Satellite Photogrammetry HW4
RPC Estimation

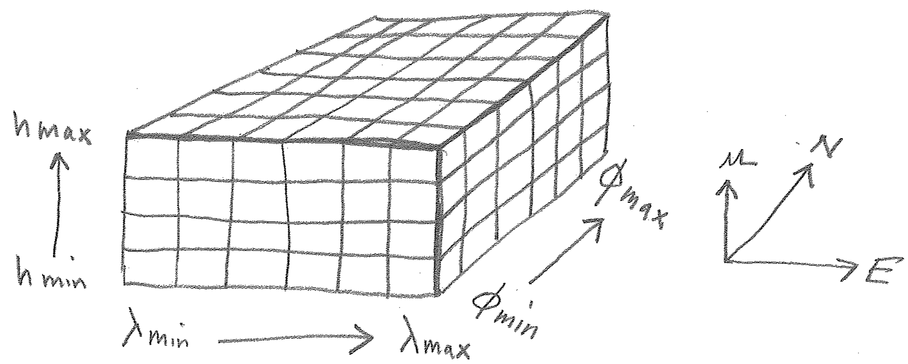
1. Make a 3D grid using

$\phi_{min} = 39.946512$, $\phi_{max} = 40.099933$ (dec. deg.)

$\lambda_{min} = -105.429866$, $\lambda_{max} = -105.104480$

$h_{min} = 1400$ m, $h_{max} = 3000$ m

subdivide into 6 intervals in ϕ, 6 intervals in λ, 4 intervals in h



So you have a $7 \times 7 \times 5$ grid of coordinates ϕ, λ, h

2. For image 1, use $\Delta q_i, \Delta q_j, \Delta q_k$: $\begin{bmatrix} 3.200615\text{e-}05 \\ 2.386764\text{e-}05 \\ 1.665507\text{e-}05 \end{bmatrix}$

Make sure you apply them correctly:
- interpolate q's
- unitize q's
- add $\Delta q_i, \Delta q_j, \Delta q_k$
- unitize q's again

3. Using refined model, project every one of the 245 grid points into the image (image 1) using our function,

$$\begin{bmatrix} l \\ s \end{bmatrix} = FG2I(im, \phi, \lambda, h, \Delta p)$$

4. make a "table"

ϕ_1	λ_1	h_1	l_1	s_1
ϕ_2	λ_2	h_2	l_2	s_2
$\vdots$				$\vdots$
ϕ_{245}	λ_{245}	h_{245}	l_{245}	s_{245}

5. make normalized variables:

$$P = (\phi - \phi_{offset}) / (\tfrac{1}{2} \, range(\phi)) \qquad P: -1 \to +1$$

$$L = (\lambda - \lambda_{offset}) / (\tfrac{1}{2} \, range(\lambda))$$

$$H = (h - h_{offset}) / (\tfrac{1}{2} \, range(h))$$

$$r_n = (l - l_{offset}) / (\tfrac{1}{2} \, range(l))$$

$$c_n = (s - s_{offset}) / (\tfrac{1}{2} \, range(s))$$

get ϕ, λ, h offsets from midpoints of 3D grid
get l, s offsets from midpoints of line & sample

6. using conventions in the published RPC specification

$$r_n = \frac{\sum_{i=1}^{20} A_i \cdot f_i(P,L,H)}{\sum_{i=1}^{20} B_i \cdot f_i(P,L,H)}, \quad c_n = \frac{\sum_{i=1}^{20} C_i \cdot f_i(P,L,H)}{\sum_{i=1}^{20} D_i \cdot f_i(P,L,H)}$$

$$A_1 + A_2 L + A_3 P + A_4 H + A_5 LP + A_6 LH + A_7 PH + A_8 L^2 +$$
$$A_9 P^2 + A_{10} H^2 + A_{11} PLH + A_{12} L^3 + A_{13} LP^2 + A_{14} LH^2 +$$
$$A_{15} L^2 P + A_{16} P^3 + A_{17} PH^2 + A_{18} L^2 H + A_{19} P^2 H + A_{20} H^3$$

use same strategy as first order case to make pseudo linear model :

$$r_n = \frac{A_1 + A_2 L + A_3 P + A_4 H}{1 + B_2 L + B_3 P + B_4 H}$$

$$r_n + r_n B_2 L + r_n B_3 P + r_n B_4 H = A_1 + A_2 L + A_3 P + A_4 H$$

$$r_n = A_1 + A_2 L + A_3 P + A_4 H - r_n B_2 L - r_n B_3 P - r_n B_4 H$$

$$r_n = \begin{bmatrix} 1 & L & P & H & -r_n L & -r_n P & -r_n H \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}$$

except you have to do for $A_1 \to A_{20}$, $B_2 \to B_{20}$ & r_n & c_n

$$\begin{bmatrix} V_{rn} \\ V_{cn} \end{bmatrix} - \begin{bmatrix} 1 & L & P & H & -r_n L & -r_n P & -r_n H & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & L & P & H & -C_n L & -C_n P & -C_n H \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ B_2 \\ B_3 \\ B_4 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \begin{bmatrix} -r_n \\ -C_n \end{bmatrix}$$

$$\boxed{V + B\Delta = f}$$

indirect observation model

each point generates 2 equations.

B will have dimensions 490 × 78, f is 490 × 1

solve by $\Delta = B \backslash f$ to avoid singularity issues.

look @ residuals (scale up to pixels!)

$v = f - B\Delta$, confirm $\ll$ 1 pixel

7. Make a new table at points __between__ the grid points

This grid will be 6 × 6 × 4 = 144 × 5

ϕ	λ	h	l	s

compute l, s using RPC model (you have to un-normalize r_n, c_n). compare l, s phys. model and l, s RPC model, They should all be < 0.5 pixels.

<next page shows full equation & linearized equation for __1 point__, fill B matrix and f vector with 245 instances of these 2 lines.>

$$r_n = \frac{A_1 + A_2L + A_3P + A_4H + A_5LP + A_6LH + A_7PH + A_8L^2 + A_9P^2 + A_{10}H^2 + A_{11}PLH + A_{12}L^3 + A_{13}LP^2 + A_{14}LH^2 + A_{15}L^2P + A_{16}P^3 + A_{17}PH^2 + A_{18}L^2H + A_{19}P^2H + A_{20}H^3}{1 \quad B_2L + B_3P + B_4H + B_5LP + B_6LH + B_7PH + B_8L^2 + B_9P^2 + B_{10}H^2 + B_{11}PLH + B_{12}L^3 + B_{13}LP^2 + B_{14}LH^2 + B_{15}L^2P + B_{16}P^3 + B_{17}PH^2 + B_{18}L^2H + B_{19}P^2H + B_{20}H^3}$$

$$c_n = \frac{C_1 + C_2L + C_3P + C_4H + C_5LP + C_6LH + C_7PH + C_8L^2 + C_9P^2 + C_{10}H^2 + C_{11}PLH + C_{12}L^3 + C_{13}LP^2 + C_{14}LH^2 + C_{15}L^2P + C_{16}P^3 + C_{17}PH^2 + C_{18}L^2H + C_{19}P^2H + C_{20}H^3}{1 + D_2L + D_3P + D_4H + D_5LP + D_6LH + D_7PH + D_8L^2 + D_9P^2 + D_{10}H^2 + D_{11}PLH + D_{12}L^3 + D_{13}LP^2 + D_{14}LH^2 + D_{15}L^2P + D_{16}P^3 + D_{17}PH^2 + D_{18}L^2H + D_{19}P^2H + D_{20}H^3}$$

parameter vector : $[A_1\ A_2\ A_3\ A_4\ A_5\ A_6\ A_7\ A_8\ A_9\ A_{10}\ A_{11}\ A_{12}\ A_{13}\ A_{14}\ A_{15}\ A_{16}\ A_{17}\ A_{18}\ A_{19}\ A_{20}\ B_2\ B_3\ B_4\ B_5\ B_6\ B_7\ B_8\ B_9\ B_{10}\ B_{11}$
$B_{12}\ B_{13}\ B_{14}\ B_{15}\ B_{16}\ B_{17}\ B_{18}\ B_{19}\ B_{20}\ C_1\ C_2\ C_3\ C_4\ C_5\ C_6\ C_7\ C_8\ C_9\ C_{10}\ C_{11}\ C_{12}\ C_{13}\ C_{14}\ C_{15}\ C_{16}\ C_{17}\ C_{18}\ C_{19}\ C_{20}\ D_2\ D_3\ D_4\ D_5$
$D_6\ D_7\ D_8\ D_9\ D_{10}\ D_{11}\ D_{12}\ D_{13}\ D_{14}\ D_{15}\ D_{16}\ D_{17}\ D_{18}\ D_{19}\ D_{20}]^T$, (78×1)

pseudo linear equations for a point :

$$\begin{bmatrix} r_n \\ c_n \end{bmatrix} = \left[\begin{matrix} 1 & L & P & H & LP & LH & PH & L^2 & P^2 & H^2 & PLH & L^3 & LP^2 & LH^2 & L^2P & P^3 & PH^2 & L^2H & P^2H & H^3 & -r_nL & -r_nP & -r_nH & -r_nLP & -r_nLH & -r_nPH & -r_nL^2 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \end{matrix}\right.$$

$$\begin{matrix} -r_nP^2 & -r_nH^2 & -r_nPLH & -r_nL^3 & -r_nLP^2 & -r_nLH^2 & -r_nL^2P & -r_nP^3 & -r_nPH^2 & -r_nL^2H & -r_nP^2H & -r_nH^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & L & P & H & LP & LH & PH & \cdots \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ L^2 & P^2 & H^2 & PLH & L^3 & LP^2 & LH^2 & L^2P & P^3 & PH^2 & L^2H & P^2H & H^3 & -c_nL & -c_nP & -c_nH & -c_nLP & -c_nLH & -c_nPH & -c_nL^2 & \cdots \end{matrix}$$

$$\left.\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_nP^2 & -c_nH^2 & -c_nPLH & -c_nL^3 & -c_nLP^2 & -c_nLH^2 & -c_nL^2P & -c_nP^3 & -c_nPH^2 & -c_nL^2H & -c_nP^2H & -c_nH^3 \end{matrix}\right] \begin{bmatrix} A \\ \vdots \\ B \\ \vdots \\ C \\ \vdots \\ D \\ \vdots \end{bmatrix}$$

$$\boxed{V + B\delta = f} \qquad \boxed{\underset{(2,1)}{\begin{bmatrix} V_{r_n} \\ V_{c_n} \end{bmatrix}} + \underset{(2,78)}{\begin{bmatrix} -1 & -L & -P & -H & \cdots \\ 0 & 0 & 0 & 0 & \cdots \end{bmatrix}} \underset{(78,1)}{\begin{bmatrix} A \\ \vdots \\ B \\ \vdots \\ C \\ \vdots \\ D \\ \vdots \end{bmatrix}} = \underset{(2,1)}{\begin{bmatrix} -r_n \\ -c_n \end{bmatrix}}}$$