Revised: November 20, 2002

U.S. Geological Survey
National Mapping Division

PROCEDURE FOR COMPENSATION OF AERIAL CAMERA LENS DISTORTION AS COMPUTED BY THE SIMULTANEOUS MULTIFRAME ANALYTICAL CALIBRATION (SMAC) SYSTEM

INTRODUCTION:

The following procedure is an example of how to use the parameters given in the USGS Camera Calibration Report for correcting a photos x and y-coordinate for radial and decentering lens distortion. The parameters used in the following formulas are given in the USGS Calibration Reports issued after October 1993.

TRANSLATION TO POINT OF SYMMETRY:

All measured points (x,y) needing a distortion correction must be referenced to the intersection (0,0) of the lines connecting either the midside fiducials or the corner fiducials (indicated principal point, IPP) and then translated to the principal point of autocollimation (PPA). The measured points must then be translated to the calibrated point of symmetry (POS), (x_P, y_P).

$$\overline{x} = (x + x_{IPP}) - x_P \qquad \overline{y} = (y + y_{IPP}) - y_P \tag{1}$$

Once $\overline{x}$ and $\overline{y}$ are found the radial distance (r) is computed by equation (2):

$$r = \sqrt{\overline{x}^2 + \overline{y}^2} \tag{2}$$

RADIAL DISTORTION:

To compute the correction for the radial distortion (*x,*y) of the measured point, use the coefficients of radial distortion (K_0', K_1', K_2', K_3', K_4') in the following formulas:

$$*x = \overline{x}\,(K_0' + K_1'r^2 + K_2'r^4 + K_3'r^6 + K_4'r^8 \ldots) \tag{3a}$$

$$*y = \overline{y}\,(K_0' + K_1'r^2 + K_2'r^4 + K_3'r^6 + K_4'r^8 \ldots) \tag{3b}$$

DECENTERING DISTORTION:

To compute the correction for decentering distortion ()x,)y) of the measured point, use the coefficients of the decentering distortion (P_1, P_2, P_3, P_4) in the following formulas:

$$)x = (1 + P_3r^2 + P_4r^4 \;.\;.\;.)(P_1(r^2 + 2\overline{x}^2) + 2P_2\overline{x}\,\overline{y}) \tag{4a}$$

$$)y = (1 + P_3r^2 + P_4r^4 \;.\;.\;.)(2P_1\overline{x}\,\overline{y} + P_2(r^2 + 2\overline{y}^2)) \tag{4b}$$

FINAL CORRECTED COORDINATES:

Adding the radial and decentering distortion corrections to the translation of the measured point, results in a corrected final x and y-coordinate for the measured point (x_C, y_C).

$$X_C = \overline{x} + *x +)x \qquad y_C = \overline{y} + *y +)y \tag{5a \& 5b}$$

REFER TO EXAMPLE ON FOLLOWING PAGE

SAMPLE PARAMETERS OBTAINED FROM USGS CAMERA CALIBRATION REPORT:

INDICATED PRINCIPAL POINT | POINT OF SYMMETRY

$x_{IPP} = 0.009$ mm $y_{IPP} = 0.006$ mm $x_P = 0.003$ mm $y_P = -0.001$ mm

COEFFICIENTS OF RADIAL DISTORTION

$K'_0 = -.2165 \times 10^{-3}$ $K'_2 = -.1652 \times 10^{-11}$

$K'_1 = .4230 \times 10^{-7}$ $K'_3{}^* = .2860 \times 10^{-19}$

$K'_4{}^* = .5690 \times 10^{-26}$

COEFFICIENTS OF DECENTERING DISTORTION

$P_1 = -.1483 \times 10^{-6}$ $P_2 = .1558 \times 10^{-6}$ $P_3{}^* = -.1464 \times 10^{-18}$ $P_4{}^* = .1233 \times 10^{-38}$

* Non-significant   Set P_3 & $P_4 = 0$ K'_3 & $K'_4 = 0$

MEASURED POINT (x,y) TO BE CORRECTED FOR LENS DISTORTION:

Example: $x = 62.142$ mm   (x,y-coordinates referenced to fiducial
$y = -62.336$ mm   intersection and corrected for comparator errors)

Translate x,y to point of symmetry using equation (1):

$\overline{x} = (x + x_{IPP}) - x_P$ $\overline{y} = (y + y_{IPP}) - y_P$

$\overline{x} = 62.148$ mm $\overline{y} = -62.329$ mm

$r^2 = \overline{x}^2 + \overline{y}^2$ $r^2 = 7747.278 \text{ mm}^2$ (2)

Using radial distortion correction equations (3a) and (3b):

$$*x = \overline{x}\,(K'_0 + K'_1 r^2 + K'_2 r^4 + K'_3 r^6 + K'_4 r^8 \ldots)$$

$$*y = \overline{y}\,(K'_0 + K'_1 r^2 + K'_2 r^4 + K'_3 r^6 + K'_4 r^8 \ldots)$$

$*x = 7.4927 \times 10^{-4}$ mm (.0007 mm) $*y = -7.5146 \times 10^{-4}$ mm (-.0008 mm)

Using decentering distortion correction equations (4a) and (4b):

$$)x = (1 + P_3 r^2 + P_4 r^4 \;.\;.\;.)(P_1(r^2 + 2\overline{x}^2) + 2P_2\overline{x}\,\overline{y})$$

$$)y = (1 + P_3 r^2 + P_4 r^4 \;.\;.\;.)(2P_1\overline{x}\,\overline{y} + P_2(r^2 + 2\overline{y}^2))$$

$)x = -3.5015 \times 10^{-3}$ mm (-.0035 mm) $)y = 3.5665 \times 10^{-3}$ mm (.0036 mm)

Computing the final corrected value (x_c, y_c) using equations (5a) and (5b):

$x_C = \overline{x} + *x +)x$ $y_C = \overline{y} + *y +)y$

$x_C = 62.145$ mm $y_C = -62.326$ mm

The final x_c, y_c are suitable for precision photogrammetry.