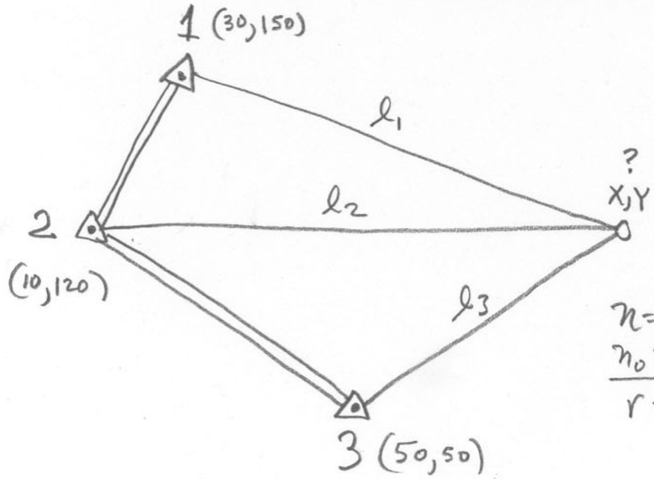
Trilateration Example (Nonlinear problem!)



#	l	σ
1	125.0	0.5
2	133.5	0.2
3	98.6	0.2

choose $\sigma_0 = 0.5$

$w_1 = 1$

$w_2 = w_3 = \frac{(0.5)^2}{(0.2)^2} = 6.25$

$w_i = \sigma_0^2 / \sigma_i^2$

$n = 3$
$n_0 = 2$
$r = 1$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6.25 & 0 \\ 0 & 0 & 6.25 \end{bmatrix}$$

<u>Method of Indirect Observations</u>, choose $u = n_0 = 2$ parameters: X, Y

initial approximation: from graph paper plot, or from autocad, drawing 3 circles centered @ 3 control points $(X^0, Y^0) \approx (140, 90)$

we need $n = 3$ condition equations, all will look like,

$$l_i = \left[(X - x_i)^2 + (Y - Y_i)^2\right]^{1/2} \quad \text{or} \quad F_i = l_i - \left[(X - x_i)^2 + (Y - y_i)^2\right]^{1/2} = 0$$

evaluate

$$B = \begin{bmatrix} \partial F_1/\partial X & \partial F_1/\partial Y \\ \partial F_2/\partial X & \partial F_2/\partial Y \\ \partial F_3/\partial X & \partial F_3/\partial Y \end{bmatrix} \quad \text{and} \quad f = -\begin{bmatrix} F_1^0(X^0, Y^0) \\ F_2^0(X^0, Y^0) \\ F_3^0(X^0, Y^0) \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{-(X - X_1)}{[(X-X_1)^2 + (Y-Y_1)^2]^{1/2}} & \frac{-(Y - Y_1)}{[(X-X_1)^2 + (Y-Y_1)^2]^{1/2}} \\ \frac{-(X - X_2)}{[(X-X_2)^2 + (Y-Y_2)^2]^{1/2}} & \frac{-(Y - Y_2)}{[(X-X_2)^2 + (Y-Y_2)^2]^{1/2}} \\ \frac{-(X - X_3)}{[(X-X_3)^2 + (Y-Y_3)^2]^{1/2}} & \frac{-(Y - Y_3)}{[(X-X_3)^2 + (Y-Y_3)^2]^{1/2}} \end{bmatrix}_{(X^0, Y^0)}$$

$$f = -\begin{bmatrix} l_1 - [(X - X_1)^2 + (Y - Y_1)^2]^{1/2} \\ l_2 - [(X - X_2)^2 + (Y - Y_2)^2]^{1/2} \\ X_3 - [(X - X_3) + (Y - Y_3)^2]^{1/2} \end{bmatrix}$$

<u>Iteration One</u>:

$$B = \begin{bmatrix} -.8779 & .4789 \\ -.9744 & .2249 \\ -.9138 & -.4061 \end{bmatrix}, \quad f = \begin{bmatrix} .2996 \\ -.0834 \\ -.1114 \end{bmatrix}$$

$$\Delta = (B^T W B)^{-1} B^T W f = \begin{bmatrix} .0669 \\ -.1682 \end{bmatrix}$$

$$\begin{bmatrix} X^0 \\ Y^0 \end{bmatrix}_{new} = \begin{bmatrix} 140.0 \\ 90.0 \end{bmatrix} + \begin{bmatrix} .0669 \\ -.1682 \end{bmatrix} = \begin{bmatrix} 140.0662 \\ 90.1739 \end{bmatrix}$$

<u>Iteration Two</u>:

$$B = \begin{bmatrix} -.8786 & .4776 \\ -.9747 & .2235 \\ -.9133 & -.4074 \end{bmatrix}, \quad f = \begin{bmatrix} .2745 \\ -.0579 \\ .0197 \end{bmatrix}$$

(note: almost same)

$$\Delta = \begin{bmatrix} -.000120 \\ .000042 \end{bmatrix} \quad \text{(getting smaller)}$$

$$\begin{bmatrix} X^0 \\ Y^0 \end{bmatrix}_{new} = \begin{bmatrix} 140.0662 \\ 90.1739 \end{bmatrix} + \begin{bmatrix} -.000120 \\ .000042 \end{bmatrix} = \begin{bmatrix} 140.0660 \\ 90.1739 \end{bmatrix}$$

Iteration three

$$B = \begin{bmatrix} -.8786 & .4776 \\ -.9747 & .2235 \\ -.9133 & -.4074 \end{bmatrix}, \quad f = \begin{bmatrix} .2545 \\ -.0580 \\ .0197 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} .00000000699 \\ -.00000001682 \end{bmatrix}$$

$$\begin{bmatrix} X^o \\ Y^o \end{bmatrix}_{new} = \begin{bmatrix} 140.0660 \\ 90.1739 \end{bmatrix} + \begin{bmatrix} .0000 \\ .0000 \end{bmatrix} = \begin{bmatrix} 140.0660 \\ 90.1739 \end{bmatrix}$$

Δ is very small so let's stop.

$$V = \underbrace{f - B\Delta}_{\text{from last iteration}} = \begin{bmatrix} .2745 \\ -.0580 \\ .0197 \end{bmatrix}, \quad \hat{l} = l + V = \begin{bmatrix} 125.2745 \\ 133.4420 \\ 98.6197 \end{bmatrix}$$

(l ↑ original; V ↑ last iteration)

```
% trilati.m  1-oct-02
% do ce506 homework problem, indirect obs.
% modify trilat.m for this homework problem

% define observations

d1=125.0;
d2=133.5;
d3=98.6;
l=[d1;d2;d3];

% define the sigmas and weights

sigd1=0.5;
sigd2=0.2;
sigd3=0.2;
% define a priori sigma-naught squared
sig0=0.5;
W=eye(3);
W(1,1)=sig0^2/sigd1^2;
W(2,2)=sig0^2/sigd2^2;
W(3,3)=sig0^2/sigd3^2;
W

% the points

xa=30;
ya=150;
xb=10;
yb=120;
xc=50;
yc=50;

xf=140;
yf=90;

B=zeros(3,2);
f=zeros(3,1);
max_iter=10;
iter=1;
keep_going=1;
% convergence variables
phi=10;
last_phi=20;
threshold=1.0e-06;

while keep_going == 1
  d_af=sqrt((xa-xf)^2 + (ya-yf)^2);
  d_bf=sqrt((xb-xf)^2 + (yb-yf)^2);
  d_cf=sqrt((xc-xf)^2 + (yc-yf)^2);

  % make coefficients of the condition equations
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```
  B(1,:)=[(xa-xf)/d_af  (ya-yf)/d_af];
  B(2,:)=[(xb-xf)/d_bf  (yb-yf)/d_bf];
  B(3,:)=[(xc-xf)/d_cf  (yc-yf)/d_cf];

  f(1)=-(d1 - sqrt((xa-xf)^2 + (ya-yf)^2));
  f(2)=-(d2 - sqrt((xb-xf)^2 + (yb-yf)^2));
  f(3)=-(d3 - sqrt((xc-xf)^2 + (yc-yf)^2));

  if iter == 1
    disp('iteration 1 B,f,W');
    B
    f
    W
    end

  % now solve and update and check convergence

  N=B'*W*B;
  t=B'*W*f;
  iter
  del=inv(N)*t
  xf=xf + del(1);
  yf=yf + del(2);
  v=f-B*del;
  phi=v'*W*v;

  % use fractional change in the quadratic form vTWv as
  % the convergence criterion, units-free alternative to
  % checking magnitude of delta

  if( abs(phi-last_phi)/last_phi < threshold )
    keep_going=0;
    disp('we have converged');
    end
  last_phi=phi;
  if iter > 10
    keep_going=0;
    disp('too many iterations');
    end
  iter=iter+1;
  end;

disp('final coordinates');
[xf yf]
disp('residuals');
v=f - B*del
lhat=l + v
```

```
trilati
W =
    1.0000         0         0
         0    6.2500         0
         0         0    6.2500
iteration 1 B,f,W
B =
   -0.8779    0.4789
   -0.9744    0.2249
   -0.9138   -0.4061
f =
    0.2996
   -0.0834
   -0.1114
W =
    1.0000         0         0
         0    6.2500         0
         0         0    6.2500
iter =
     1
del =
    0.0662
    0.1739
iter =
     2
del =
   1.0e-03 *
   -0.1204
    0.0419
iter =
     3
del =
   1.0e-07 *
    0.0699
   -0.1682
we have converged
final coordinates
ans =
  140.0660   90.1739
residuals
v =
    0.2745
   -0.0580
    0.0197
lhat =
  125.2745
  133.4420
   98.6197
diary off
```