Unified LS (AKA Bayesian estimation, Ridge regression, stochastic constraints, ...)

$F(\ell, x) = 0$ nonlinear condition equations with obs & params.

with prior $\Sigma_{\ell\ell}$ and Σ_{xx} for the observations and parameters

choose σ_0^2 (can be anything)

$$\Sigma = \sigma_0^2 Q$$

$$Q = \frac{1}{\sigma_0^2}\Sigma$$

$$W = \sigma_0^2 \Sigma^{-1}, \text{ therefore}$$

$$Q_{\ell\ell} = Q = \frac{1}{\sigma_0^2}\Sigma_{\ell\ell}$$

$$W_{xx} = \sigma_0^2 \Sigma_{xx}^{-1}$$

$$A = \frac{\partial F(\ell, x)}{\partial \ell}, \quad B = \frac{\partial F(\ell, x)}{\partial x}$$

Linearized condition equations are

$$A v + B \Delta = f$$

where $f = -F(\ell^0, x^0) - A(\ell - \ell^0)$

$f_x = x - x^0$ (diff from OLS)

ℓ^0: current est. of obs.
ℓ: original obs.
x^0: current est. of param
x: original val. of param.

$$\Delta = (N + W_{xx})^{-1}(t + W_{xx} f_x)$$

$$N = B^T W_e B$$

$$W_e = Q_e^{-1}$$

$Q_e = A Q A^T$ (Q without subscripts is $Q_{\ell\ell}$)

$t = B^T W_e f$ (f shown above)