CE 506 Exam 2 — 15 December 2006

(1 letter size sheet of notes allowed)

Name ______________________

1. A random vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ has covariance $\Sigma_{xx} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$.

We obtain $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ via the following transformation:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

z is obtained from $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ by: $z = 2y_1 - y_2 + 1$. What are σ_{y_1}, σ_{y_2}, and σ_z?

2. Following an adjustment, the residuals are $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 2 \\ -3 \\ 1 \end{bmatrix}$,

$W = I_5$, a priori $\sigma_0 = 3$, redundancy $= 4$.

Make the 2-sided global test at $\alpha = 0.10$ for the hypotheses,

$$H_0: \sigma^2 = \sigma_0^2$$
$$H_1: \sigma^2 \neq \sigma_0^2$$

3. $\Sigma_{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}$ from a GPS pseudorange LS solution is

$$\begin{bmatrix} 60 & -30 & 20 \\ -30 & 180 & -70 \\ 20 & -70 & 140 \end{bmatrix} m^2, \quad \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix} = \begin{bmatrix} 1000 \\ 2000 \\ 3000 \end{bmatrix} m$$

Assume we have <u>passed</u> the global test. Construct a 50% confidence interval for Z. Construct a 50% confidence ellipse for the coordinate pair $\begin{pmatrix} X \\ Z \end{pmatrix}$.

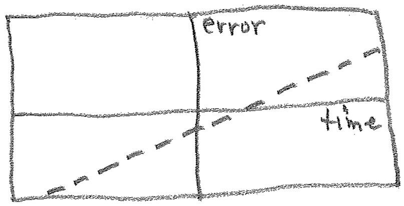
4. We wish to solve for the parameters of a 4-parameter, 2D, coordinate transformation of the form,

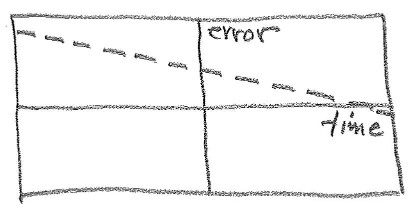
$$X = ax + by + c$$
$$Y = -bx + ay + d$$

in which coordinates of <u>5 points</u> are observed in <u>both</u> <u>systems</u>. Give n, n_0, and r. What would you choose for parameters? How many condition equations are needed? Write the condition equation for <u>1 point</u> in the matrix form,

$$Av + B\Delta = f.$$

5. The clock error in a <u>new</u> GPS receiver is known to be a linear function of time as in the following graphs,





The linear function is stable within an observing session, but changes between sessions. What would be the appriate pseudorange condition equation? For a 5 epoch LS solution how many parameters are required?

6. Seth Chandler's main discovery about latitude variation was that there are ________ (number) components to the effect, which have individual periods of ____________ (months).
What was the common <u>innovative</u> <u>feature</u> that Chandler used in the optical instruments he developed (Chronodeik & almucantar)?

CE 597 Adjustment of Geospatial Observations

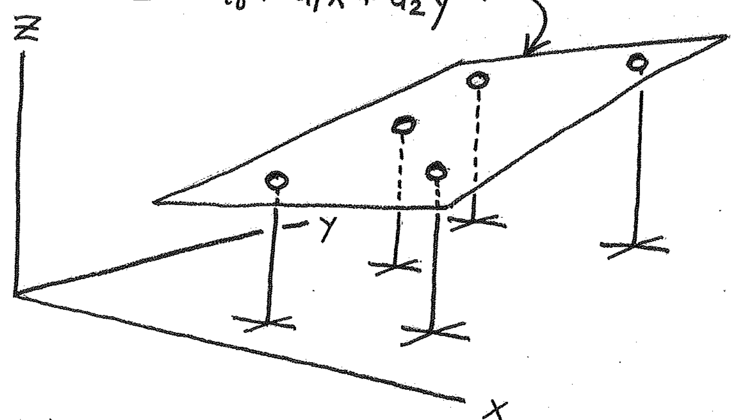
EXAM 2

15 Dec 2008, 2 hours, 1 crib sheet allowed, all questions equally weighted, show your work.

Name ________________

1. For a LS adjustment we have $W = I_4$, $\sigma_0^2 = 0.09$, $n = 4$, $n_0 = 2$, $V^T = [0.1 \quad 0.2 \quad -0.3 \quad -0.2]$, Make a 2-sided global test at $\alpha = 0.05$.

2. $z = a_0 + a_1 x + a_2 y$

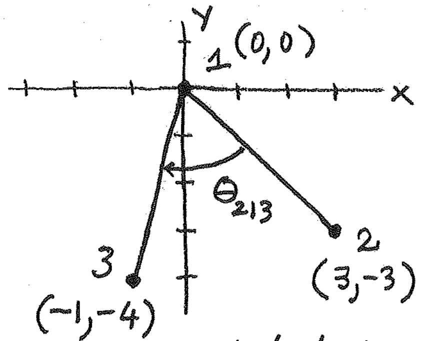


Five points are observed in all three coordinates. We wish to fit the indicated surface to these points. What are n, n_0, and r? Show the linearized condition equation for one point.

3. For a LS adjustment $Q = I_3$ and

$$Q_{\hat{\ell}\hat{\ell}} = \begin{bmatrix} 0.7143 & 0.4286 & -0.1429 \\ 0.4286 & 0.3571 & 0.2143 \\ -0.1429 & 0.2143 & 0.9286 \end{bmatrix}$$

what is the redundancy number for observation # 1 ?

4. 

Angle θ_{213} is measured as $60.0°$. For the angle condition equations, $F_\theta = \theta_{213} - (\alpha_{13} - \alpha_{12})$, where α_{ij} is the azimuth from i to j, what is the value of $\underline{\underline{f}}$ for the condition equation, $V + B\Delta = f$, using the approximate coordinates shown?

(Recall $\alpha_{ij} = \tan^{-1}\left(\frac{x_j - x_i}{y_j - y_i}\right)$)

5. The condition equations for the 7-parameter transformation are:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \lambda M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \lambda M_\kappa M_\phi M_\omega \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

for $\omega = 10°$, $\kappa = 180°$, $\phi = 0°$, $\lambda = 2$,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}, \quad \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -199.0 \\ -299.7 \\ 165.2 \end{bmatrix}$$

what is

$$\begin{bmatrix} \partial F_x / \partial \omega \\ \partial F_y / \partial \omega \\ \partial F_z / \partial \omega \end{bmatrix}$$

numerically?

6. $\Sigma_{\binom{X}{Y}} = \begin{bmatrix} 0.4 & -0.6 \\ -0.6 & 1.5 \end{bmatrix}$ in a LS problem where we passed the global test. Sketch the 70% confidence ellipse.

Useful facts:

$M_\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix}$, $M_\phi = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$,

$M_K = \begin{bmatrix} \cos k & \sin k & 0 \\ -\sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix}$