CE 597 Exam 1 7 March, 2013

75 minutes allowed, 1 sheet of notes allowed

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Name

1. The given matrix is constructed by first applying a z-rotation (κ), and then applying a y-rotation (ϕ). What are κ and ϕ ?

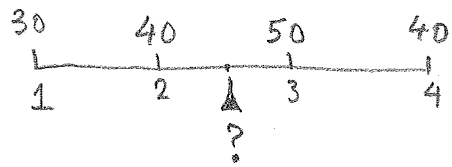
$$\begin{bmatrix} -.4981 & .8627 & -.0872 \\ -.8660 & -.5 & 0 \\ -.0436 & .0755 & .9962 \end{bmatrix}$$

2. A frame photograph has the following attributes :

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}_m, \quad M = \begin{bmatrix} .9848 & -.1736 & 0 \\ 0 & 0 & 1 \\ -.1736 & -.9848 & 0 \end{bmatrix}, \quad x_0 = y_0 = 0, \quad f = 100\,mm$$

for the image point $(x,y) = (30, 25)$, find the intersection with the plane in object space : $Y = 200\,m$.

3. The intensities at pixels 1,2,3,4 are 30, 40, 50, 40. Use the cubic interpolation model to find the intensity at pixel 2.5 .



4. Form the coplanarity condition equation with

$$b = \begin{bmatrix} 10 \\ 0.4 \\ 0.2 \end{bmatrix} \quad a_1 = \begin{bmatrix} 6 \\ -0.5 \\ -10 \end{bmatrix} \quad a_2 = \begin{bmatrix} -4 \\ -0.9 \\ -10.2 \end{bmatrix} \quad b = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

numbers are computed from initial approximations

Find $\frac{\partial F}{\partial b_z}$ analytically and numerically.

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}, \quad M_2 = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}, \quad M_3 = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$f_1(x) = |x|^3 - 2|x|^2 + 1 \qquad 0 < x \le 1$$

$$f_2(x) = -|x|^3 + 5|x|^2 - 8|x| + 4 \qquad 1 < x \le 2$$

$$M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}, \quad \frac{\partial |M|}{\partial p} = \begin{vmatrix} \partial M_1/\partial p \\ M_2 \\ M_3 \end{vmatrix} + \begin{vmatrix} M_1 \\ \partial M_2/\partial p \\ M_3 \end{vmatrix} + \begin{vmatrix} M_1 \\ M_2 \\ \partial M_3/\partial p \end{vmatrix}$$

$\boxed{\text{useful facts}}$

$$F_{CP} = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix} = 0$$

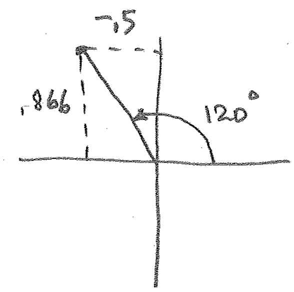
CE 597 Photo 1 Exam Solution
19 March 2013

1. $M = M_y(\phi) \cdot M_z(k) = \begin{bmatrix} c\phi & 0 & -s\phi \\ 0 & 1 & 0 \\ s\phi & 0 & c\phi \end{bmatrix} \begin{bmatrix} ck & sk & 0 \\ -sk & ck & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} c\phi ck & c\phi sk & -s\phi \\ -sk & ck & 0 \\ s\phi ck & s\phi sk & c\phi \end{bmatrix} = \begin{bmatrix} -.4981 & .8627 & -.0872 \\ -.8660 & -.5 & 0 \\ -.0436 & .0755 & .9962 \end{bmatrix}$$

$\phi = a\sin(-m_{13}) = a\sin(.0872) = \boxed{5°}$

$K = a\sin(-m_{21}) = a\sin(.8660)$
$= a\cos(m_{22}) = a\cos(-0.5)$ $\boxed{120°}$



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2. $\begin{pmatrix} x \\ y \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$ $\quad \frac{1}{\lambda} M^T \begin{pmatrix} x \\ y \\ -f \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$

$\frac{u}{v} = \frac{X - X_L}{Y - Y_L}$

$\frac{w}{v} = \frac{Z - Z_L}{Y - Y_L}$

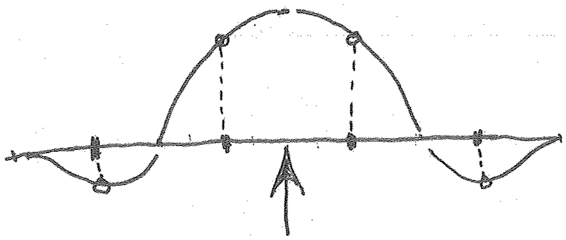
$$\begin{bmatrix} .9848 & 0 & -.1736 \\ -.1736 & 0 & -.9848 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 30 \\ 25 \\ -100 \end{bmatrix} = \begin{bmatrix} 46.904 \\ 93.272 \\ 25 \end{bmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$\frac{u}{v}(Y - Y_L) + X_L = X \quad , \quad .5029 \cdot (200 - 10) + 5 = 100.5$

$\frac{w}{v}(Y - Y_L) + Z_L = Z \quad , \quad .2680 \cdot (200 - 10) + 15 = 65.9$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 100.5 \\ 200 \\ 65.9 \end{pmatrix}$$

3.



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$$f_1(\pm 0.5) = (0.5)^3 - 2(0.5)^2 + 1 = .625$$

$$f_2(\pm 1.5) = -(1.5)^3 + 5(1.5)^2 - 8(1.5) + 4 = -.125$$

$$-.125(30) + .625(40) + .625(50) - .125(40) = 47.0$$

4. $F = \begin{vmatrix} b_x & b_y & b_z \\ a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \end{vmatrix} = 0$, $\begin{vmatrix} 10 & 0.4 & 0.2 \\ 6 & -0.5 & -10 \\ -4 & -0.9 & -10.2 \end{vmatrix} = 0$

$$\frac{\partial F}{\partial b_z} = \begin{vmatrix} 0 & 0 & 1 \\ 6 & -.5 & -10 \\ -4 & -.9 & -10.2 \end{vmatrix} + 0 + 0 = (1)\left[6\times(-.9) - 4\times(.5)\right]$$
$$= \boxed{-7.4}$$

or numerically, $\Delta b_z = 0.01$

$$\frac{\partial F}{\partial b_z} \approx \frac{F(b_z^\circ + \Delta b_z) - F(b_z^\circ)}{\Delta b_z} = \frac{-.074 - 0}{.01} = \boxed{-7.4}$$