ ($m,1 = m,n\ n,1$)

$\vec{x}$: Rand. vector
A: Constant matrix
Σ_{xx}: have this

$$\boxed{\Sigma_{yy} = A\Sigma_{xx}A^T}$$

Mar 28-4:29 PM

22-2

Concept of weight

$$w_i = \frac{\sigma_0^2}{\sigma_i^2}$$

1,1,2,3
2,2,4,6

$$W = \begin{bmatrix} w_1 & & \phi \\ & w_2 & \\ & & w_3 \\ \phi & & & w_4 \end{bmatrix}$$

σ_0^2: reference variance

variance of unit weight

σ_0^2 prior

apriori

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22-3

Mathematical model

functional model (equations)

Stochastic model

unknown $\sigma = \infty$
constant $\sigma = 0$
obs σ = finite

n: # of observations
$-\ n_0$: min # of obs to fix model
r: redundancy

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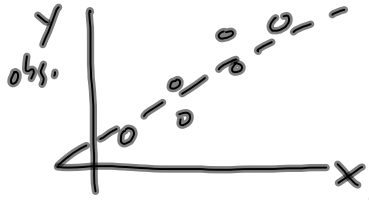
22-4



$n = 6$
$n_0 = 2$
$r = 4$

$c = n$

constant $y = mx + b$

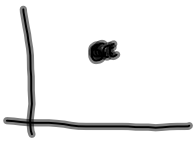
linear regression

$y + v_y - mx - b = 0$

$v_y - mx - b = -y$

$v + B\Delta = f$

$\begin{bmatrix} \hat{m} \\ \hat{b} \end{bmatrix} = \Delta = (B^T W B)^{-1} B^T W f$



$n = 6$
$n_0 = 2$
$r = 4$ ✗ ⇒ obs must fix model

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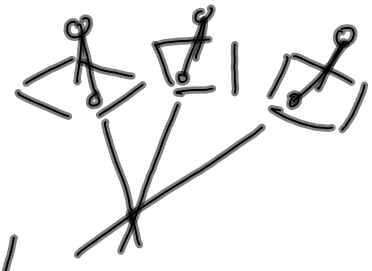
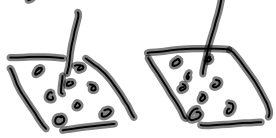
intersection



22-5

$n = 6$
$n_0 = 3$
$r = 3$

RO:



GenLS

$n = 8 \times 2 \times 2 = 32$ $\quad c = r + u = 8$
$n_0 = 5 \times 2 \times 2 + 3 \times 2 + 3 + 1 = 29$ ←
$r = 3$
$u = 5$

u: # params,
c: # cond eqn's

Mar 28-4:29 PM

Indirect observations: cond. eqn: any equation w/ observation

22.6

- $c = n$
- $u = n_0$
- one obs. per cond. equation

$V + B_0 = f$

$W, Q: \quad Q = W^{-1}$

$\breve{\Sigma} = \sigma_0^2 \breve{Q}$
$W = \sigma_0^2 \Sigma^{-1}$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix} \qquad W = \begin{bmatrix} \sigma_0^2/\sigma_1^2 & & & \\ & \sigma_0^2/\sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_0^2/\sigma_n^2 \end{bmatrix}$$

$$Q = \begin{bmatrix} \sigma_1^2/\sigma_0^2 & & & \\ & \sigma_2^2/\sigma_0^2 & & \\ & & \ddots & \\ & & & \sigma_n^2/\sigma_0^2 \end{bmatrix} = \frac{1}{\sigma_0^2} \Sigma$$

Mar 28-4:29 PM

$V + B\Delta = f$

$\Delta = \underbrace{(B^T W B)^{-1}}_{N} \underbrace{B^T W f}_{t}$ Q: scaled covariance matrix 22-7

$\Delta = N^{-1} t$ Estimation

$V = f - B\Delta$

$\hat{l} = l + v$ (adj obs.)

$Q_{\Delta\Delta} = N^{-1}$

$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}$ Error Propagation

Mar 28-4:29 PM

$Q_{VV} = B N^{-1} B^T$, $\Sigma_{VV} = \sigma_0^2 Q_{VV}$ 22-8

$Q_{\hat{l}\hat{l}} = Q - Q_{VV}$

↑ Ind. Obs.

GenLS (mixed model)

if more than 1 obs per equation

$Av + B\Delta = f$, W, Q $Q = W^{-1}$

$Q_e = A Q A^T$

$W_e = Q_e^{-1}$

$N = (B^T W_e B)$, $t = B^T W_e f$

$\Delta = N^{-1} t$, $v = Q A^T W_e (f - B\Delta)$

$\hat{l} = l + v$

Mar 28-4:29 PM

$Q_{\ell\ell} = N^{-1}$, $\Sigma_{\ell\ell} = \sigma_0^2 Q_{\ell\ell}$ 22-9

$Q_{vv} = Q A^T W_e (I - B N^{-1} B^T W_e) A Q$

$Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}$

Q, W without subscripts : original obs.

Evaluate consistency of obs + model

look @ magn. of residuals ⇒ global test

Mar 28-4:29 PM

global test

22-10

σ_0^2 : prior var of unit weight

$\hat{\sigma}_0^2 = \frac{v^T W v}{r}$

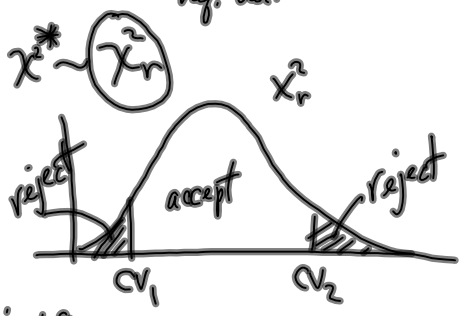
$\hat{\sigma}_0^2$: post adjustment estimate of ref. var.

Hyp. test: $\frac{v^T W v}{\sigma_0^2} = \chi^{2*} \sim \chi^2_r$

$H_0 : \sigma^2 = \sigma_0^2$

$H_1 : \sigma \neq \sigma_0^2$

α : prob of Type I error : .05, level of significance



Mar 28-4:29 PM

decision rule

22-11

$CV_1 < \chi^{2*} < CV_2$ accept H_0

otherwise reject H_0

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