CE 697 Sat. Photogr. Homework 1

assigned Wed. 18 Jan 2017, due 1 week (25th)

uncertainty of single ray intersection using simplified projection model

1. find image (+ pyramid) and support data at

   $\boxed{\text{ftp://ftp.ecn.purdue.edu/bethel/laf01 \& /laf02}}$

   for this assignment we use image # 1

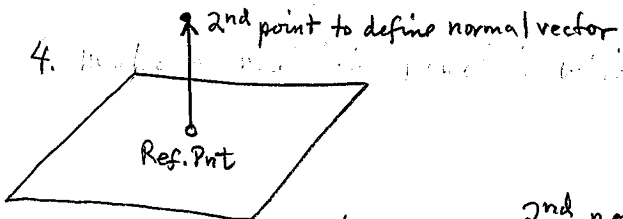
2. for ephemeris point no. 1, find X_L, Y_L, Z_L & Σ in laf01.eph

   find q_i, q_j, q_k, q_s & Σ in laf01.att

3. we use pixel no. 0 at that time ($S = 0$), so the image space vector

$$\vec{V}_c = \begin{bmatrix} x_0 \\ y_0 - S \times \text{det. pitch} \\ P.D. \end{bmatrix} \quad \text{units} = \text{meters}$$

4.



plane for intersection

$$\begin{bmatrix} \phi_0 \\ \lambda_0 \\ h_0 \end{bmatrix}_{Ref} = \begin{bmatrix} 40°-25'-47.98'' \\ -(86-54-51.43) \\ 172.33 \text{ (m)} \end{bmatrix}$$

$$\text{2nd point} \begin{bmatrix} \phi_0 \\ \lambda_0 \\ h_0 + 100 \end{bmatrix}$$

convert 2 points to cartesian (ECF) by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ ((1-e^2)N+h)\sin\phi \end{bmatrix}$$

$$N = a \Big/ \left[1 - e^2 \sin^2\phi\right]^{1/2}$$

WGS 84: $a = 6378137.0$, $f = 1/298.257223563$, $f = (a-b)/a$

$e^2 = (a^2 - b^2)/a^2$, $e^2 = 2f - f^2$, $b = a(1-f)$

compute the normal vector & unitize, that will be $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ in equation of plane $a_0 + a_1 X + a_2 Y + a_3 Z = 0$

Solve for $a_0 = -(a_1 X_{Ref} + a_2 Y_{Ref} + a_3 Z_{ref})$

this defines the plane that we intersect with the ray from camera.

5. make a matlab function: $\begin{bmatrix} e \\ n \end{bmatrix} = f1(X_L, Y_L, Z_L, q_i, q_j, q_k, q_s)$

this function intersects the camera ray at pixel 0, with the reference, tangent plane. Inside the function:

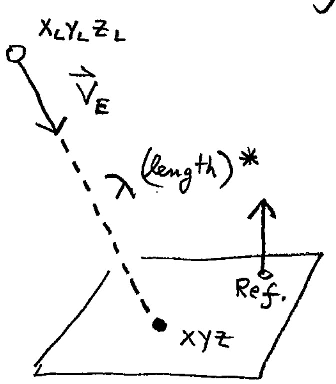
(a) normalize (unitize $\vec{q}$)

(b) compute the corresponding rotation matrix:

$$M = \begin{bmatrix} q_s^2 + q_i^2 - q_j^2 - q_k^2 & 2(q_j q_i - q_s q_k) & 2(q_i q_k + q_s q_j) \\ 2(q_j q_i + q_s q_k) & q_s^2 - q_i^2 + q_j^2 - q_k^2 & 2(q_j q_k - q_s q_i) \\ 2(q_i q_k - q_s q_j) & 2(q_j q_k + q_s q_i) & q_s^2 - q_i^2 - q_j^2 + q_k^2 \end{bmatrix}$$

(c) transform image vector to object space $\vec{V}_E = M \vec{V}_C$
unitize $\vec{V}_E$

(d) intersect the ray with the reference plane:



$X = X_L + \lambda V_x$

$Y = Y_L + \lambda V_y$

$Z = Z_L + \lambda V_z$

* NOT longitude!

Solve for λ:

$$a_0 + a_1(X_L + \lambda V_x) + a_2(Y_L + \lambda V_y) + a_3(Z_L + \lambda V_z) = 0$$

$$\lambda = -\left(\frac{a_0 + a_1 X_L + a_2 Y_L + a_3 Z_L}{a_1 V_x + a_2 V_y + a_3 V_z} \right)$$

(e) convert to e, n:

$$\begin{bmatrix} e \\ n \\ u \end{bmatrix} = M_x(90^\circ - \Phi_0)\, M_z(\lambda_0 + 90^\circ) \left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \right],$$

$\Phi_0, \lambda_0, X_0, Y_0, Z_0$ all refer to the reference point

(6) evaluate partial derivatives numerically:

$$J = \begin{bmatrix} \frac{\partial e}{\partial X_L} & \frac{\partial e}{\partial Y_L} & \frac{\partial e}{\partial Z_L} & \frac{\partial e}{\partial q_i} & \frac{\partial e}{\partial q_j} & \frac{\partial e}{\partial q_k} & \frac{\partial e}{\partial q_s} \\ \frac{\partial n}{\partial X_L} & \frac{\partial n}{\partial Y_L} & \frac{\partial n}{\partial Z_L} & \frac{\partial n}{\partial q_i} & \frac{\partial n}{\partial q_j} & \frac{\partial n}{\partial q_k} & \frac{\partial n}{\partial q_s} \end{bmatrix}$$

$\Delta X, Y, Z = .001$

$\Delta q = 1 \times 10^{-8}$

(7) make a composite of position and attitude covariance matrices:

$$\Sigma_{\begin{bmatrix} pos \\ att \end{bmatrix}} = \underset{(7,7)}{\begin{bmatrix} \underset{(3,3)}{\Sigma_{pos}} & \underset{(3,4)}{0} \\ \underset{(4,3)}{0} & \underset{(4,4)}{\Sigma_{att}} \end{bmatrix}}$$

(8) compute covariance of intersected point:

$$\Sigma_{\begin{bmatrix} e \\ n \end{bmatrix}} = J \, \Sigma_{\begin{bmatrix} pos \\ att \end{bmatrix}} J^T$$

what are σ_e, σ_n ?

D.G. says CE90 using support data is $< 4\,m$. Are we consistent with that (approximately) ?